



THÈSE DE DOCTORAT
DE L'UNIVERSITÉ PSL

Préparée à l'École Normale Supérieure

Exploring Galactic CO emission with *Planck* data
Methodology and new full-sky maps

Soutenue par

Clément Walter

Le 3 Juillet 2023

École doctorale n°127

**Astronomie et Astrophysique d'Île
de France**

Spécialité

Astronomie et Astrophysique

Composition du jury :

Michel PERAULT École Normale Supérieure	<i>Président</i>
Isabelle RISTORCELLI Institut de Recherche en Astrophysique et Planétologie, Toulouse	<i>Rapporteuse</i>
Mark H. HEYER University of Massachusetts	<i>Rapporteur</i>
Juan MACIAS PEREZ Lab. de Physique Subatomique & Cos- mologie Grenoble	<i>Examineur</i>
Mark ASHDOWN University of Cambridge	<i>Examineur</i>
Maryvonne GERIN Lab. d'Études du Rayonnement et de la Matière en Astrophysique et Atmosphères	<i>Examinatrice</i>
Jean-Marc DELOUIS Lab. d'Océanographie Physique et Spa- tiale	<i>Invité</i>
François LEVRIER École Normale Supérieure	<i>Directeur de thèse</i>
Jean-Loup PUGET École Normale Supérieure	<i>Invité, co-directeur</i>

Remerciements

Il n'est pas tâche aisée d'estimer la liste des contributeurs du bon déroulement et de la réussite d'une aventure aussi vaste et riche qu'une thèse. Il ne l'est pas plus d'estimer la mesure desdites contributions, et de remercier chacune et chacun à la hauteur de ce qu'ils m'ont apporté. En effet, bien que le travail de recherche nécessaire à la progression de mon projet de thèse et des résultats associés ait été déployé par «moi», je ne peux que reconnaître la nature *plurielle* qui caractérise ce «moi» en nature comme en origine, et ce qui a rendu possible ses accomplissements. En cela, célébrer la réussite de cette aventure de thèse signifie *en même temps* la célébration des personnes et occasions qui l'ont rendu possible, et qui lui ont donné la forme qu'elle a prise. Pour célébrer cette page qui se tourne pour moi, ouvrant sur de nouveaux chapitres palpitants, je vais ici lister certains noms, ceux de personnes que je remercie d'avoir participé à leur manière, et à leur échelle, à l'arc «*thèse*» de l'histoire de ma vie. Cette liste risque fort de ne pas être exhaustive, et j'en demande pardon d'avance à ceux qui n'y apparaîtront pas nommément. J'invite ces derniers à se reconnaître sans hésitation dans ces quelques lignes de remerciements, quand bien même ils n'y liraient pas leur nom.

Je remercie mes encadrants François Levrier et Jean-Loup Puget, pour m'avoir donné l'occasion de me lancer dans cette aventure qu'a été la thèse, et de me pencher sur ce sujet complexe, mais aussi stimulant qu'intéressant.

My PhD work burnt a huge amount of CPU hours, graciously provided to me by the CMB data analysis (mp107) group at the National Energy Research Scientific Computing Center (NERSC), to which I owe the possibility of obtaining any of the results presented in this work. For this reason, I deeply thank the NERSC, and more personally Julian Borrill, for his availability and kind generosity when helping me with my computing resources related issues and needs.

Je remercie les membres de mon jury de thèse d'avoir généreusement accepté ce rôle exigeant et éprouvant : merci à Isabelle Ristorcelli, Mark H. Heyer, Juan Macias Perez, Mark Ashdown, Maryvonne Gerin, and Michel Perault. Je suis tout particulièrement reconnaissant envers les 2 rapporteur.e.s de ce manuscrit, Isabelle Ristorcelli et Mark H. Heyer, pour avoir fait l'effort de lire en détail ces presque 300 pages riches en technicités souvent assez indigestes. J'ai également une attention particulière aux 2 membres de mon comité de suivi, Juan Macias Perez et Mark Ashdown, qui ont su se montrer rassurants et encourageants quand j'en avais cruellement besoin, et ont participé à m'aider à garder les pieds sur Terre.

Il est tout bonnement inconcevable de ne pas remercier nommément, et profondément, Jean-Marc Delouis, sans qui cette thèse n'aurait jamais abouti. Tu as été pour moi un phare dans la nuit, et ton aide m'a été aussi précieuse que les instants d'échanges avec toi (aussi bien professionnels que personnels) ont été vivants, stimulants, et sympathiques. Pour cela et plus encore, je te remercie.

De même, je tiens à remercier Sylvain Mottet, qui m'a été d'une aide aussi précieuse que généreuse dans la première année de ma thèse (et même avant !), pour me mettre sur les

rails de ce sujet techniquement intimidant par sa taille et sa complexité.

Merci à Delphine Rolland pour sa pétillante sympathie, sa générosité, et son efficacité déconcertante à assurer une bonne partie des démarches administratives nécessaire à ma fonction de chercheur. Elle a été pour moi le contact avec cet arrière-plan de la recherche bien trop invisibilisé, en dépit de son rôle absolument consubstantiel au bon fonctionnement de la recherche, et elle l'a été en maintenant une chaleureuse humanité malgré les conditions difficiles de sa tâche, ce pour quoi je la remercie doublement.

Dans un registre plus personnel, je tiens à remercier toutes les personnes qui m'ont accompagné, soutenu, et humainement nourri durant ces années de thèse. Je sais que cette liste est celle que je ne pourrai pas exhaustivement couvrir dans ces quelques lignes, mais dont je vais ici faire quelques mentions honorables.

Merci à mes amis et collègues du LPENS, et particulièrement mes camarades de bureau Pierre Dell'Ova, Thibaud Richard, Bruno Regaldo, Mélanie Armante, Constant Auclair, et Pablo Richard. Merci pour les longues discussions aussi riches que diverses, pour les pinsas, pour les burgers parce que «ça doit être les burgers !», mais surtout pour m'avoir aussi généreusement et patiemment supporté durant tant d'années, malgré mes sujets de discussions bizarres, mes bruits et regards étranges et impromptus, ma tendance à parler tout seul, et fort. Merci d'avoir été, et d'être, des personnes bienveillantes et chaleureuses, de m'avoir inspiré et enrichi aussi bien scientifiquement qu'humainement. Mais surtout, merci d'avoir illuminé mon quotidien à toutes occasions, et de m'avoir donné l'envie de me lever le matin quand celle-ci me manquait fort. La forte promiscuité imposée par notre bureau, souvent sujet d'occasionnelles râleries, va me manquer sensiblement, car y cohabiter avec vous a été pour moi un pur bonheur. J'espère que la vie m'offrira de recroiser vos chemins à maintes occasions, mais sachez que quoi qu'il advienne, je vous emporte avec moi dans la musette de mes souvenirs sucrés et savoureux, et que la place que vous avez trouvé dans mon cœur ne s'estompera qu'avec les derniers battements de ce dernier. Mon ami Pierre, je ne peux que pudiquement résumer la place que tu a pris dans cette thèse comme dans ma vie, tant notre amitié prend racine dans les espaces intimes et vulnérables de qui nous sommes. Je te considère autant un ami qu'un frère tant nos similarités et la nature de notre lien auraient pu naître d'une parenté en commun. Mais ce lien n'est que renforcé par son origine fermement enracinée non pas dans la sérendipité, mais bien dans le choix délibéré et maintenu, même à travers le doute, la frustration, l'inconfort, ou l'appréhension, d'un désir de tenter l'un avec l'autre de nous offrir ce que nous avons tant attendu d'autres personnes, et que nous avons eu (ou avons encore) tant de mal à nous offrir nous-même : la douce et rassurante promesse d'un amour sans condition, sincèrement entretenu, et sans date limite. Je te remercie pour ce cadeau précieux.

Merci à mes amis et collègues d'autres laboratoires, avec mentions particulières à Daniela Galarraga, Florian Regnault, et Thibault Vieu. Daniela je t'ai bien trop peu vu ces dernières années, mais je sais que j'en suis le premier responsable. Malgré cela nous avons partagé plusieurs années de la thèse aussi bien sur la scène de la recherche que dans les coulisses de la vie, construisant une longue amitié qui m'a beaucoup apporté en maturité et m'a soutenu dans des périodes pas toujours faciles. Tu es une étincelante boule de vie et chaque rencontre avec toi est un rafraîchissement existentiel. Florian, merci pour toutes ces belles tranches de vie que nous avons partagé, pour les séances d'escalade, les vacances et les balades en vélo, les infinies discussions avec notre camarade commun Pierre, aussi bien autour d'une bière, un burger, un jeu vidéo ou de société, que sous un ciel étoilé. Mais surtout, merci d'être pour moi l'incarnation d'un espoir renouvelé dans le potentiel humain à la transformation pour le meilleur, même venant de là où je me serais bien moins attendu à le trouver. Voir la personne

que tu es devenu le long de ces années où nous nous sommes côtoyés (et que tu continues de devenir) m'emplit d'admiration et d'optimisme, et je suis heureux et honoré d'avoir pu faire partie de bribes de ce chemin. Thibault, mon cher camarade, merci de m'avoir offert une place aussi privilégiée dans ta vie. Ma relation avec toi est aussi singulière qu'elle est chère à mon cœur, et aussi riche et nourrissante que tu es unique. Je te considère comme l'une des personnes les plus extraordinaires qu'il m'ait été donné de rencontrer, à tel point que cela peut en devenir intimidant. Malgré cela, tu maintiens une simplicité et une humilité qui forcent le respect et l'admiration. Chaque expérience de vie que j'ai partagé avec toi a été pour moi une véritable transformation. Ta compagnie fait vibrer l'existence avec une intensité qui fait tourner la tête, mais paradoxalement ancre les pieds droit dans l'instant présent, dans la vie, avec un duo de fermeté et de quiétude déconcertant. Ton amitié est un cadeau inestimable dont je te suis infiniment reconnaissant. Merci pour les heures de prises de tête et de rigolades, de travail acharné et d'expériences uniques, de la découverte des montagnes sous un nouvel angle (dans la douleur comme dans l'extase), de nouvelles expériences en littérature, et j'en passe.

Merci à mes amis non collègues, qui rendent la vie plus pleine et colorée. Milan, mon ami de toujours, colloc' et compagnon du quotidien, dont la folie n'a d'égal que la mienne, merci d'être aussi débordant d'originalité, de curiosité, et surtout d'authenticité. Merci pour tous ces repas, aussi bien les soirées crêpes que les plats de pâtes imposés, pour ta disponibilité et ton soutien, mais surtout pour cette relation décalée et en même temps si riche et vivante, dont la saveur n'a aucun égal. Merci pour ces années passées en ta compagnie et pour toutes celles à venir. Mes vieux amis de collège Hugo, Martin, Lucas, et Pio, merci d'être toujours aussi sympathiques après tant d'années, et de m'avoir gardé dans vos vies malgré des passages parfois compliqués dans nos relations. Merci de rendre chaque retrouvailles aussi inattendues qu'inoubliables, offrant des instants de répit bienvenus au milieu du chahut de l'existence. Votre compagnie est une respiration, et votre amitié un oxygène au milieu d'un monde parfois étouffant. Olivier et Sodany, merci pour ces soirées de jeux vidéos offrant une détente oh combien nécessaire et bienvenue. Merci pour votre accueil Lyonnais toujours aussi chaleureux, bien que généralement trop léger en sustentation, où j'ai fini par me faire aux joues creuses et au ventre qui gargouille. Merci pour votre simplicité et votre générosité : même dans mes pires moments, venir vous voir est toujours un profond bonheur et un véritable refuge, et je connais peu d'endroit aussi chaleureux, accueillant et rassurant que votre foyer. Ainsi, je vous remercie du fond du cœur de m'en garder les portes grandes ouvertes avec une telle constance et une telle bienveillance. Jeanne, merci pour avoir initié des retrouvailles que je ne m'attendais pas à autant apprécier. Merci pour toutes ces soirées bières, sushi et discussions qui sont toujours une véritable bouffée d'air frais.

Merci à mes amis de prépa, que je ne vois que trop rarement, mais adore intensément. Elio, merci pour ces soirées Walking Dead, fléchettes, et dégustations de cocktails. Merci pour ta générosité et ta simplicité qui sont pour moi autant un repère qu'une inspiration. Les retrouvailles annuelles, ou biennuelles, avec toi, Joris, Camille, Julien, et Faustin, sont des rendez-vous que j'attends toujours avec la même excitation et la même intensité qu'un enfant attend et adore les grandes vacances. Nous nous voyons si rarement, et pourtant chaque retrouvaille se fait si naturellement, comme si nous nous étions quittés la veille, et sont toujours des expériences si vivantes et pleines. Merci d'avoir été les premiers à m'offrir de m'aimer simplement et entièrement pour qui j'étais, sans masque ni artifice, et d'être pour moi le repère et l'assurance d'un amour sans condition et intemporel.

Merci à mes parents, dont la somme des contributions serait trop longue pour être adéquatement décrite et saluée ici, et pour lesquelles je vais me limiter à celles plus immé-

atement reliées à ma thèse. Merci d'avoir enduré avec patience et bienveillance les longues périodes de silence que je vous ai imposé durant cette thèse. Merci de m'avoir malgré tout toujours accueilli avec la même chaleur et le même naturel lorsque j'y mettais fin. Merci à eux de m'avoir transmis le cœur des qualités qui font de moi un chercheur et un scientifique, mais aussi celles qui me font garder la tête, les pieds, et le cœur à leur juste place. Merci à ma mère, Daniel et Raphaël, de m'avoir généreusement et chaleureusement accueilli et entretenu durant un des mois les plus intenses de la rédaction de ce manuscrit. Merci à mon père pour ses généreuses relectures de quelques parties de ce manuscrit, ainsi que pour son soutien discret mais toujours aussi palpable qu'omniprésent. Merci à ma sœur, dont l'inaccessibilité m'attriste et le contact me manque, mais qui est et sera toujours pour moi un exemple, et une source d'inspiration, que cela lui plaise ou non. Merci à elle d'avoir été la première à m'offrir un regard sur ce que la vie peu produire de plus beau, et d'avoir été pour moi un phare inébranlable même dans mes périodes les plus sombres et désespérées.

Merci à ma psychologue Valérie Benoit, dont l'accompagnement a été aussi essentiel que riche et précieux. Merci de m'avoir aidé à faire de cette thèse une aventure bien plus que seulement scientifique. Merci de m'avoir aidé à apprendre comment en saisir chaque occasion, et comment y transformer chaque expérience en une création ou une rencontre positive et vivante. Merci de m'avoir accompagné dans la reconquête du goût de la vie, et d'une incarnation dans cette dernière pleine et ancrée. Merci d'avoir été avec moi pour l'entièreté de cette boucle essentielle de ma vie, qui teintera l'orientation, les couleurs et le goût du reste de mon existence.

Enfin, je tiens ici à honorer, et remercier, tou.te.s celles et ceux que je ne nommerai pas, mais qui ont malgré tout participé significativement aux presque 4 années de ma thèse. Ceux que j'ai vu régulièrement comme ceux avec qui j'ai à peine échangé, car l'important pour moi est la place qu'ils ont occupé et occupent encore dans mon quotidien, ne serait-ce qu'à travers ce qu'ils m'ont laissé emporter d'eux après m'avoir fait le cadeau de marquer le chemin de ma vie. Tous et toutes ces ami.e.s que j'ai eu la chance d'aimer, ces enseignant.e.s qui m'ont marqué par leur passion, leur humanité, et leur bienveillance, ma famille si aimante et soutenante. Je remercie toutes ces personnes qui font aujourd'hui partie de moi, et à travers moi ont fait partie de cette thèse et de sa réussite. Je remercie toutes ces personnes qui, je l'espère, savent qu'ils ont une place immuable dans mon cœur et mes souvenirs, et qui tout à la fois font de moi qui je suis aujourd'hui, et donnent à la vie ses infinies couleurs et son ineffable saveur.

Contents

Remerciements	i
Contents	v
Introduction	ix
Index of abbreviations	xi
I Foreground emissions in the <i>Planck</i> data: a crossroad of physical motivations	1
I.1 Cosmic microwave background	1
I.2 The science of Planck	3
I.3 The foreground emissions in the CMB frequency range	7
I.4 The CO emission in the ISM from the Milky Way	12
I.4.1 CO as one of the main tracers of cold molecular gas in the galaxy . . .	12
I.4.2 Physics of the CO emission from dense clouds	14
I.4.3 Potential CO from turbulence dissipation in low density region	16
I.5 Main goals of this thesis	16
II The <i>Planck</i> mission: instruments & data processing	19
II.1 The bolometer instruments	19
II.1.1 What is a bolometer ?	19
II.1.2 The bolometers aboard the <i>Planck</i> satellite	21
a) Bandpass	24
b) Scanning beam	26
II.1.3 The bolometers response chain	27
II.2 The scanning strategy	30
II.3 Data structure and processing pre-SRoll	33
II.3.1 TOI processing pipeline	33
II.3.2 Shaping the data for SRoll	38
a) The TOI compression into HPRs	38
b) Calibration and the dipoles	41
II.4 CO maps from the <i>Planck</i> mission	45
II.4.1 CO conversion coefficient determination	46
II.4.2 Component separation	47
a) Type 1	47
b) Type 2	49
c) Type 3	49
II.4.3 ^{13}CO contamination in the <i>Planck</i> CO maps	50

IIISRoll mapmaking algorithm	53
III.1 Basic nature, objective, and history	53
III.1.1 Philosophy of the mapmaking algorithm	54
a) Absolute photometric calibration from the orbital dipole	57
b) Analogue-to-digital converter non-linearity (ADCNL)	58
c) Foreground bandpass mismatch	59
d) Ring offset	62
e) Additional empirical transfer function	63
f) The need for a <i>mask</i>	65
III.1.2 History of evolutions	66
III.2 The SRoll2 version	68
III.2.1 Structure of algorithm	69
a) Inputs	69
b) Outputs	79
c) Looping structure	80
III.2.2 Data model	83
III.2.3 Mapmaking parameters computation (Step#1)	89
III.2.4 SRoll2 final residuals	95
III.3 SRoll2 simulations	97
III.3.1 Simulation pipeline	98
III.3.2 Sky model	100
III.3.3 SRoll2 updates & peculiarity	101
IV Building new CO maps	103
IV.1 SRoll2.1, SRoll2 made for the CO maps	103
IV.1.1 Addition of the 143GHz	104
a) What is gained from this added frequency band	106
b) The need for a new template : DustVar	108
IV.1.2 Modification of the SRoll2 sky mask	113
a) The problem of <i>decorrelation</i>	114
b) SRoll2.1 masks construction	114
IV.1.3 Modifications of SRoll2 model and equations	116
a) Input and output changes and addition	116
b) Equations and model changes	119
c) Closure condition modification	121
d) Simulations modifications and limitations	123
IV.1.4 Results on CO, dust, and DustVar coefficients from SRoll2.1	126
a) Methods to compute the uncertainty	127
b) The response difference coefficients	131
c) Comparison to CO results from <i>Planck</i> 2013	133
d) The sky model consistency validation	137
e) The issue of the $J = 2 \rightarrow 1$ CO templates	138
IV.2 ^{12}CO and ^{13}CO maps computed from SRoll2.1 outputs	142
IV.2.1 Method	142
a) Separation philosophy	142
b) Separation model	146
IV.2.2 Resulting maps	148
IV.2.3 Error maps and noise	156
a) Impact on noise of the use of the 143GHz band	160

IV.3 Risks of systematic errors and biases	160
IV.3.1 Leakage dust $\rightarrow$ CO	160
IV.3.2 Leakage $^{12}\text{CO} \rightarrow ^{13}\text{CO}$	165
IV.3.3 Bolometers monopole effect	167
IV.3.4 Beam effect	169
IV.3.5 CO correlated molecular emission in the 143GHz band	170
a) Origin & nature	170
b) Impact on the SRoll2.1 RDCs and CO maps	170
IV.4 Attempts at an iterative solution on SRoll2.1	175
IV.4.1 Philosophy and method of iteration	176
IV.4.2 First results and intensity drift	178
IV.4.3 Toy model exploration	183
a) Toy model definition	183
b) Impact of noise	185
c) Dust template error	187
d) Templates' low resolution	187
e) Toy model conclusion	189
V New CO maps' qualification & exploration	191
V.1 New CO maps validation from state-of-the-art distinct CO observations . . .	191
V.1.1 Comparison to the survey of Goldsmith et al. (2008)	192
a) Maps from SRoll2.1	193
b) <i>Planck</i> CO maps from the legacy release	196
V.1.2 Comparison to the FCRAO 2011 survey	202
a) Perseus	202
b) Ophiuchus	206
V.1.3 Comparison to the ^{12}CO survey of Dame & Thaddeus (2022)	210
V.1.4 $J = 2 \rightarrow 1$ comparison to the <i>Planck</i> CO maps	212
V.2 $J = 1 \rightarrow 0$ <i>composite</i> CO maps to correct the peaked signal loss	215
V.2.1 Limit of the 143GHz band to compute the CO maps	215
V.2.2 A straightforward solution: <i>composite</i> CO maps	218
a) Composite maps building method	219
b) Correction results	220
V.2.3 Leakage $^{12}\text{CO} \rightarrow ^{13}\text{CO}$ from composite maps	229
V.2.4 ^{13}CO power spectrum: the "bump"	230
V.3 High latitude & low column density exploration	230
V.3.1 Exploration method	232
a) Evaluate random and <i>spurious</i> correlation level using simulations	232
b) Definition of the detection threshold	233
V.3.2 Results on ^{12}CO	234
V.3.3 Results on ^{13}CO	234
Conclusion	243
A HEALPix scheme: a pixelisation of the sphere	247
B ^{12}CO and ^{13}CO maps computation: complete matrix form	251

C	Maps spectral analysis & sky spectra masking	257
C.1	Spherical harmonics decomposition	257
C.2	The <i>sky fraction</i> masks	260
D	Foregrounds response <i>difference</i> coefficients from SRoll2.1	263
D.1	Error from data: additional checks	263
D.2	The 100-143GHz SRoll2.1 run	263
D.3	The 217-143GHz SRoll2.1 run	266
D.4	Free-free and synchrotron SRoll2.1 RDCs	266
D.5	Monopole effect: limits of the uncertainty estimation	274
E	The Stokes parameters: the 3 map components	277
	Bibliography	279

Introduction

My thesis work stands at the frontier between astrophysics and cosmology, although my focus and results are turned towards astrophysical interests. Indeed, this work is based on the exploitation of the data produced by the *Planck* mission, an international effort of major scale which led to the most advanced spatial observation of the cosmic microwave background radiation to this day. This radiation is a central direct cosmological observable provided by our Universe.

Although my work does not focus on the part of the *Planck* data coming from the cosmic microwave background radiation, any result enriching the knowledge of the signal present in the *Planck* data, and any new method developed for this data cleaning and processing, can prove to be a useful contribution to the scientific endeavours towards a more accurate observation of the aforementioned radiation.

My central object of interest in the *Planck* data is the signal of Galactic origin produced by the molecule of carbon monoxide, or CO. This emission is of great astrophysical interest, especially for the study of the interstellar medium, since it is a very efficient tracer of the cold molecular gas in the Galaxy, and provides a window to many physical and chemical processes occurring in the gas present throughout the Milky Way.

I structured this manuscript as follows:

- Firstly, I present an overview of the scientific context in which my work is set, and introduce the main goals and stakes of my PhD thesis,
- Secondly, I present the main properties of the *Planck* mission, its instruments, and its collected data, in the scope of what is needed to satisfactorily achieve the goals set in the first chapter,
- Thirdly, I detail the nature, properties, and operation of the SRoll mapmaking algorithm, and more precisely its second version, which is at the base of my work. Indeed, this algorithm, which I slightly adapted to better fit my objectives, provides me with the data and fitted parameters which I use to produce new fullsky CO maps,
- Fourthly, I present and qualify my adapted version of the SRoll algorithm, as well as the method I developed to build new ^{12}CO and ^{13}CO maps from the algorithm's outputs,
- Lastly, I qualify my newly constructed fullsky maps, and use them to explore the regions of high latitude and low column density in search of a CO signal detection.

Note that all the original contributions provided by my work, either in the form of development of new methods, or scientific results, are gathered in the last 2 chapters (i.e. Chapters IV and V).

Index of abbreviations

CMB	Cosmic Microwave Background (cf. Section I.1)
ISM	Interstellar Medium (cf. Section I.3)
SED	Spectral Emission Distribution (cf. Section I.3)
CIB	Cosmic Infrared Background (cf. Section I.3)
G08	Refers to the CO survey from Goldsmith et al. (2008) (first introduced in subsection I.4.1 , later presented in subsection III.2.1.a))
D22	Refers to the CO survey from Dame & Thaddeus (2022) (first introduced in subsection I.4.1 , later presented in subsections IV.1.2.b) and subsection V.1.3)
LFI	Low-Frequency Instrument (cf. Section II.1)
HFI	High-Frequency Instrument (cf. Section II.1)
PSB	Polarization-Sensitive Bolometer (cf. subsection II.1.2)
SWB	Spider-Web Bolometer (cf. subsection II.1.2)
FWHM	Full Width at Half Maximum (cf. subsection II.1.2.b))
FSL	Far-Side Lobes (cf. subsection II.1.2.b))
ADC	Analog-to-Digital Converter (cf. subsection II.1.3)
ADU	Analog-to-Digital Unit (cf. subsection II.1.3)
rpm	rotation per minute (cf. Section II.2)
TOI	Time-Ordered Information (cf. Section II.3)
ADCNL	Analog-to-Digital Converter Non-Linearity (cf. subsection II.3.1)
HPR	HealPix Ring (cf. subsection II.3.2)
HM	Half-Mission, referring to <i>Planck</i> mission subsurveys (cf. subsection II.3.2)
SNR	Signal-to-Noise Ratio (cf. subsection II.3.2.b))
MBB	Modified Black-Body (cf. subsection II.3.2.b))
RDC	Response Difference Coefficient (cf. subsection III.1.1.c))
PSM	<i>Planck</i> Sky Model (cf. Section III.3)
FFP	Full Focal Plane (simulations) (cf. Section III.3)

Chapter I

Foreground emissions in the *Planck* data: a crossroad of physical motivations

In this first chapter, I will introduce the scientific context of my thesis, and the framework in which my main subject is set: the CO emission as a foreground in the *Planck* mission data. It aims to introduce how the *Planck* data, and more specifically the so-called "foreground emissions", are a crossroad of physical motivations.

I will first introduce the cosmic microwave background, central object of the *Planck* mission and main cosmological observable. Then, I will present the main objectives of the *Planck* mission, and the scope of the science behind it. I will subsequently define the notion of "foreground emission", and summarize the ones present in the *Planck* data. Afterwards, I will focus on the CO emission in the ISM, presenting an overview of its nature, origins, characteristics, and the main interests it presents for the field of astrophysics. Finally, once the framework has been set, I will introduce the main goals I followed during my thesis, and aimed to fulfill through the results provided by the present manuscript.

Note that the scientific concepts presented and discussed in this chapter will not be explored in detail, and that their introduction is only meant to provide the reader with an overview of the scientific context in which this work took place.

I.1 Cosmic microwave background

The Cosmic Microwave Background (CMB) radiation is a blackbody emission originating from our early Universe. First predicted in 1948 by [Alpher & Herman \(1948\)](#), it was later measured for the first time *accidentally* (in 1965) by [Penzias & Wilson \(1965\)](#) as an excess antenna temperature estimated at $3.5 \pm 1.0\text{K}$, which could only be due to a detected radiation of unexplained origin at the time. This observation was quickly linked to the predictions of a cosmic black body radiation (cf. [Dicke et al. \(1965\)](#)).

As one of the first and central cosmological observables, this first detection was then followed by numerous ground-based as well as spatial missions aiming at improving the mapping of this emission on the sky. Some of the major contributors to the CMB observation were the RELIKT missions (the first spatial observations of the CMB, [Klypin et al. \(1992\)](#)), followed by the Cosmic Background Explorer, also called COBE ([Bennett et al. \(1993\)](#)), the Balloon Observations Of Millimetric Extragalactic Radiation And Geophysics, or BOOMERanG experiment ([De Bernardis et al. \(2000\)](#), [Crill et al. \(2003\)](#)), the Wilkinson Microwave Anisotropy Probe, called WMAP ([Bennett et al. \(2013\)](#)), and the *Planck* mission. Each new mission brought new improvements to the CMB measurements, most no-

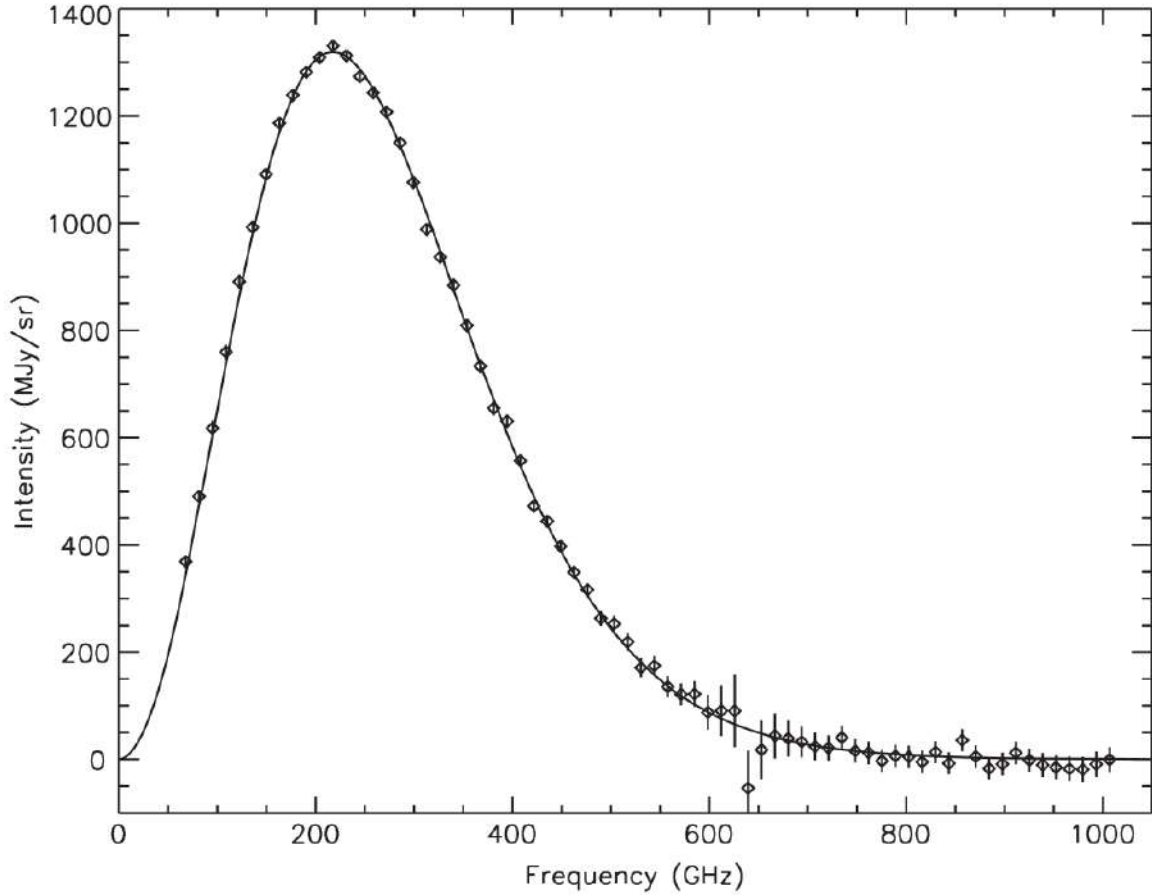


Fig. I.1 CMB spectrum from the FIRAS instrument of the COBE mission, recalibrated and fitted from the results of the WMAP mission at a temperature of $T = 2.7260 \pm 0.0013\text{K}$.

Taken from [Fixsen \(2009\)](#), Fig.3.

tably in terms of accuracy of its small anisotropies on the sky. The CMB is, to this day, the most precisely measured and closest to an *ideal* blackbody radiation found in nature. I display the CMB spectrum in Fig.I.1 (cf. [Fixsen \(2009\)](#)).

At early times, the Universe was filled by a hot and dense plasma of particles, and later formed nuclei, such that no photon could freely travel without being immediately absorbed or scattered (mainly through Thomson scattering by electrons). Because of the expansion and cooling of the Universe, it reached a point where the protons and electrons could combine to form *neutral* atoms of hydrogen, interacting far more weakly with the photons which could for the first time travel freely in space. This quick transition, called *recombination*, is estimated to have happened at a redshift of $z \sim 1100$ (when the Universe was ~ 379000 years old). It is indicated on the schematic of the history of the Universe's evolution displayed in Fig.I.2. The CMB corresponds to the photons released at the time of the recombination, emitted from all points in the Universe and in all directions, therefore detectable in all directions on the sky today. However, due to redshifting of the photons from the expansion of the Universe, affecting the temperature of the CMB T_{CMB} through a factor of $(1+z)$, the blackbody emission of the CMB appears today at a temperature of $T_{\text{CMB}} = 2.72548 \pm 0.00057\text{K}$ (cf. [Fixsen \(2009\)](#)).

The CMB radiation is highly isotropic, with temperature variations on the sky of the

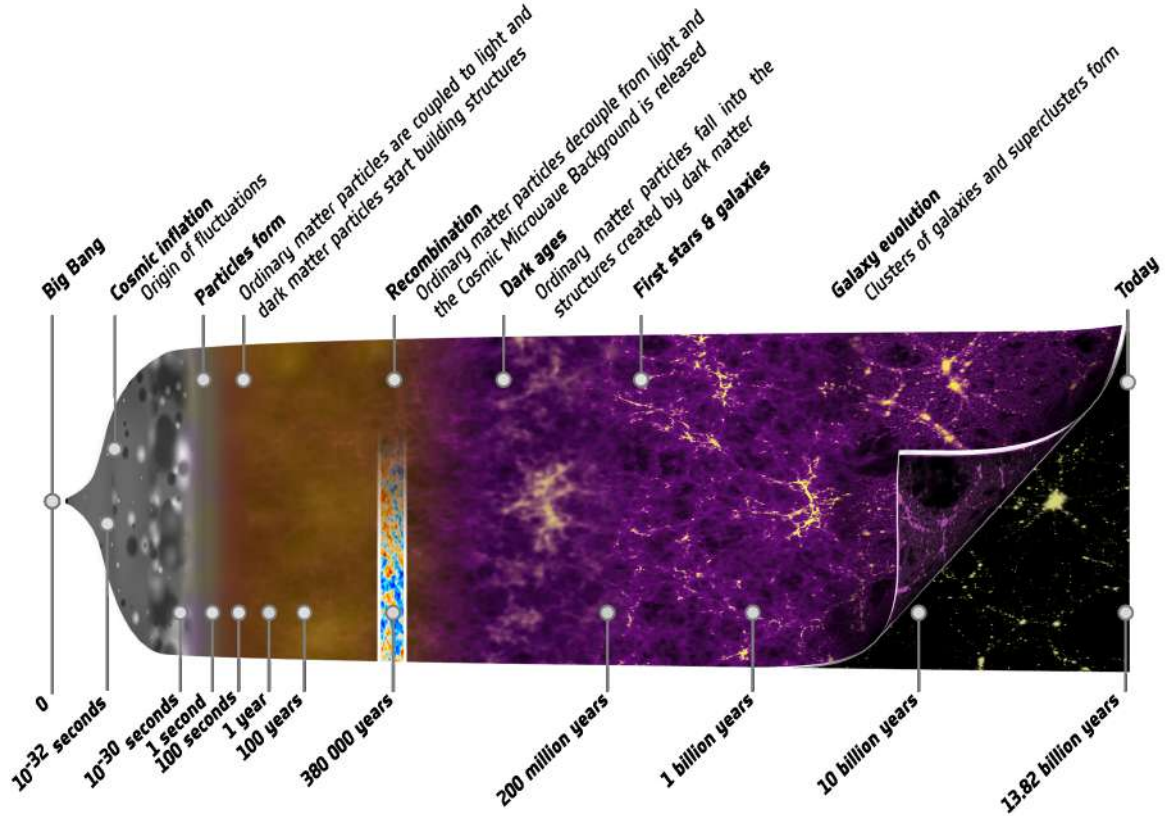


Fig. I.2 *History of the Universe.*
Credit: ESA, C.Carreau.

order of $\frac{\Delta T_{CMB}}{T_{CMB}} \sim 10^{-5}$ (cf. [Bennett et al. \(2013\)](#)), once the dipole has been subtracted (see subsection [II.3.2.b](#)) for more details). Nevertheless, the accurate measurement of these CMB anisotropies, however small they may be, is a critical observable for cosmology: not only are they imprinted with the state and characteristics of the Universe at an early epoch, but it also informs us of the structure and history of the Universe through the perturbations subjected to the CMB photons by the matter populating it during their journey before reaching us. This is why the missions like COBE, WMAP, and *Planck* always looked to improve the accuracy of the CMB anisotropies, in order to better estimate and constrain the various cosmological parameters defined by our models describing the Universe and its evolution.

I.2 The science of Planck

Mapping the sky in a very large spectral range, from $\sim 27\text{GHz}$ to $\sim 1000\text{GHz}$ over nine broad frequency bands, the *Planck* mission produced the most accurate map of the anisotropies to this day, as displayed in [Fig.I.3](#), and provided us with the corresponding power spectrum of CMB anisotropies, plotted in [Fig.I.4](#) (see [Appendix.C](#) for details on the power spectrum definition and computation). These observables, associated with the corresponding maps and power spectra of the linearly polarized part of the CMB anisotropies, compose the centre of the *Planck* products, from which all other results are determined.

Indeed, the CMB anisotropies come from 2 types of sources:

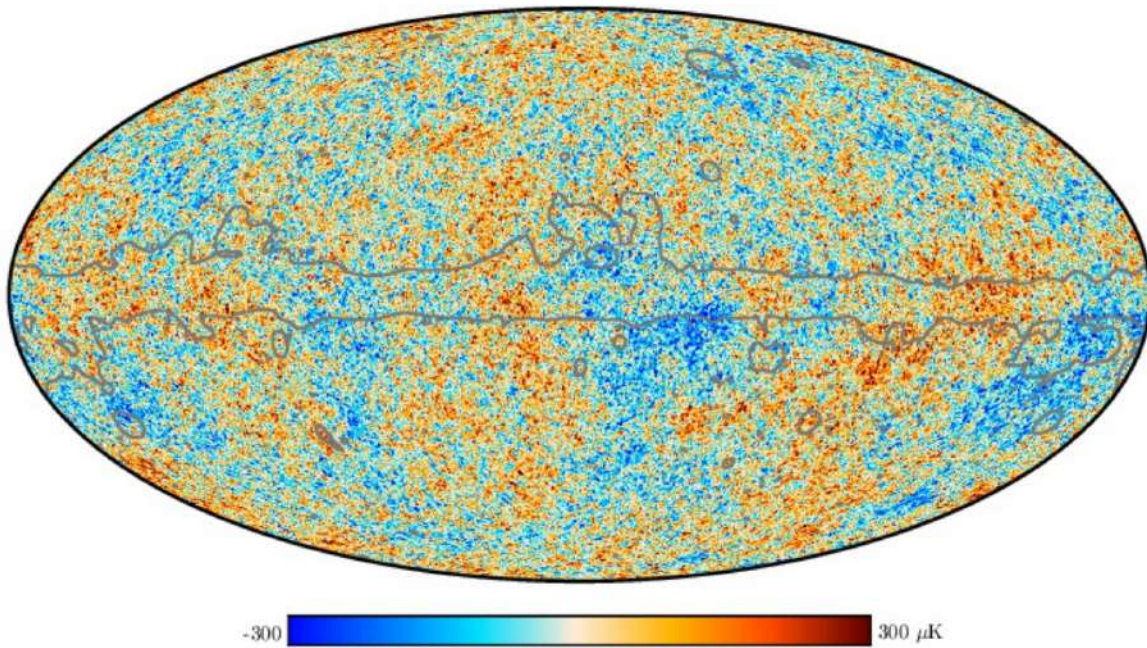


Fig. I.3 *Planck CMB sky 2018 temperature map. The map has been masked and inpainted in regions where residuals from foreground emissions are expected to be substantial. This mask, mostly around the Galactic plane, is delineated by a grey line.*

Taken from Planck Collaboration I (2020), Fig.6.

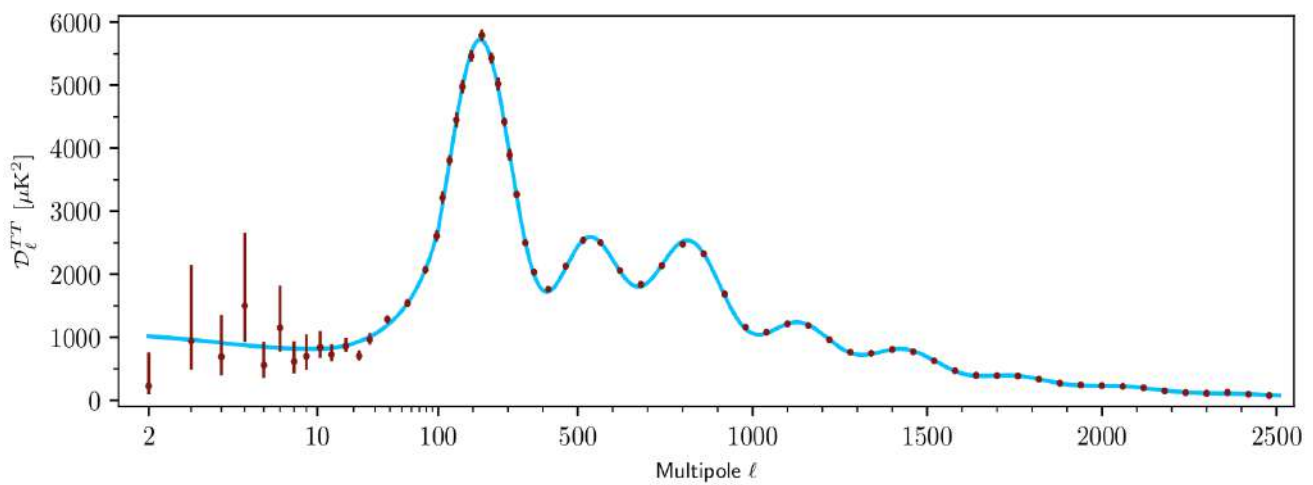


Fig. I.4 *Planck CMB angular power spectrum of the temperature map. Taken from Planck Collaboration I (2020), Fig.9.*

- (i) The primary anisotropies: they correspond to the contribution of all anisotropy sources originating from the properties of the Universe and its contents **up to** the recombination, such as:
- Quantum fluctuations enlarged during the inflation period: these quantum fluctuations, initially present and enlarged during the inflation era (between $\sim 10^{-36}$ s and $\sim 10^{-32}$ s after the Big Bang) up to macroscopic scales, are the origin of the primordial density fluctuations of *both* photons and baryons throughout the Universe,
 - Primordial gravitational waves: these are gravitational waves originating from the inflation and expected to be imprinted in the CMB polarization,
 - Acoustic oscillations (Peebles & Yu (1970), Sunyaev & Zeldovich (1970)): these are due to a "competition" between the effect of the radiative pressure from the photons, tending to smooth and reduce the density anisotropies, and the gravitational attraction of the baryons, tending to attract the matter towards overdensities. This 2 competing effects create the aforementioned acoustic oscillation, which are at the origin of the successive peaks observed in the CMB power spectrum of Fig.I.4,
- (ii) The secondary anisotropies: the sum of all effects and sources impacting the CMB photons **after** the recombination, during their journey through the Universe, such as:
- Gravitational lensing (Planck Collaboration XVII (2014)): this corresponds to the lensing effect of massive clusters on the CMB radiation, i.e. the deflection of the CMB photon's trajectory by the gravitational field from massive clusters. This allows a mapping of the distribution of mass in the Universe in the form of a map of this lensing effect, which I displayed in Fig.I.5,
 - Sunyaev-Zeldovich effect (Sunyaev & Zeldovich (1972), Planck Collaboration XX (2014)): this effect corresponds to the inelastic *inverse* Compton scattering of the CMB photons by high-energy electrons from the hot gas bathing galaxy clusters. It is often divided into 2 main parts: the thermal effect stemming from the *inverse* Compton scattering from electrons due to their thermal energy (I plot the *Planck* map of this effect in Fig.I.6), and the kinetic effect causing a scattering of the CMB photons by electrons, being dragged by the galaxy cluster they belong to, along its relative motion with respect to the CMB frame,
 - Sachs-Wolfe effect (Sachs & Wolfe (1967), Planck Collaboration XIX (2014)): it corresponds to the gravitational redshift of the CMB photons. The redshift occurring at the time of the recombination is called the *non-integrated* Sachs-Wolfe effect, while the one occurring *after* the recombination, along the path of the photons throughout the Universe, is called the *integrated* Sachs-Wolfe effect (I plot the *Planck* map of the integrated part of this effect in Fig.I.7).

From the CMB anisotropies' map and its power spectrum, the respective contributions from these sources can be fitted, or at least upper limits can be deduced, providing a wide set of observational results and constraints for cosmological models.

The main results of cosmological parameters fitted from *Planck* are summarized in Tables 6 and 7 of Planck Collaboration I (2020). The data distributed publicly from *Planck*, and the later developed improvements such as that of the mapmaking algorithm (see Delouis et al. (2019), detailed in later Chapter III), allowed even further refinement of some cosmological parameters, such as the optical depth to reionization (cf. Pagano et al. (2020)). This

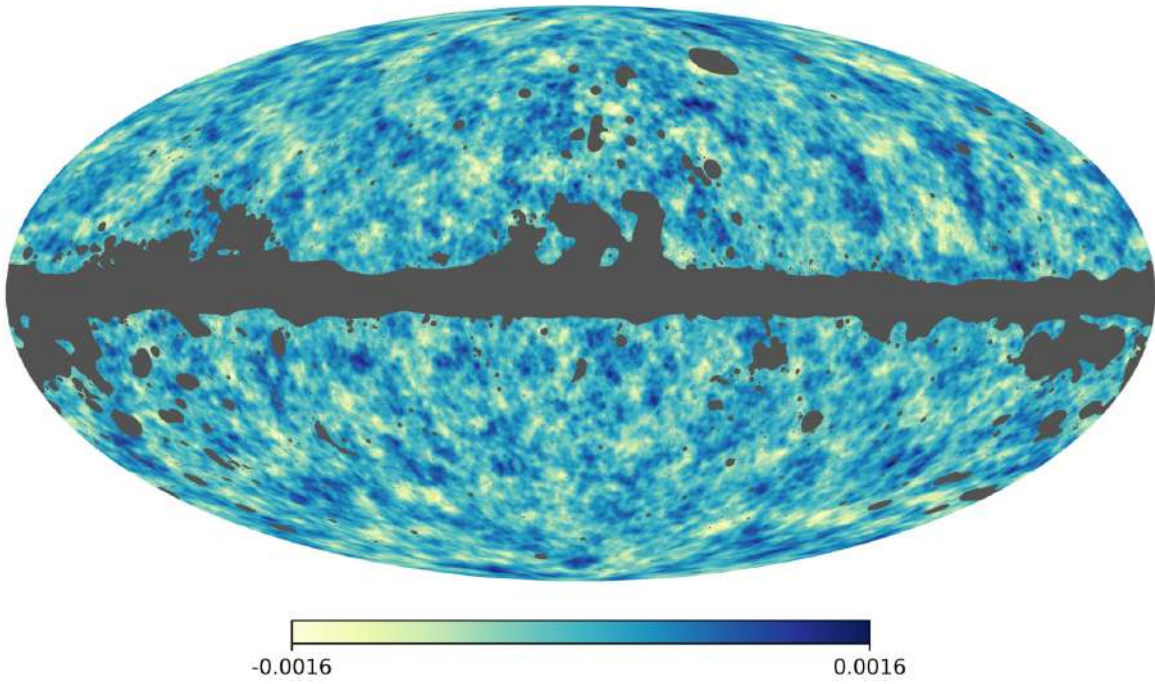


Fig. I.5 *Planck* lensing map. Taken from *Planck Collaboration I (2020)*, Fig.6.

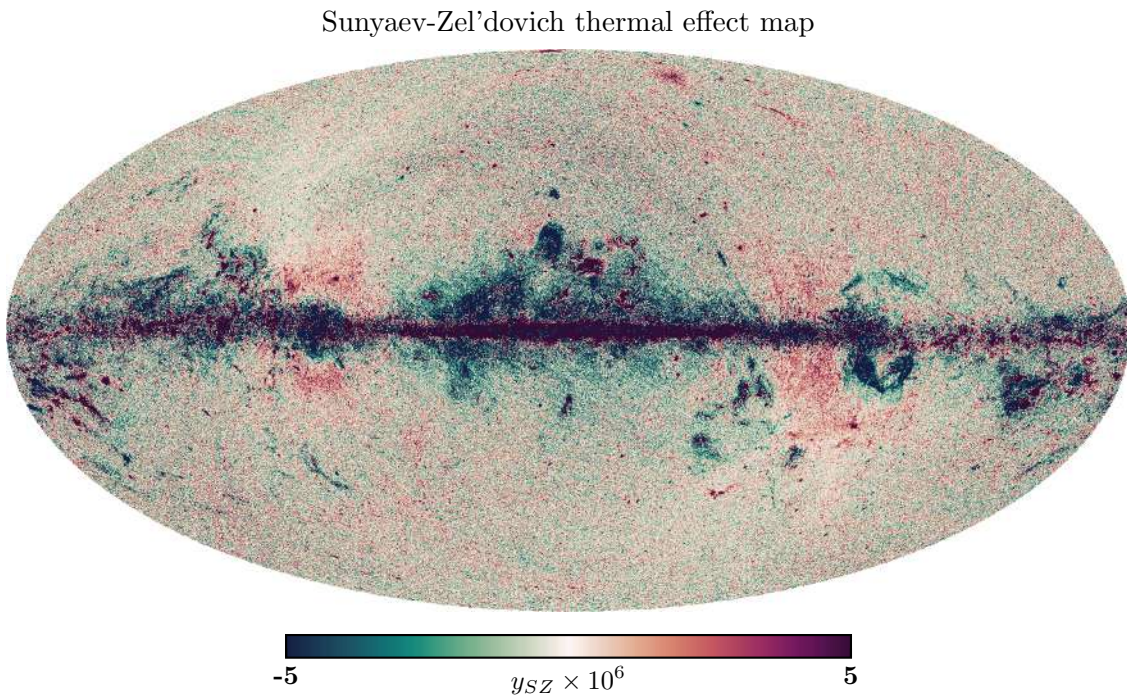


Fig. I.6 *Planck* map of the thermal Sunyaev-Zeldovich effect. See *Planck Collaboration XXII (2016)* for details on the nature and construction of this map.

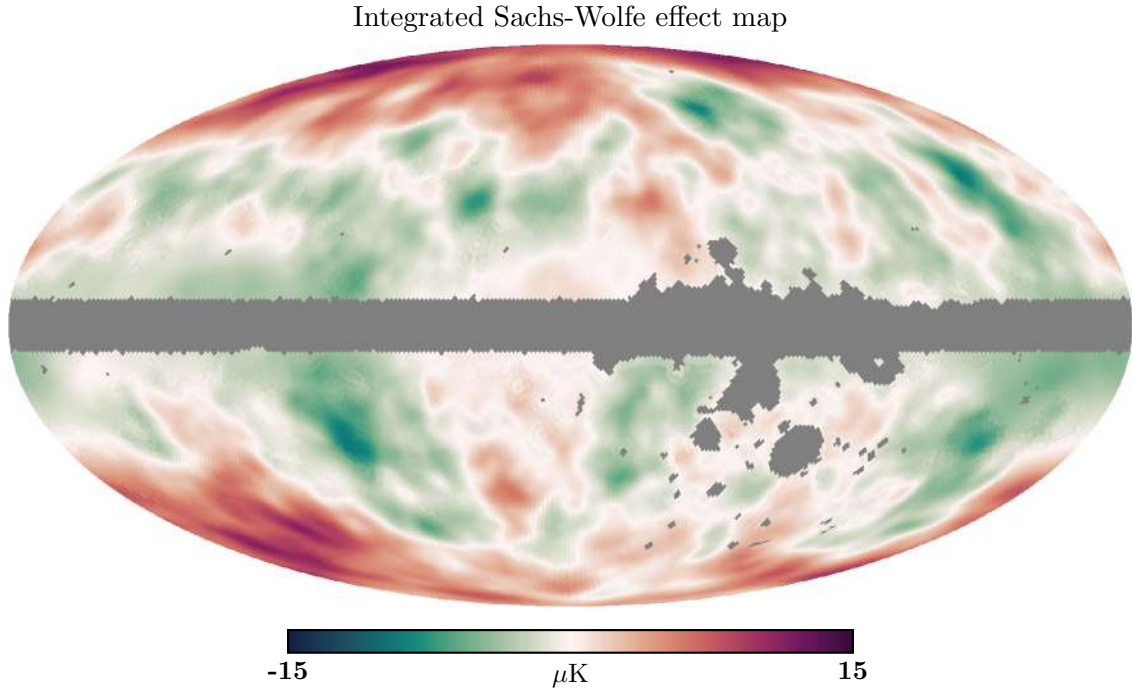


Fig. I.7 *Planck* map of the integrated Sachs-Wolfe effect. See [Planck Collaboration XXI \(2016\)](#) for details on the nature and construction of this map.

parameter is an alternative representation of the position in time of the reionization period, which is the time when the first stars formed in the Universe, causing the matter to reionize.

The CMB polarization can be decomposed into 2 geometric components: the E-modes (which are curl-free), and the B-modes (which are divergence free). These B-modes of the CMB polarization are less intense than the E-modes, as can be seen in Fig.I.8, which is why it has yet to be detected at medium and large scales. They represent nonetheless a key observational goal for future cosmological endeavours: indeed, only in these B-modes (at large scales) is imprinted a characteristic parameter of the primordial gravitational waves produced during the inflation and called the tensor-to-scalar ratio (cf. [Kamionkowski & Kovetz \(2016\)](#)). The measurement of the B-modes, in turn allowing a determination of this parameter, would be the next decisive step to constrain the various scenarii and models of the cosmic inflation. This is the main objective of the next "spiritual" successor to the *Planck* mission: the Lite (Light) satellite for the studies of B-mode polarization and Inflation from cosmic background Radiation Detection, or LiteBIRD ([Hazumi et al. \(2019\)](#), [Sugai et al. \(2020\)](#)), selected and approved by the Japan Aerospace Exploration Agency (JAXA) and expected for a launch in 2028.

I.3 The foreground emissions in the CMB frequency range

In order to accurately explore the CMB radiation, the instruments aboard the *Planck* satellite collectively covered a very broad spectral range. Consequently, it was expected that such wide spectral range would contain emission from other sources, coming from other galaxies as well as from the matter within our Milky Way. These emissions coming from other sources and falling in the frequency range of the CMB radiation are referred to as *foreground emissions*, or simply foregrounds, as they are often more intense than the CMB

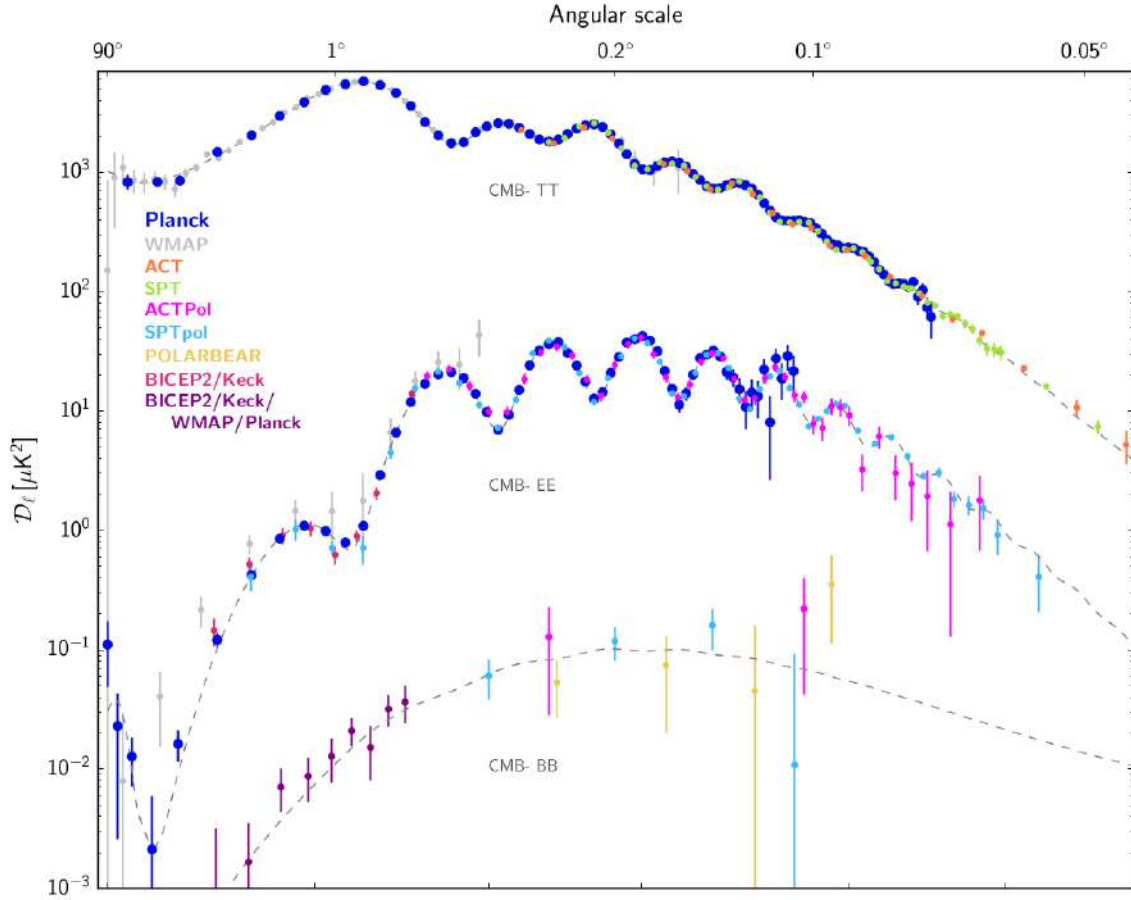


Fig. I.8 Compilation of recent CMB angular power spectrum measurements from which most cosmological inferences are drawn, for the temperature, E-mode and B-mode polarization signals. Different colours correspond to different experiments, each retaining its original binning. For Planck, ACTPol, and SPTpol, the EE points with large error bars are not plotted (to avoid clutter). The dashed line shows the best-fit Λ CDM model to the Planck temperature, polarization, and lensing data. See *Planck Collaboration I (2020)* for details and references. Taken from *Planck Collaboration I (2020)*, Fig.18.

anisotropies (at least for the emissions from the Milky Way’s galactic disk) and overlap the signal from the anisotropies in the final fullsky maps. This is visible in Fig.I.9 where I report the fullsky maps of the entire signal collected from each of the frequency bands observed by the *Planck* instruments. At lower frequencies, we can see the CMB anisotropies at high latitude, but they are covered around the galactic disk by the multiple foreground emissions. At higher frequencies (353GHz and above), the anisotropies are no longer even visible and the entire sky is covered and dominated by the foregrounds.

For this reason, an extensive and accurate *component separation* must be carried out on the sky maps to disentangle all the various sources of signal on the sky, and separate precisely the CMB radiation from all the other foreground emissions. The methods used for this separation, and the produced results, are summarized for the last *Planck* release in *Planck Collaboration IV (2020)*.

This is where the foregrounds become a crossroad of physical motivations: the construction of an accurate map of the CMB anisotropies requires an efficient and accurate

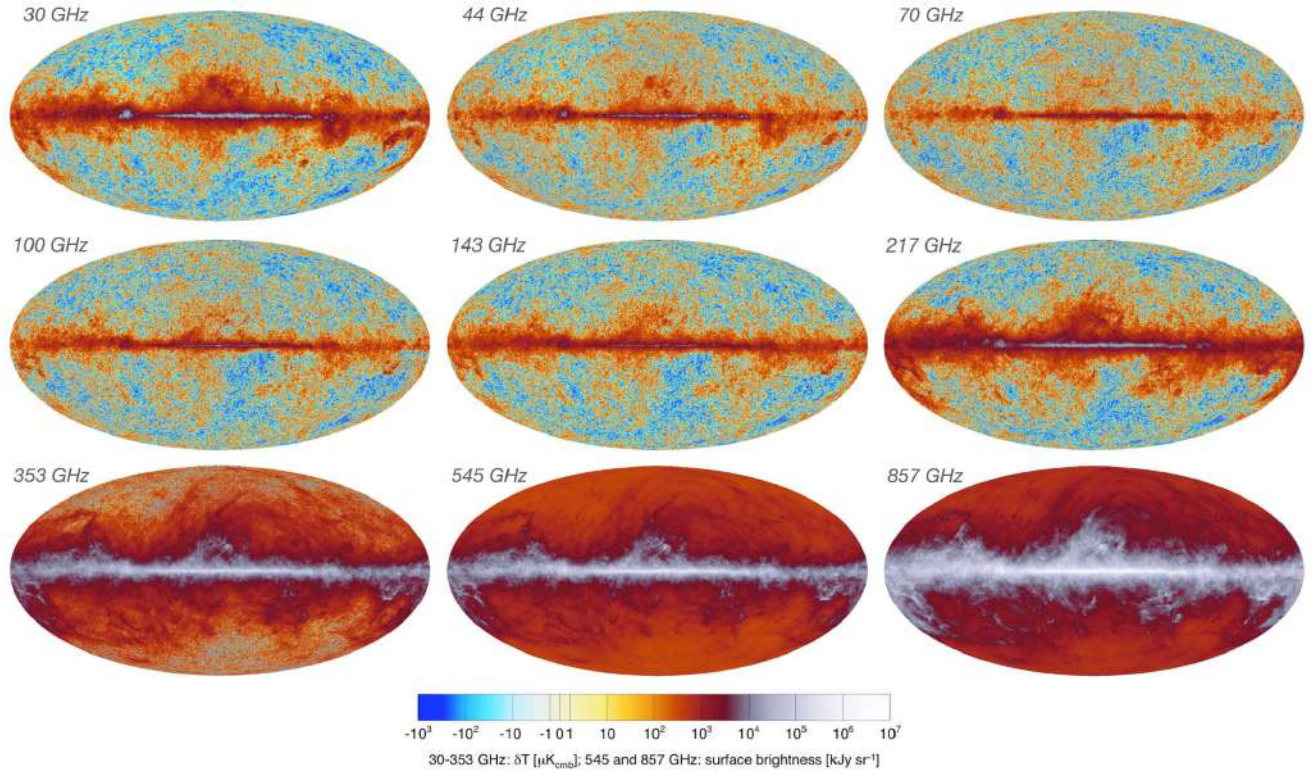


Fig. I.9 Fluctuations of sky emission in each of the nine *Planck* frequency bands, after removal of a common dipole component (see subsection II.3.2.b) for definition and details on the dipole). The fluctuations are expressed as equivalent temperature variations at each of the seven lowest frequencies, so that the CMB fluctuations will appear the same in each map (except for the effects of the varying resolution of the maps, see subsection II.3.2.b) for details on the units definition). The highest frequencies, which monitor the dust emission, are expressed in more conventional units ($\text{MJy} \cdot \text{sr}^{-1}$). Taken from *Planck Collaboration I* (2020), Fig.1.

component separation, which in turn requires an extensive and precise knowledge of the nature and origin of the foreground emissions. Such a study of the various astrophysical sources of emission overlaps with the interests of another field of study: that of the Interstellar Medium (ISM) physics. Additionally, for many of these astrophysical sources of foregrounds, the *Planck* data represent the best, if not the only way of producing *fullsky* maps of the corresponding emissions. This is also a central reason for mapping the sky even at spectral ranges where the CMB anisotropies are *completely* dominated by the foreground emissions, as these frequencies provide us with a good knowledge of these dominant foregrounds to be later separated at frequencies where the anisotropies are better visible.

The main sources of foreground emissions present in the *Planck* data are plotted in Fig.I.10, where I reported the frequency dependence of the main foreground contributors in the *Planck* frequency ranges. I summarize and introduce below the main sources of foreground emission in the *Planck* data (see *Planck Collaboration X* (2016) for more details about this list of foreground contributions; see Chapters III and IV for figures of template maps of the main foregrounds in *Planck*).

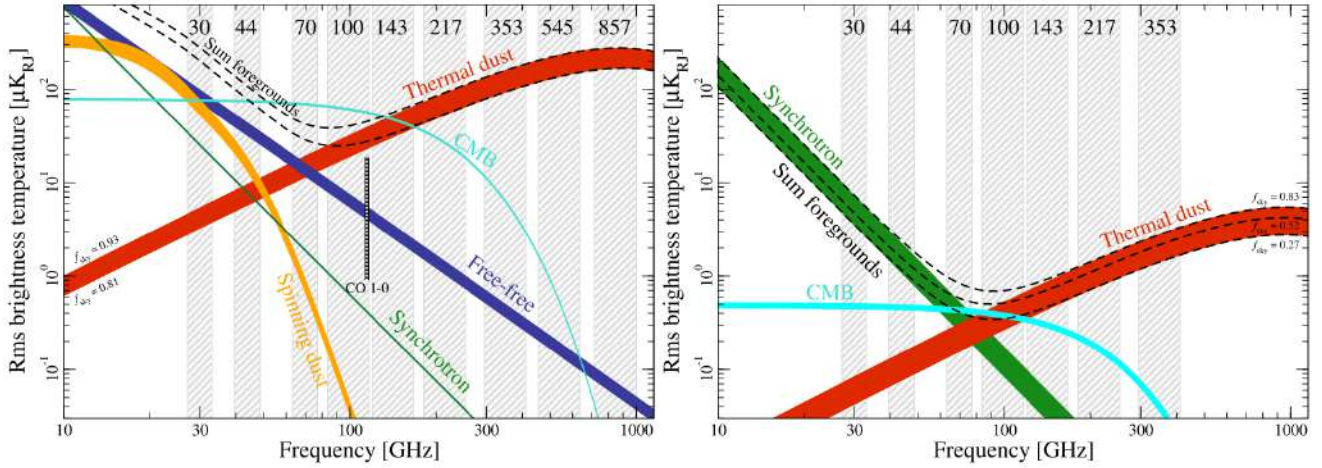


Fig. I.10 Frequency dependence of the main components of the submillimetre sky in temperature (**left**) and polarization (**right**; shown as $P = \sqrt{Q^2 + U^2}$), see Appendix.E for details on the Stokes parameters Q and U). The vertical grey bands show the Planck frequency bands, with the coloured bands indicating the major foreground components. For temperature, the components are smoothed to 1° and the widths of the bands show the range for masks with 81–93% sky coverage. For polarization, the smoothing is $40'$ and the mask range is 27–83%. Taken from *Planck Collaboration I* (2020), Fig.4.

Thermal dust The *thermal dust* emission corresponds to the cooling of the dust grains from the ISM, which have been heated by starlight, and radiate a thermal emission in the infrared (cf. [Draine \(2011\)](#)). This foreground dominates the *Planck* data for frequencies above 100GHz, as seen in Fig.I.10. The thermal dust Spectral Energy Distribution (SED) may reasonably well be described by a modified blackbody emission (see later subsection II.3.2.b) for a definition, written in Eq.II.3) which was fitted in *Planck* to be at a temperature of about $\sim 19\text{K}$, even though recent efforts have been made to develop more extensive and more representative models of this emission (cf. [Désert \(2022\)](#)). This emission has a significant polarization part, as the aspherical dust grains tend to align their major axes perpendicularly to the local direction of the magnetic field, leading to observed polarization fractions of up to 22.0% (cf. [Planck Collaboration XII \(2020\)](#)).

Spinning dust The dust grains can also emit a detectable signal from their rotation. Indeed, the rotation of dust grains displaying a non-zero electric dipole moment lead to the emission of an electromagnetic wave, with non-negligible intensities spanning a frequency range of about $[10\text{GHz}, 100\text{GHz}]$ (cf. [Draine & Lazarian \(1998\)](#), [Planck Collaboration Int. XV \(2014\)](#)). This emission displays significant correlation with the synchrotron and free-free emissions, in addition to its naturally high correlation with the thermal dust, increasing the uncertainty on the fit of its associated parameters in *Planck* (cf. [Planck Collaboration X \(2016\)](#)). The polarization part of this emission is neglected in *Planck* since the grains at the origin of the spinning dust are small and generally align weakly with the local magnetic field. This leads to an estimation of a polarization fraction of the spinning dust emission that may go up to 1-3% between 10GHz and 30GHz, but falls below 1% at higher frequencies (cf. [Planck Collaboration X \(2016\)](#)). The *Planck* map of the spinning dust emission is plotted in Fig.I.11.

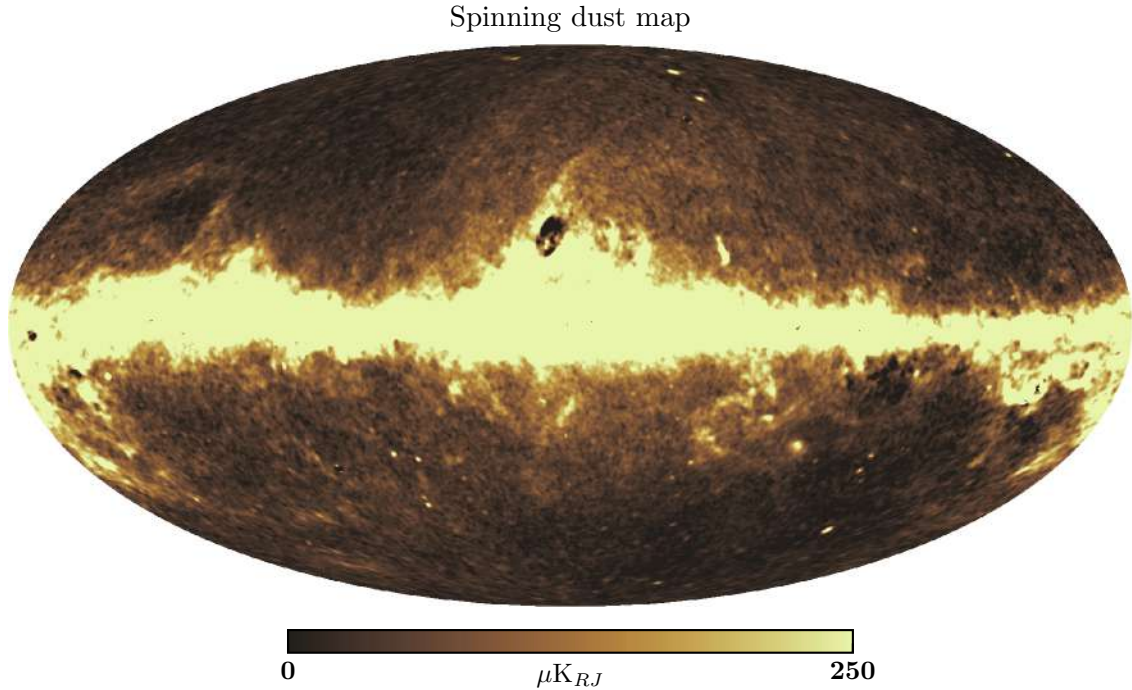


Fig. I.11 *Planck* map of the spinning dust emission. See [Planck Collaboration X \(2016\)](#) for details on the nature and construction of this map.

Synchrotron In the ISM roams relativistic electrons, which get deflected by the Galactic magnetic fields and in turn emit the so-called *synchrotron* emission. This emission, which is correlated on the sky with the free-free and spinning dust emissions, is intrinsically highly polarized, with a polarization fraction going as high as 0.75. Theoretical models as well as observation suggest that this emission’s SED can be described by a power law with an index of $\beta_s \approx -3$ at frequencies above 20GHz, but displaying a knee and a flattening at low frequencies (cf. [Planck Collaboration X \(2016\)](#)).

Free-free In thermal plasma and ionized regions, the mutual acceleration between charged particles (i.e. the electron-ion collisions) emit radiations. The sum of all Bremsstrahlung radiations from such free particles interactions is called the *free-free* emission ([Rybicki & Lightman \(1979\)](#)). Its emission is intrinsically independent of direction and therefore naturally unpolarized, since the random distribution of free particles trajectory does not favour any direction. However, this emission can be polarized by Thomson scattering at the edges of bright free-free regions (i.e. strong H II regions), with strong local polarization of up to $\sim 10\%$. Nonetheless, at higher latitudes (outside the Galactic disk), and with a relatively low resolution, the residual polarization is expected to be less than $< 1\%$ (cf. [Macellari et al. \(2011\)](#)).

Molecular lines Molecular line emissions from various species are also expected within the *Planck* frequency range. These lines are expected to be dominated in intensity by the carbon monoxide (CO) lines, at least for frequencies above 100GHz. The CO emission will be presented and discussed further in Section I.4 as this foreground from the *Planck* data is the central interest of this work, and some other molecular emission will be the subject of discussion at other occasion in this work (notably, see subsection IV.3.5).

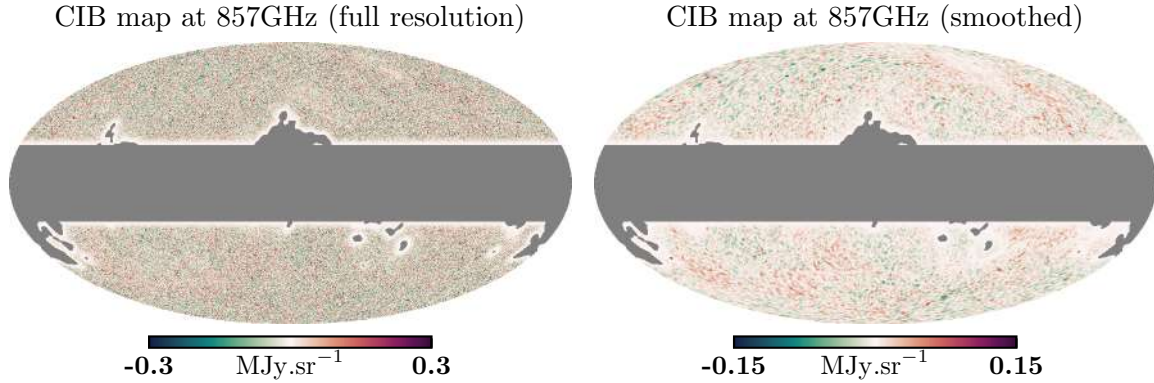


Fig. I.12 GNILC map of the CIB from the *Planck* data at the frequency 857GHz. **Left:** map at original resolution; **Right:** map smoothed to a resolution of 1° . Grey area is masked. See [Planck Collaboration Int. XLVIII \(2016\)](#) for details on the nature and construction of this map.

Cosmic Infrared Background A background of redshifted thermal dust emission from distant galaxies is also present in the signal, called the Cosmic Infrared Background (CIB). As a cosmological emission, it is essentially isotropic on the sky, with small fluctuations coming from the distribution of column density of the emitting dust integrated over the redshift z . The CIB anisotropies' intensity is far smaller than that of the CMB up to frequencies of at least ~ 400 GHz, 5000 times smaller at 143GHz and 500 times smaller at 217GHz at scales of $\ell = 100$ (cf. [Planck Collaboration XXX \(2014\)](#)). A map of the CIB produced from the *Planck* data at the frequency 857GHz is plotted in Fig.I.12.

Note that the aforementioned Sunyaev-Zeldovich effect (both thermal and kinetic parts) are often considered as foreground emission, despite not being independent emission sources, but *scattering* of the CMB photons.

I.4 The CO emission in the ISM from the Milky Way

The rotational lines from the CO are the dominant molecular lines in the *Planck* data, and one of the main foreground emissions. I will detail later in Section II.4 how the CO emission was extracted from the rest of the signal in *Planck*, prior to this work. Also, this emission is of great interest in the study of the ISM since it is one of the main tracers of cold molecular gas in the Galaxy. For this reason, further discussed in Section I.5, the CO foreground in the *Planck* data is the central subject of interest for this work.

In this section, I will introduce the CO lines' emission in the ISM. I will first discuss why this emission is so essential in the observation and study of the ISM today. Then, I will summarize the main characteristics and underlying mechanisms leading to the CO emission. Finally, I will introduce a secondary mechanism leading to the production (and excitation) of CO, which has been modelled but barely explored observationally.

I.4.1 CO as one of the main tracers of cold molecular gas in the galaxy

Since its first observation in emission (as opposed to absorption) in 1970 by [Wilson et al. \(1970\)](#), the CO molecule has quickly become a pillar of the ISM exploration as one of the main tracers of matter, and especially molecular clouds, in the Galaxy. Indeed, while it is

only the second most abundant molecule in the Galaxy at only about $\sim 10^{-4}$ that of the H_2 , it is a far more suitable candidate for observation.

The molecule of H_2 is highly symmetrical and therefore does not exhibit a permanent electric dipole moment. This greatly limits the range of possible transitions, as this imposes that only quadrupolar (or higher order) transitions are allowed, i.e. $\Delta J = \pm 2$. This leads to a very dim rotational and vibrational radiation from H_2 . Additionally, these lines correspond to high energy transitions, requiring high temperatures to excite the molecule and populate the corresponding quantum upper levels (more than $> 500\text{K}$). I will present in the next subsection the physics of the rotational transitions and the associated emission (or absorption).

Conversely, dipole transitions are allowed by the structure of the CO and the excitation temperatures are very low (5.5 K for the first transition), as can be seen from Table I.1, causing the CO to radiate intensely in the physical conditions characteristic of the molecular clouds (i.e. low temperatures and high density, see in next section), making it an *almost* perfect source of observation to explore these structures. Of course, this interest is slightly countered by the CO rotational lines being almost always optically thick (cf. Lequeux et al. (2002)), at least for the first lines of the ^{12}CO which are the most common and most surveyed, and requiring the knowledge of the abundance ratio $^{12}\text{CO}/\text{H}_2$ (or $^{13}\text{CO}/\text{H}_2$ depending on the conditions) to be able to estimate the total gas column density. Additionally, the critical densities necessary to be in local thermodynamic equilibrium conditions are not always reached (about $n \gtrsim 10^3\text{-}10^4 \text{ cm}^{-3}$, cf. Dell'Ova (2021)), except in the inner parts of dense clouds.

For these reasons, CO is used as a mean to estimate the mass of observed CO-bright structures, and especially molecular clouds. However, the accuracy of such a method of mass estimation has been discussed recently as a significant fraction of molecular mass from the dense clouds may lie in regions deprived of CO emissions, at the outskirts of the clouds. Indeed, along the photodissociation region, a significant volume of the cloud contains significant amounts of self-shielded H_2 in a quasi-total absence of CO, which gets photodissociated. Such regions are called "dark" molecular gas, as they do not radiate in CO, and since the H_2 is a very weak emitter, and their mass is neglected when estimating the clouds' mass only from the CO radiation (see Wolfire et al. (2010)).

The high latitudes While a more thorough review of the CO observation history in the Galaxy was provided by Heyer & Dame (2015), it is worth noting that numerous CO surveys covered the search of molecular clouds and CO emission at higher latitudes (cf. Gir et al. (1994), Blitz et al. (1984), Magnani et al. (1985), Hartmann et al. (1998), Magnani et al. (2000)). These surveys catalogue new small clouds at high latitudes, and additionally, CO emission from regions of unusual physical conditions was detected in absorption: notably, in regions of very low column density, impossible to provide the necessary conditions for the apparition of the self-shielding mechanism (cf Hennebelle & Falgarone (2012)).

Note that 2 recent CO surveys were carried out over large regions of the sky: the Taurus region ^{12}CO and ^{13}CO survey of Goldsmith et al. (2008), more precisely introduced in subsection III.2.1.a), and the very large sky-coverage survey of Dame & Thaddeus (2022) in ^{12}CO , presented in Section V.1. These 2 surveys will be extensively exploited in this work and I will respectively refer to them, and the maps they provided, as G08 and D22.

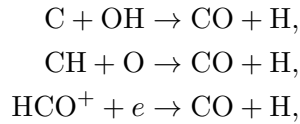
I.4.2 Physics of the CO emission from dense clouds

I will give here a simplified picture of the physics of CO in the dense clouds from the ISM by summarizing:

- How CO is formed and stays present in the molecular clouds,
- How CO is excited and emits lines of wavelength in the millimeter range,
- How the CO signal from the molecular clouds is characterized.

For a more in depth description of the physics of the CO, I encourage the reader to see the works of [Draine \(2011\)](#), [Lequeux et al. \(2002\)](#), and [Dell’Ova \(2021\)](#).

Production of CO The molecule of CO is mainly seen in regions called molecular clouds, or giant molecular clouds, characterized by a low average temperature $T \sim 10\text{K}$ and high density $n > 10^3\text{cm}^{-3}$ (cf. [Hennebelle & Falgarone \(2012\)](#)). In these high density regions, shielded from most of the dissociative UV radiation, a rich chemistry takes place, leading to the formation of multiple molecular species. The main reactions leading to the production of CO are the following (cf. [Draine \(2011\)](#), [Lequeux et al. \(2002\)](#)):



The reactions and processes producing CH and HCO^+ are rich and diverse, mainly consisting in successive reactions starting from the most represented species C^+ , H_2 and OH.

Well within the dense clouds, this chemistry can thrive with very few dissociation of the molecules since only a few cosmic rays still manage to penetrate the cloud, but this is only due to a mechanism occurring at the edges of the giant clouds and called *shielding*. Indeed, at the cloud’s periphery, in a layer called the photodissociation region, the incoming flux of UV radiation is strong enough to break the molecules present. In these regions, molecular species are essentially represented by the molecules H_2 and CO, displaying a significant presence for column densities greater than $N_H \geq 1 \times 10^{21}\text{cm}^{-2}$ (cf. [Lequeux et al. \(2002\)](#)), while the rest of the gas is populated by atoms. Therefore, a competition takes place between the rate of synthesis and photodissociation of the molecular species. The deeper the UV light enters the cloud, the fainter it becomes, as it has already been absorbed by the dust and by the gas along the way, and the lower its dissociative power becomes. More precisely, the UV radiation is indiscriminately reduced by absorption from the dust, but it is reduced at specific frequencies from the photodissociation of encountered molecules. This is due to the photodissociation being an energetically *quantified* process, meaning that it can only occur at specific energies for each molecular species. As the UV radiation penetrates the cloud, photons at the specific energies corresponding to the photodissociation of molecules present are rarefied, whereby the lower dissociative power of said radiation. As the local density increases, a point is reached where the synthesis prevails over the photodissociation from the UV, and the UV radiation is entirely absorbed at this layer (for column densities higher than $N_H \geq 5 \times 10^{21}\text{cm}^{-2}$, cf. [Lequeux et al. \(2002\)](#)). This provides a shielding for the gas beyond this depth (deeper within the cloud) which is no longer exposed to the incoming UV radiation. The main contributors of this shielding are the dust, and the molecules H_2 and CO (cf. [Stephán \(2016\)](#)), and since the CO at the periphery participates in shielding the CO present within the cloud, it is said that the CO is *self-shielding*.

Table I.1: Summary of the first rotational lines for the ^{12}CO and ^{13}CO . E_u/k is the energy of the upper level of the corresponding transition. Values taken from [Lequeux et al. \(2002\)](#) and [Dell’Ova \(2021\)](#).

Species	Transition	Frequency [GHz]	E_u/k [K]
^{12}CO	$J = 1 \rightarrow 0$	115.271	5.5
	$J = 2 \rightarrow 1$	230.538	16.6
	$J = 3 \rightarrow 2$	345.796	33.2
^{13}CO	$J = 1 \rightarrow 0$	110.201	5.3
	$J = 2 \rightarrow 1$	220.399	15.9
	$J = 3 \rightarrow 2$	330.588	31.7

We will see that another source of production of CO has be modelled, notably taking place in regions of column densities too low to display self-shielding, as will be discussed in subsection [I.4.3](#).

Excitation and emission The CO molecules are first excited, essentially via collisions. In the dense cloud, the molecules gain energy, and notably rotational energy through collisions with other surrounding molecules and atoms. This rotational kinetic energy can then be dissipated through radiation of rotational lines, corresponding to the emission of a photon of same energy as that of the transition between 2 states of angular momentum J . Indeed, the Schrödinger’s equation gives eigenvalues of the energy of the following form (for *purely* rotational transitions, and *without* any deformation of the molecule, cf. [Lequeux et al. \(2002\)](#)):

$$E_r = BJ(J + 1),$$

where J is the quantum number corresponding to the angular momentum, h is the Planck constant, and $B = \frac{h^2}{8\pi^2 I}$ with I the moment of inertia ($B/k = 2.77\text{K}$ for the CO). Since the CO molecule displays a non-zero permanent electric dipole moment, the dipole transitions are allowed, and radiative transitions can then follow a variations of the type $\Delta J = \pm 1$ (cf. [Lequeux et al. \(2002\)](#)). The radiative emission corresponding to any transition of the type $\Delta J = -1$ is what I call a *rotational line* in this work, as I focus on CO emission and *not* its absorption. I summarized the first rotational lines for the ^{12}CO and ^{13}CO in [Table.I.1](#).

Signal properties The CO rotational lines’ emission occurs only at the specific energies of the allowed transitions, therefore leading to a spectral distribution in the form of a very narrow line (natural width, estimated from Heisenberg’s uncertainty principle, of the order of $\sim 10^{-8}$ Hz, $\sim 10^{-7}$ Hz, and $\sim 10^{-6}$ Hz for the first 3 lines of both ^{12}CO and ^{13}CO) at the corresponding frequency (see [Table.I.1](#)). Also, the CO rotational lines are mostly optically thick (especially the first lines of the ^{12}CO , as the ^{13}CO is less abundant and the higher level of rotational energy are less populated, i.e. they need a higher column density to become optically thick), leading to only the shell of the molecular cloud being theoretically visible in ^{12}CO . However, because of varying velocity distributions of the molecules within the clouds, the CO rotational lines are broadened by the associated Doppler effect, allowing to probe some of the cloud’s depth. Such velocity distributions within the clouds are typically of the order of a few km.s^{-1} (cf. [Gir et al. \(1994\)](#), [Blitz et al. \(1984\)](#)).

The CO emission was predicted to be weakly polarized (cf. [Goldreich & Kylafis \(1982\)](#)),

and this polarization was first observed by Greaves et al. (1999). The rotating CO molecules display a magnetic moment and interact with the local magnetic field, which can in turn lead to polarized emission if the cloud displays anisotropies. However, observation only detected polarization fractions of about $\sim 1\%$ (cf. Greaves et al. (1999)). Nevertheless, a recent simulation model predicted that even such low levels of polarization fraction from the CO emission can have non-negligible impact on the observation of the CMB polarization, and especially the search for the B-modes (cf. Puglisi et al. (2017)). This means that a better knowledge of the CO polarization and an implementation of its correction may prove necessary to achieve a detection of the B-modes of polarization in the CMB.

Note that in radioastronomy surveys, the rotational lines brightness is usually expressed in units of *brightness temperature*, in units of K, defined from the Rayleigh-Jeans approximation (which is valid at these "low" frequencies, i.e. $h\nu \ll kT$) as:

$$T_B^* = \frac{c^2}{2k_B\nu^2} I_\nu,$$

where c is the speed of light in vacuum, k_B is the Boltzmann constant, ν is the frequency of the observed radiation, and I_ν is the specific intensity of the radiation. Additionally, the lines width due to the velocity dispersion is often expressed in velocity units, i.e. in km.s^{-1} , by using $\Delta\nu = \frac{\nu_0 \Delta v}{c}$. Consequently, when the total line intensity is given, i.e. integrated over the whole line, it is usually expressed in units of K.km.s^{-1} .

I.4.3 Potential CO from turbulence dissipation in low density region

In more recent works, a second mechanism of CO formation has been proposed and modelled, distinct from the one introduced in subsection I.4.2. It consists in the occurrence of warm chemistry stemming from local heating and compression of the ISM induced by the energy dissipation at the end of the turbulent cascade. Indeed, in the ISM, turbulence can form in both the diffuse ISM as well as denser regions, leading to the energy being transferred between scales through the Kolmogorov energy cascade, finally dissipating at the lowest scales of the cascade: the dissipation scale.

In these turbulent regions, the energy dissipates in either extremely elongated filamentary structures ($1'$ width for more than $> 20'$ length; cf. Falgarone & Puget (1995)), or sheet-like structures (Lesaffre et al. (2020), Richard et al. (2022)) and the warmed and compressed medium can lead to exhibit specific warm chemistry, able to produce CO molecules (Hennebelle & Falgarone (2012)).

In models of the turbulent dissipation regions, describing the produced warm chemistry and specific physical conditions, they managed for the first time to recover large abundances of CO in the conditions of the diffuse ISM and in regions of low H_2 column density, hinting that turbulent dissipation may play a crucial role in the formation of diffuse interstellar CO (cf. Godard et al. (2014), Lesaffre et al. (2020)).

I.5 Main goals of this thesis

In this thesis, I have 2 interdependent goals:

- The production of new and improved fullsky maps of the rotational lines of the ^{12}CO and ^{13}CO isotopologues, by extracting their signal from the *Planck* mission data,
- The exploration of regions at high latitudes and low column densities from my newly constructed maps, to search these regions for CO emission, which may originate from turbulence dissipation structures in the diffuse ISM.

The *Planck* data is currently the only available dataset allowing the construction of CO emission maps on the entire sky. The objective is to use a new and recent algorithm for the *Planck* mapmaking, SRoll2, which also improves the data cleaning with respect to the last *Planck* release. Using this algorithm, I aim to produce new maps of CO, improved compared to the ones provided by the last *Planck* release. I also aim to separate the 2 isotopologues ^{12}CO and ^{13}CO , which was not done in the *Planck* collaboration. These new maps could help improve our knowledge of the CO lines foreground, and thereby help to improve the component separation step, especially for future CMB observation missions.

The main strength of these maps, which presents an astrophysical interest, is that they cover *the entire sky*. While I do not expect the final resolution of the maps to be able to compete with that of radioastronomy surveys, these maps will provide us with a wealth of statistical information throughout the sky. This widespread information from the CO emission can then be explored using statistical tools to look for CO in regions which have not been searched before, and also by gathering information at larger scales than it is possible from radio surveys. Finally, while I do not expect to be able to resolve the structures of turbulent dissipation on the sky, I may detect CO emission originating from such structures by a statistical gathering of signal from the whole sky.

Chapter II

The *Planck* mission: instruments & data processing

This chapter presents the main characteristics of the *Planck* mission and its instruments. I will focus on the elements essential to the presentation of the method and results in this manuscript, and necessary to understand the shape, nature, and content of the *Planck* data handled in my work.

Firstly, I introduce the *Planck* instrument which collected the data I use, called HFI. Secondly, I present the main characteristics of the *Planck* mission operation, and more specifically its scanning strategy. Then, I summarize the data analysis and processing pipeline applied before the intervention of the mapmaking algorithm, which will be thoroughly detailed in the next chapter. After this summary, I introduce a few main characteristics of the data, as well as processing and reshaping steps it undergoes before it can be used by the aforementioned mapmaking algorithm. Finally, I present the *Planck* CO maps, last provided by the 2015 release, which I will use as comparison reference and which my work aims to improve.

II.1 The bolometer instruments

The *Planck* satellite was fitted with two main instruments: the Low Frequency Instrument (LFI), and the High Frequency Instrument (HFI). LFI is made of an array of 11 radiometric receivers of bandpasses centred at frequencies ~ 30 GHz (2 receivers), ~ 44 GHz (3 receivers), and ~ 70 GHz (6 receivers). However, this work only exploits the data collected by HFI, and I will therefore only describe HFI and its collected data. For more details on LFI, see [Bersanelli et al. \(2010\)](#) and [Mennella et al. \(2011\)](#).

HFI is composed of a collection of 52 bolometers, distributed along 6 frequency bands, approximately centred on the frequencies 100 GHz, 143 GHz, 217 GHz, 353 GHz, 545 GHz, 857 GHz. In this section, I will introduce all the essential characteristics of the HFI bolometers needed in the scope of my work. First, I will sum up the main nature and function of the bolometer instrument. Then, I will present the characteristics specific to the bolometers from HFI.

II.1.1 What is a bolometer ?

First invented around 1880 by Samuel Pierpont Langley (cf. [T. \(1881\)](#)), the bolometer is an instrument made to measure incoming radiation. More precisely, it measures the power of said incoming radiation by monitoring the heat increase of the apparatus induced by its

absorption of the incoming radiation. The bolometer instrument, being a thermal radiation detector, consists of:

- (i) An absorbing material (or absorber), which absorbs all incoming radiations to be measured, inducing an increase of its temperature, therefore transforming radiative energy into heat,
- (ii) A thermometer linked to the absorber, measuring the increase of its temperature, which is typically performed by relying on the change in the semi-conductor's resistivity implied by its temperature increase,
- (iii) A weak thermal link, which connects the bolometer to a heat sink of very stable temperature.

A strong asset of the bolometer is that the temperature dependence of semi-conductors' resistivity has been thoroughly studied, for a large panel of materials, and can vary widely. This offers a large flexibility in the instrument design, and a high adaptability of the various aforementioned elements comprising the instrument to meet the requirements set by its specific goals. Most typical bolometers' designs for observation in the infrared and millimetre waves, as well as the computation of its main components' characteristics are presented in the work of [Richards \(1994\)](#).

One main characteristic quantity of the bolometer is its noise-equivalent-power (NEP), which is a way of expressing the noise in the detector in terms of incoming power. It is defined as the incident signal power required to obtain a signal equal to the noise in a one Hz bandwidth ([Richards \(1994\)](#)). It is computed as the square-root of the noise spectral density, leading to its expression in units of $\text{W}\cdot\text{Hz}^{-\frac{1}{2}}$. This NEP can be computed for each source contributing to the total bolometer noise, of which the main constituents are:

- The photon noise: due to the random arrival of photons, resulting in a variance in the absorbed power ([Johnson \(2018\)](#)),
- The thermal fluctuation noise: due to the fluctuation of the energy within the bolometer ([Richards \(1994\)](#)),
- The electronic noise: coming from various sources within the electronics linked to the bolometer and part of its response chain, with one known notable contributor being the Johnson noise, corresponding to the tension fluctuations in the thermometer.

For a detailed discussion of these noise sources and the computation of their respective NEP see [Richards \(1994\)](#).

Another main characteristic feature of the bolometer is its time response, corresponding to the time needed for the incoming radiation to be absorbed and fully raise the absorber's temperature. This characteristic response time depends on the heat capacity C of the bolometer and its thermal conductance G (i.e. that of its link to the heat sink), and is expressed as:

$$\tau = \frac{C}{G}.$$

The optimization of the bolometers time response is an essential step of its design, and is linked to many other aspects of its performances, notably to the final NEP as will be discussed in the next subsection. The optimization of the parameter τ is also linked to the temperature of the heat sink and the elements composing the instrument, generally leading to the need of operating the instrument at low temperature.

As presented and discussed in the work of [Richards \(1994\)](#), the optimal conditions of operation for a bolometer are met when:

- The absorber has a large absorptivity over the frequency range of interest and a low heat capacity (leading to small time response),
- The substrate supporting the absorber has a low heat capacity and large thermal conductivity, so that it remains isothermal during the bolometer operation,
- The thermometer has low heat capacity, low electrical noise, and an adequate temperature dependence of its electrical resistance to operate efficiently at the (low) temperature of the bolometer,
- The thermal link, which connects the thermally active portions of the bolometer to the heat sink has low heat capacity and an appropriate thermal conductance G for the required application (see discussions in the next subsection),
- The mechanical support for the thermally active portion of the bolometer has low heat capacity, low thermal conductance, and must be stiff enough that the mechanical resonant frequencies are higher than the operating frequency of the bolometer,
- The heat sink has a stable temperature appropriate for the application.

All these aspects of the bolometers' optimization have been thoroughly investigated and addressed in the design, construction, and characterization of HFI, which is mostly gathered in the following papers: [Lamarre et al. \(2010\)](#), [Ade et al. \(2010\)](#), [Maffei et al. \(2010\)](#), [Rosset et al. \(2010\)](#), and [Pajot et al. \(2010\)](#). I will not go into the details of this optimization of the *Planck* bolometers in the next subsections, and will only address briefly some of its aspects when needed to introduce other characteristic features of HFI and its bolometers.

II.1.2 The bolometers aboard the *Planck* satellite

Planck's HFI is composed of 52 bolometers distributed along 6 frequency bands. However, all data from 2 bolometers out of these 52 were discarded, one from the 143GHz band (the bolometer 143-8) and one from the 545GHz band (the bolometer 545-3), because they displayed at all times a non-Gaussian noise structure called random telegraphic signal (see [Lamarre et al. \(2010\)](#) and [Planck Collaboration VI \(2014\)](#) for more details). For this reason, these 2 bolometers are not considered in my work, and I only present the results from the 50 valid bolometers, for which I listed the distribution along the 6 frequency bands in the general Table.II.1, which summarizes all the main characteristic features of said 50 valid bolometers from HFI. These bolometers are built as a free-standing grid of substrate beams of Si_3N_4 . These substrate beams are covered with a deposited metallic layer forming the *absorber* (see [Jones et al. \(2003\)](#) and [Holmes et al. \(2008\)](#) for more details on design and metallization). They are divided into 2 types of bolometers: the Spider-Web bolometer (SWB) and the Polarization-sensitive bolometer (PSB).

The Spider-Web bolometer (SWB) The SWB is made to give an equal response to any incoming linearly polarized radiation, which is achieved through the spider web shape of its metallic grids. These bolometers, if ideal, should then measure the incoming radiation's intensity without providing details on its polarization. I displayed the picture of such an SWB in the left panel of Fig.II.1, which was taken from the 143GHz frequency band. We can see that the SWB slightly favours an orientation (especially through its thermometer's

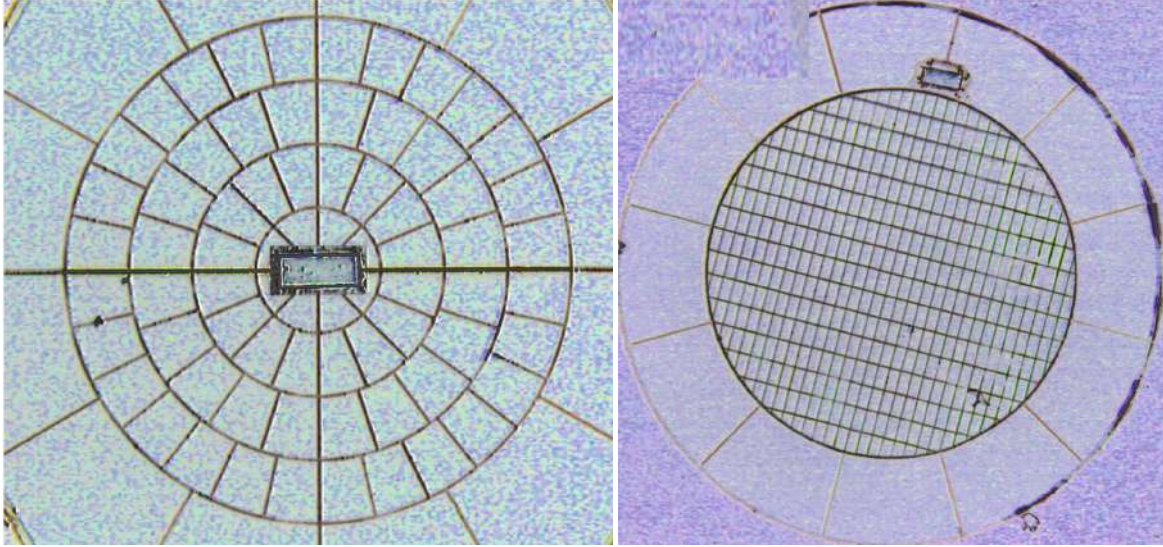


Fig. II.1 Pictures of bolometers. **Left:** SWB from the 143GHz frequency band; **Right:** PSB from the 217GHz frequency band. The temperature sensor can be seen in the center of the SWB and the upper edge of the PSB.
Taken from Fig.7 of [Lamarre et al. \(2010\)](#).

orientation) leading to a small effective sensitivity to polarization: the SWBs polarization efficiency varies from 1.6% to 8.6% along all frequency bands (see [Rosset et al. \(2010\)](#) for full lists and details on the measurement).

The Polarization-Sensitive bolometer (PSB) The PSBs are built as a rectangular grid where the metallic absorbers are deposited along only **one of the 2 preferred directions** of the grid. Linearly polarized radiations parallel to this direction are then absorbed by the metallic grid, while those polarized perpendicularly to said direction propagate through the grid with little interaction (cf. [Jones et al. \(2003\)](#)). These PSBs are set in pairs (bolometers from the same pair are differentiated by the letters either "a" or "b" in their name), and the second PSB from the pair is oriented perpendicularly to the first one to absorb the remaining polarized signal. With 2 pairs of PSBs arranged as defined and positioned along the direction of the scanning strategy, the full information (intensity *and* polarization) of any incoming linearly polarized radiation can be measured, and the 3 main Stokes parameters of the signal I , Q , and U , defined with respect to the bolometers local frame, can be determined (for more details and full definition of the Stokes parameters, see [Appendix.E](#)). Indeed, one pair of PSB is able to measure the intensity I Stokes parameter as well as the polarization Q Stokes parameter, while a second PSB pair, rotated by 45° with respect to the first pair, can provide the polarization U Stokes parameter. I displayed the picture of a PSB from the 217GHz frequency band in the right panel of [Fig.II.1](#). For various technical reasons, the PSBs behaviour deviates from the ideal case, leading to:

- Cross-polarization leakage: each bolometer from a pair is slightly sensitive to the radiation of linear polarization perpendicular to the main direction of polarization they were designed to measure,
- Alignment error, leading to increased uncertainty in the determination of the signal's polarization.

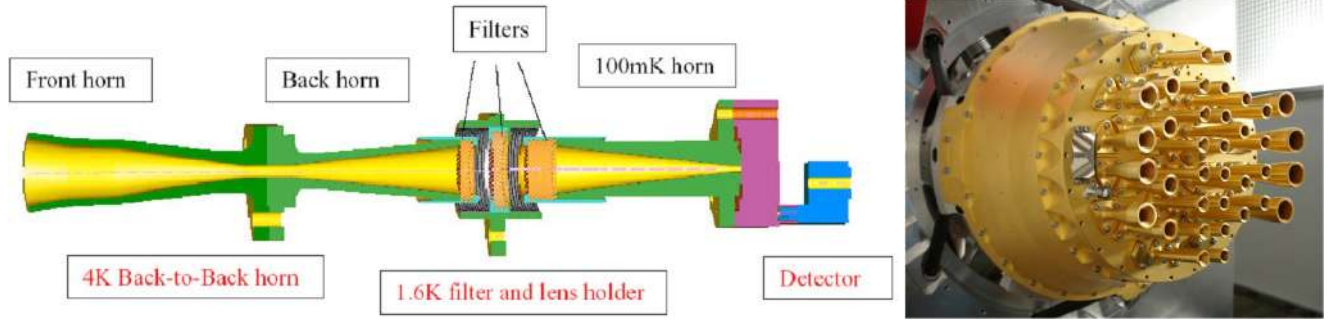


Fig. II.2 **Left panel:** Schematic of the optical chain of a bolometer, composed of a succession of feeding horns, a waveguide (present between the front horn and back horn), and filters, all cooled at low temperatures decreasing when approaching the bolometer. The 3 cooling stages are displayed: the 4K stage, the 1.6K stage, and the final 0.1K stage at the bolometer. **Right panel:** Picture of the HFI focal plane, made of the 36 front horns feeding the incoming radiations to their respective bolometers.

Schematic taken from Fig.1 of [Maffei et al. \(2010\)](#); Picture from ESA (Planck legacy archive).

While these aspects of the PSBs non-ideal behaviour have been discussed and partially addressed in the work of [Rosset et al. \(2010\)](#), some significant residual effects remain in the data, as will be discussed further in the next chapter.

Two additional bolometers are aboard the HFI instrument without belonging to any specific frequency band: the so-called *dark* bolometers. These bolometers are not optically coupled to HFI, i.e. no radiation is fed to these detectors. Their data is used to estimate the bolometers noise sources and to model some systematic effects removed in the data analysis pipeline which will be presented in Section II.3. The NEP in each frequency band was thoroughly explored and estimated, first through ground-based measurements ([Lamarre et al. \(2010\)](#), and [Pajot et al. \(2010\)](#)), followed by in-flight estimations ([Planck HFI Core Team \(2011a\)](#), [Planck HFI Core Team \(2011b\)](#)) which were updated at each new *Planck* release ([Planck Collaboration VI \(2014\)](#), [Planck Collaboration VII \(2016\)](#), [Planck Collaboration III \(2020\)](#)). The first measures of the total NEP for each frequency band are listed in Table II.1.

Before reaching the bolometers, all incoming radiations are fed into a succession of horns, waveguides, and filters, each cooled at temperatures decreasing as the waves approach the detectors. I plotted the schematic of this feeding mechanism, as well as the picture of the HFI focal plane, showing all the front horns leading to their respective bolometers, in Fig. II.2. We can see that the optical chain is divided in 3 blocks cooled at 3 different temperatures: the back-to-back front horn and back horn, composing the block called the 4K stage, are cooled down to about 4.5K, the detector horn and associated filters, called the 1.6K stage, are cooled to around 1.4K, and finally the bolometer, or more precisely its heat sink, is cooled at a temperature of 0.1K. This cooling is ensured in order to optimize the bolometers, and more precisely because of a trade-off between the time response and the NEP of the bolometers. As mentioned previously, the bolometers' time response should be kept as small as possible. However, the thermal fluctuation noise (one of the main NEP contributors) of the bolometers is proportional to the square-root of the thermal conductance, i.e. decreasing the NEP implies the decrease of G , which in turn increases τ . The only way to ensure a

low NEP while keeping a sufficiently fast response is to reduce C by cooling the bolometer to very low temperatures (cf. [Lamarre et al. \(2010\)](#)). It was determined that to achieve the goals of the *Planck* mission, the HFI bolometers needed to be cooled down to 100mK. This cooling is kept highly stable during the mission, with thermal fluctuation NEP reaching a maximum of a few $\mu\text{K}\cdot\text{Hz}^{-\frac{1}{2}}$ at time scales below 1 minute (cf. Fig.28 from [Planck Collaboration II \(2011\)](#)). The feeding horns, the waveguide and the filters are used for the spectral selection of the incoming radiations, leading to the shape of the bolometers bandpasses, also called spectral transmission, which will be presented in subsection [II.1.2.a](#)). The shape of the feeding horn also determines the shape and angular size of the beam seen by the fed bolometer. The beams of the HFI bolometers are discussed in subsection [II.1.2.b](#)).

It should be noted that the 2 PSBs from the same pair are fed by the same set of horns and filters in order to minimise the systematic effects due to potential differences of spectral and spatial response.

The last main feature characterizing the bolometers is the set of electronic devices plugged onto the bolometers to collect the measured data, amplify, digitize, and transmit it. This electronic ensemble, paired with the reception of the radiation by the bolometer, is called the bolometer response chain, and is discussed in subsection [II.1.3](#).

The HFI bolometers are named first from the frequency band they belong to, then to this frequency is attached a number differentiating the feeding horn from which they receive the incoming radiation. Finally, for the PSB pairs, an additional letter (either "a" or "b") is added to differentiate between each bolometer from the pair.

a) Bandpass

The definition of the spectral transmissions of HFI are ensured by the optical chain ensemble feeding the *filtered* radiations to the bolometers. Notably, the center and bandwidth of the bolometers' bandpasses are determined by the elements imposing the low-pass and high-pass spectral selection of the incoming radiations. The high-pass filtering, defining the start of the bolometer's bandpass, is ensured by the waveguide present in the back-to-back horn of the 4K stage, between the front horn and the back horn (see Fig.[II.2](#)). The low-pass filtering, defining the end of the bolometer's bandpass, is applied by a superposition of multiple metal mesh filters (see Fig.[II.2](#)). The selection of the respective waveguide and of the filters present in the optical chain for each bolometer therefore defines the frequency bands of HFI, of which each measured bandwidth is listed in Table.[II.1](#). For more details on the optical chain selection with regard to the spectral definition of the *Planck* frequency bands, see [Ade et al. \(2010\)](#) and [Pajot et al. \(2010\)](#). For details on the low-pass and high-pass filtering quality, see [Planck Collaboration IX \(2014\)](#).

However, the singular specificities of each optical chains and bolometers leads to significant spectral transmission variations between bolometers from the same frequency band. I plotted the spectral transmissions of each bolometers from the 100GHz, 143GHz, and 217GHz respectively in Figs.[II.3](#), [II.4](#), and [II.5](#), as they were determined by ground-based measurements with a resolution of the order of $\sim 10^8$ Hz (cf.[Planck Collaboration IX \(2014\)](#)). I only displayed the bandpasses from these 3 frequency bands because my work will only exploit the data collected by the bolometers at these frequencies, as will be further discussed in Chapter [IV](#). Also, since my work aims to build new ^{12}CO and ^{13}CO maps from the *Planck* data, I additionally represented the spectral position of the lines I will map in this work, as can be seen in Figs.[II.3](#) and [II.5](#). Note that these rotational lines' positions in frequency may vary with the lines of sight because of Doppler effects. This can have an effect on the final CO maps, as was observed in [Planck Collaboration XIII \(2014\)](#), but they observed

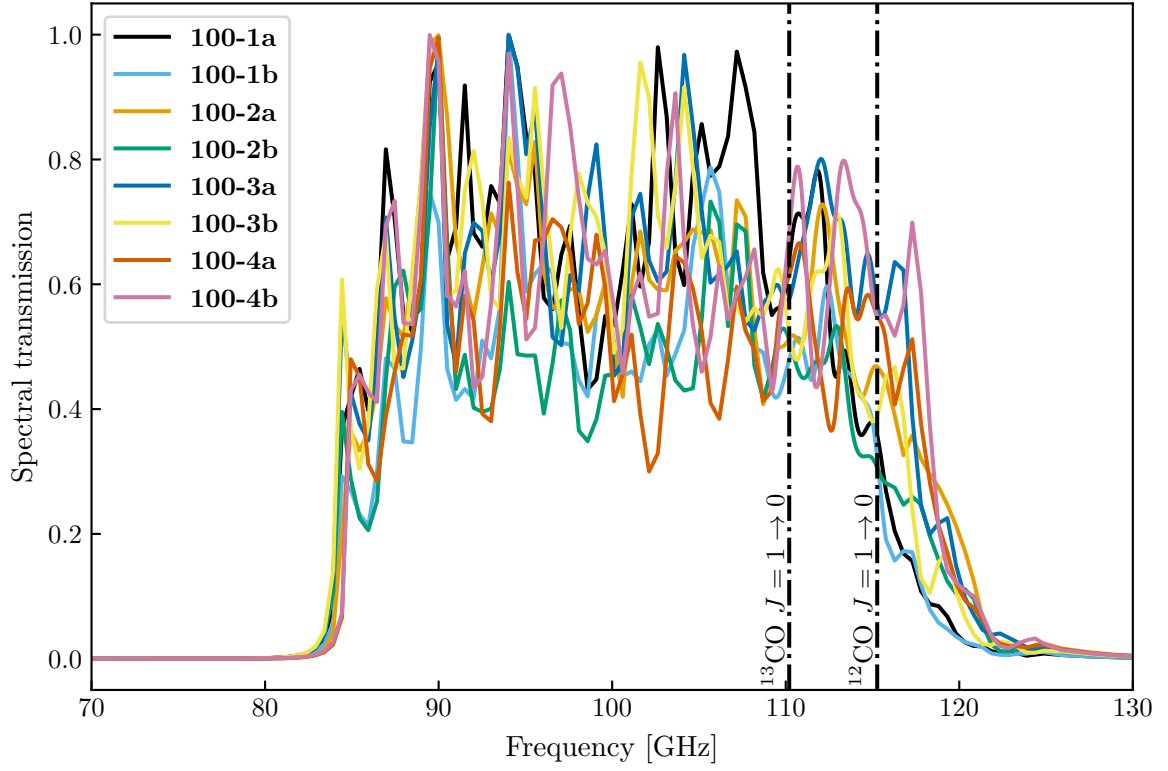


Fig. II.3 Plot of the bandpasses of all bolometers from the HFI 100GHz frequency band. The frequencies of the $J = 1 \rightarrow 0$ ^{12}CO and ^{13}CO rotational lines are also represented.

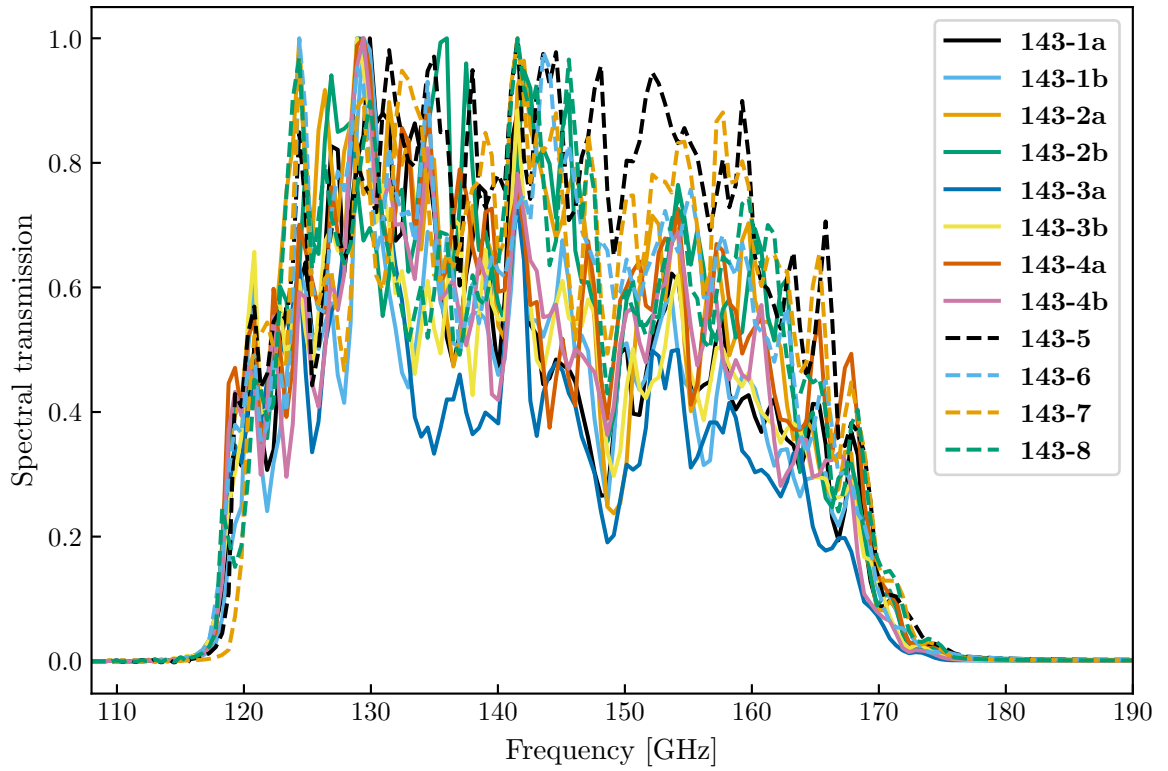


Fig. II.4 Plot of the bandpasses of all bolometers from the HFI 143GHz frequency band.

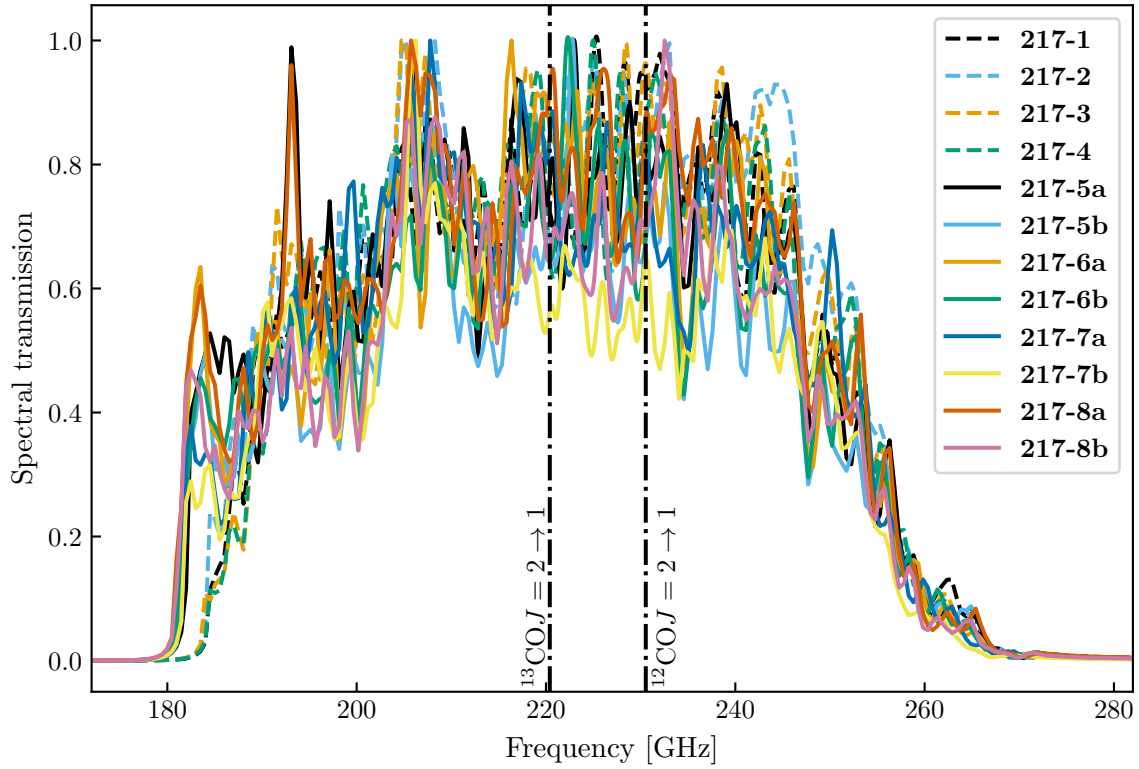


Fig. II.5 Plot of the bandpasses of all bolometers from the HFI 217GHz frequency band. The frequencies of the $J = 2 \rightarrow 1$ ^{12}CO and ^{13}CO rotational lines are also represented.

that this effect is limited to the inner galactic disk, only close to the Galactic centre, and is due to the lines' displacement caused by the rotation of the Galaxy. We can see in the figures that, while the average bandpasses of bolometers from the same frequency band are very similar, each bolometers' spectral transmission displays "wiggles", i.e. transmission variations of up to tens of percent, at the scale of the GHz or less. These "wiggles" are **not** due to spectral transmission measurement uncertainty, and are characteristic of a *genuine* bolometers transmission variation within the frequency bands. These variations proved to be essential in the quality of the sky component separation, and specifically for the ^{12}CO and ^{13}CO maps construction carried out in this work, as will be detailed in Section IV.2.

Some notable quantities which were computed using these bolometers bandpasses are the various unit conversion coefficients between relevant units encountered when working on data generated, treated, and distributed by the *Planck* collaboration. The details of the corresponding computations are presented in [Planck Collaboration IX \(2014\)](#).

b) Scanning beam

The optical chain of each bolometer, and specifically the shape and size of the feeding horns, determine the angular size and shape of the bolometer's effective beam. For consistency, the horns of the optical chains are constructed to be as similar as possible for all bolometers from the same frequency band. The Full Width at Half Maximum (FWHM) of the best average circular Gaussian fit of the effective beams along each frequency band is listed in Table II.1. While the size of the effective beam is the same for all bolometers within a frequency band, its shape slightly varies, and more notably, it deviates from ideal Gaussian beam for all frequencies, and especially for the bands 217GHz to 857GHz. The variation of effective

bolometer size between bands, as well as shape variation within each band, are visible in Fig.II.6, where the effective beam from each of the 34 horns linked to valid bolometers are represented with the same disposition found on the actual HFI focal plane. In this figure, as well as in Figs.II.7 and II.8, *co-scan* corresponds to the direction parallel to the sky scanning strategy of *Planck*, while *cross-scan* is the direction perpendicular to the scanning strategy. The beam shape differences are more closely visible in Fig.II.7, where the measured effective beams, also called "scanning beams", of one bolometer out of each HFI frequency band are plotted. We can see patterns of increasingly non-Gaussian complexity at higher frequency.

Another element we can start to notice in Fig.II.7 are long tails displayed by the bolometers along the co-scan direction. These tails are part of what are called the Far-Side Lobes (FSL), defined as non-null response of the bolometers from radiations coming from parts of the sky at angles greater than $> 5^\circ$ from the main beam centroid (cf. [Planck Collaboration VII \(2014\)](#)). Though small, these FSL can introduce spurious polarized signals at large angular scales (see [Planck Collaboration III \(2020\)](#)). The tails of the effective beams are plotted in Fig.II.8. These tails induce additional transfer functions on the collected signal, with long time constant, as will be introduced in Section II.3, and will also be again fitted in the SRoll solution presented in the next chapter.

II.1.3 The bolometers response chain

The overall performance of the bolometers is tightly linked to that of its readout electronics, connected to the instrument and used to produce the final exploited observation data. I will not detail here the composition and mode of operation of the readout electronics, which are exhaustively presented in [Lamarre et al. \(2010\)](#) and [Planck HFI Core Team \(2011b\)](#), but it can be simplified into the following parts:

- The energy deposited in the bolometer is read by measuring its bias current: it is a square wave alternative current obtained by the differentiation of a triangular voltage through a load capacitance,
- The bolometers and thermometers operate at the low temperature of 0.1K, as explained above. However, the readout electronic operates at a temperature of about $\sim 300\text{K}$, imposing a separation of the 2 systems by a physical distance of about 10m. The use of such long cables to bridge this distance, at the high displayed impedance, would render the system extremely susceptible to electromagnetic interference. This is circumvented by using JFET source followers (standing for junction-gate field-effect transistor), present in the first main unit connected to the bolometers called the JFET box, placed 2.2m from the focal plane (see [Lamarre et al. \(2010\)](#) for more details),
- The signal is then amplified a first time in the second main unit, called the preamplifier unit, or PAU box, which is placed 1.8m from the JFET box and provides an amplification of the low level voltages by a factor of 1000,
- It finally accesses the third and last main unit placed at the other end of the satellite, 5m from the PAU box, called the readout electronics unit, or REU box, which further amplifies the signal with a variable gain amplification between 1 and 22.8, and then digitizes it in an analogue-to-digital converter (ADC), sampling it at a rate of 180.37370 Hz (often referred to as the 180.4 Hz *Planck* acquisition frequency),
- Before being transmitted to the Earth, the data is submitted to a lossy compression into packets (for details on the compression and REU treatment of the signal see [Planck HFI Core Team \(2011b\)](#)).

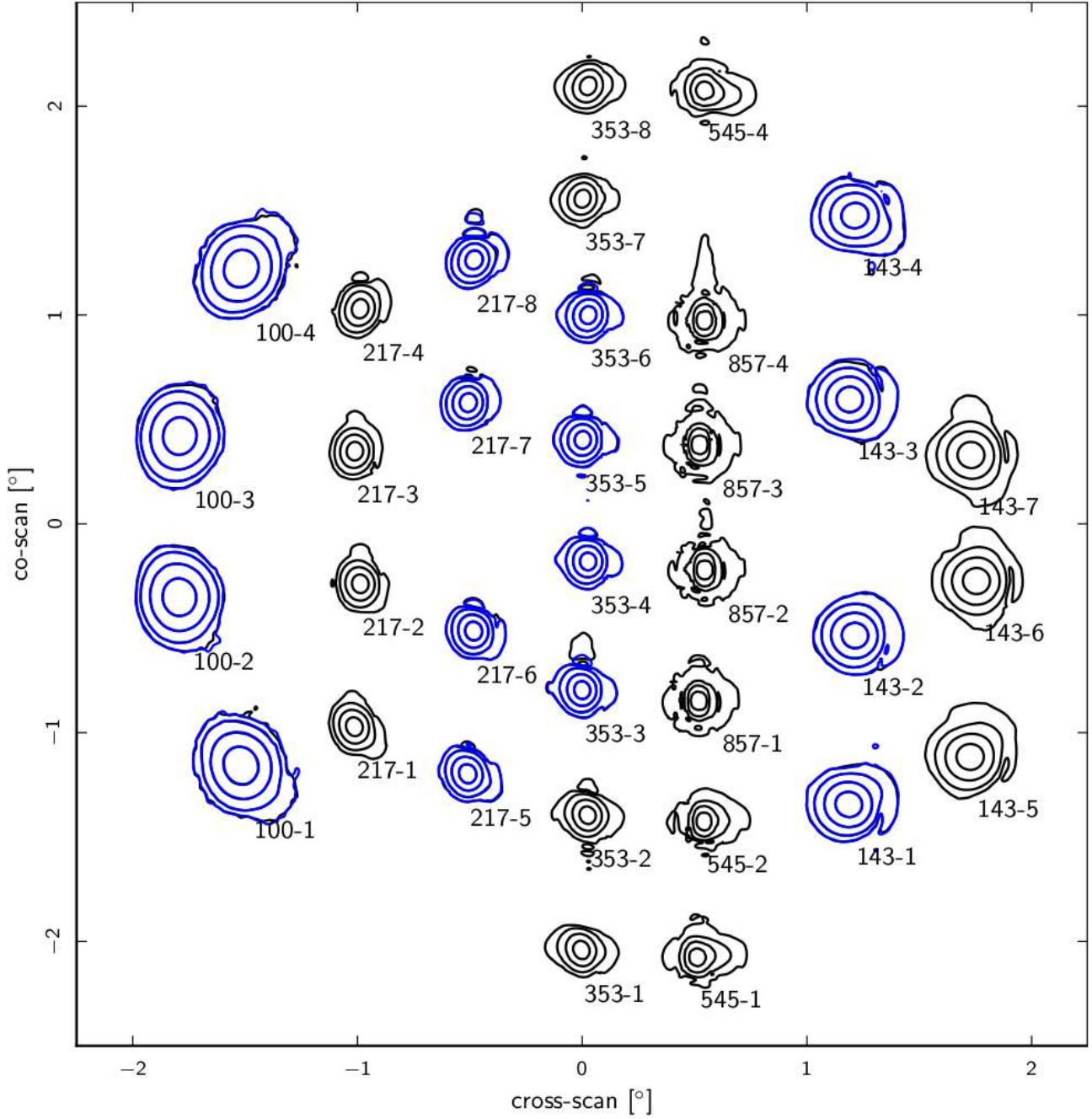


Fig. II.6 *In-flight measured effective beams as they are found on the HFI focal plane (reconstructed from Mars, Saturn and Jupiter data). The beams are plotted in logarithmic contours of -3, -10, -20 and -30 dB from the peak. PSB pairs are indicated in **blue** and SWBs are plotted in **black**. Taken from Fig.9 of [Planck Collaboration VII \(2014\)](#).*

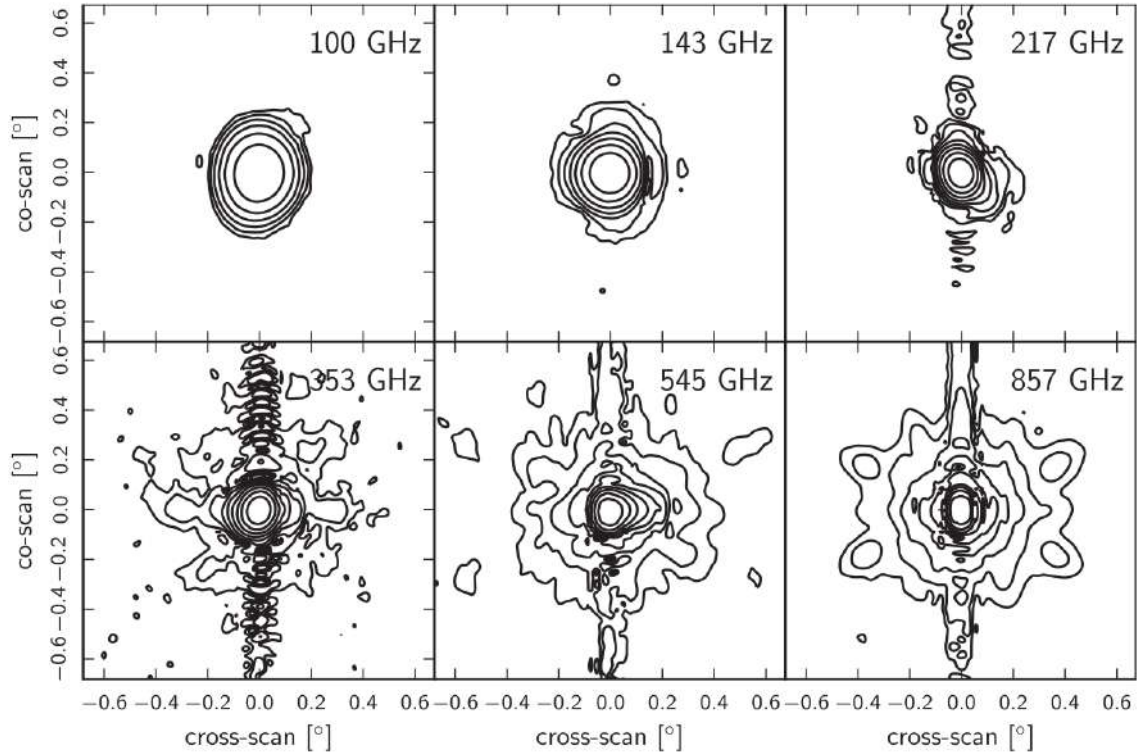


Fig. II.7 One scanning beam at each HFI frequency. **Top:** 100-3b (left), 143-6 (middle), 217-1 (right); **Bottom:** 353-7 (left), 545-1 (middle), 857-3 (right). Contours are in dB from the peak in steps of -5 dB. The lowest contours are set at -30 dB, -35 dB, -40 dB, -45 dB, -45 dB, -45 dB at 100GHz, 143GHz, 217GHz, 353GHz, 545GHz, and 857GHz, respectively. Taken from Fig.11 of *Planck Collaboration VII (2014)*.

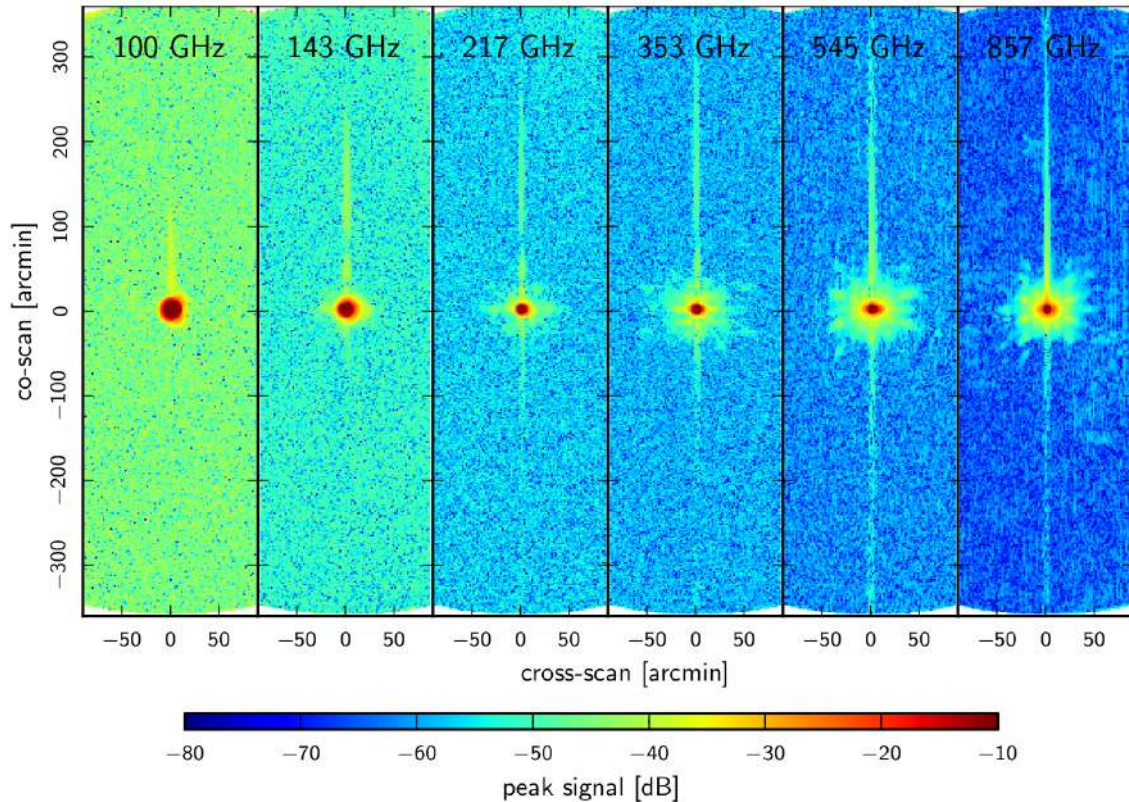


Fig. II.8 Gridded data from observations of Jupiter by each HFI frequency band. The colour scale shows the absolute value of the peak signal. Taken from Fig.12 of *Planck Collaboration VII (2014)*.

First observed in [Planck Collaboration VIII \(2014\)](#), the ADCs present a non-linearity of their analogue-to-digital units (ADU) distribution. This was detected in the form of an apparent gain variability with time from the bolometers' signal, which had never been observed in the ground-based measurements of the detectors characterization. This systematic effect has since proven to be the main instrumental systematics residual in the produced frequency maps, and one of the hardest to correct. It is further discussed in subsection [III.1.1.b\)](#) as one of the main instrumental systematic errors aimed to be corrected in SRoll.

II.2 The scanning strategy

The *Planck* satellite orbited around the sun, located on a local orbit around the 2nd Lagrange point of the Earth-Sun system, also called L2. The sky scanning is carried out thanks to the satellite spinning around its main axis, essentially oriented along the Sun-satellite direction, at a rate of 1 rotation per minute (rpm). The HFI focal plane is facing the sky at a 85° angle from the spin axis, therefore one full spin of the satellite maps a full circle on the sky. While *Planck* orbits around the sun, regular manoeuvres of the satellite are performed to move its spinning axis on average at a speed of 1° per day in steps of 2 arcmin, maintaining its orientation relative to the sun. This spinning axis rotation leads to a rotation of the full sky circle scanned by 1 satellite spin at the same speed of 1° per day. This rotation of the spin axis leads to the sky being almost entirely mapped after about 6 months. However, since the focal plane angle with the spin axis is only 85° , the satellite's orbit around the sun (same as that of the Earth) is slightly elliptical, and the spin axis is steered in a cycloid because of the satellite's rotation around L2, the symmetry of the scanning strategy is slightly imperfect. This leads to 6 months of observation only mapping about 93% of the whole sky. Nevertheless, the scanning of the next 6 months are fully complementary to the first 6 ones, leading to the sky being fully mapped *exactly* twice in one year. For more details on the satellite's orbit, spin axis orientation and stability, and on the overall scanning strategy, see [Tauber et al. \(2010\)](#) and [Planck Collaboration I \(2014\)](#).

Note that the total data acquisition time of HFI throughout the whole *Planck* mission amounts to **29 months**, brought to an end when the dilution cooler could no longer operate and ensure the stable cooling of the bolometers. The progression of the sky scanning is thoroughly monitored during the mission in the form of a continuous tracking of the satellite's *pointing* on the sky (see [Planck Collaboration I \(2014\)](#) for more details on the pointing data). This pointing data is then used at many steps of the mission assessment, as well as in the data analysis and processing pipeline, as well as later in the SRoll solution presented in Chapter [III](#).

The "stable pointing period" and the "ring" The spin axis of *Planck* is *stable* for a duration varying between 39 and 65 spins. Indeed, the time between 2 manoeuvres to rotate the spin axis varies between 2360s and 3904s, depending on which part of the cycloid *Planck* is in, conditioning the time during which the spin axis remains *fixed* with respect to distant stars (cf. [Planck Collaboration I \(2014\)](#)). This duration is called a "stable pointing period", and the set of observations made during this stable pointing period, consisting in a set of observations of the same full circle on the sky, is often referred to as a "ring". These hereby defined rings are used to compress the observation data, as will be further discussed in the next section. The whole mission produced a total of **27006** rings, and out of these 27006 rings, the valid science mission data for HFI corresponds to rings

*Animated video of the *Planck* scanning strategy: <http://www.youtube.com/watch?v=WwFXuZ0B3B8>

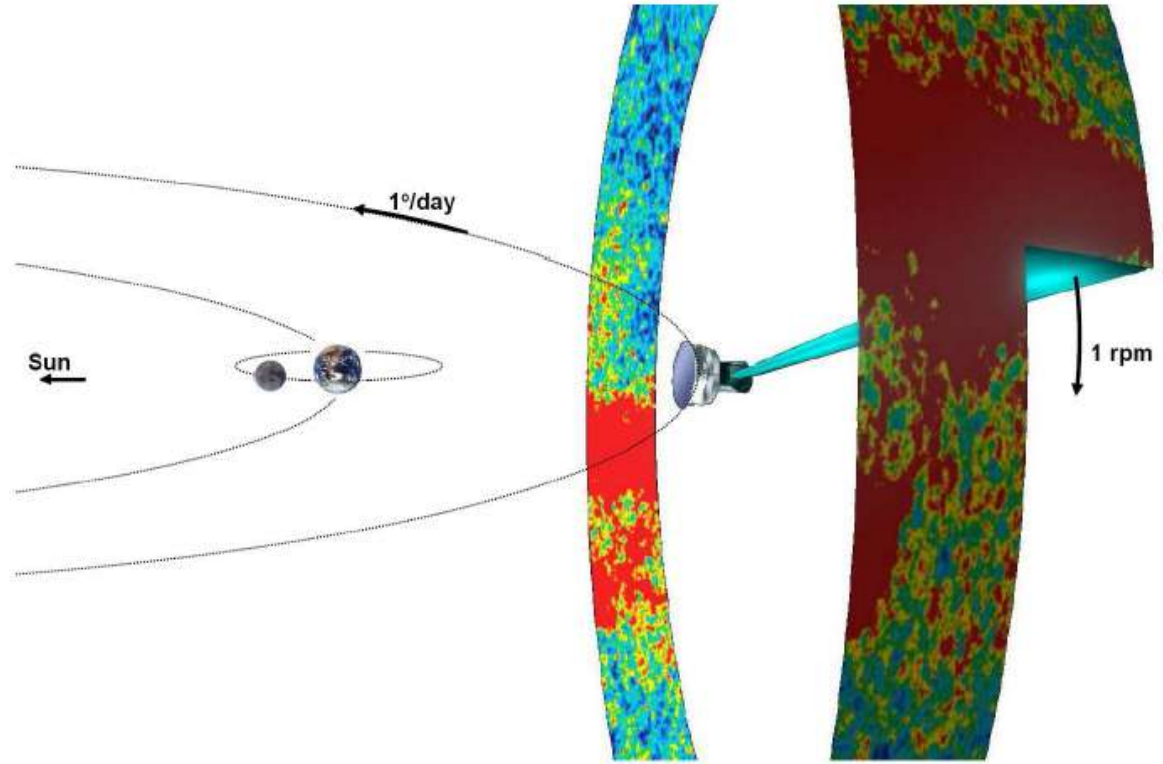


Fig. II.9 Schematic of the *Planck* scanning strategy from its orbit around L2, mapping the sky as its Field-of-View rotates at 1 rpm. The spin axis is moved on average by $1^\circ/\text{day}$ (in 2 arcmin steps) to maintain the satellite's orientation relative to the Sun. A simplified illustration of a "ring" is shown, traced by the focal plane on the sky as *Planck* spins. Note that the illustrated size of the ring's width is largely exaggerated. An animated visualization can be found [here](#)*. Taken from Fig.13 of [Tauber et al. \(2010\)](#).

from number **240** to number **26050**, leading to a total number of science data rings of $N_{\text{rings}} = 25811$. The first 239 rings are not considered as they correspond to the calibration and performance verification period taking place at the start of the mission (cf. [Planck Collaboration VII \(2016\)](#)). The rings beyond ring number 26050 are discarded because of increased temperature fluctuations of the 0.1K stage at the end of the mission, caused by low pressure in the ^3He tank used for the stage cooling (cf. [Planck Collaboration III \(2020\)](#)).

A schematic of the *Planck* scanning strategy is plotted in Fig.II.9. We can see how the satellite orbits the sun, its orientation towards it, as well as its spin axis. The scanning of a ring is also represented, as it is traced on the sky by the focal plane with the satellite's spinning motion. Do not be deceived by the illustrated width of the shown ring, as it was largely exaggerated for ease of representation.

The long duration of the *Planck* mission, leading to multiple scans of the entire sky, coupled with the high number of observations inside each ring at the high sampling rate of 180.4 Hz, implies a significant redundancy of the sky observation throughout the mission. This redundancy, for which the estimation is listed in Table.II.1, is both essential to the

Table II.1: Summary of the main characteristic features of the HFI bolometers. Only the 50 valid bolometers are considered. The bolometer identification number is used to name and differentiate the bolometers: bolometers' names are constructed by adding the identification number to the centre frequency of the band they belong to. The **FWHM** row gives the FWHM of the mean circular Gaussian best-fit of the bolometers' scanning beam along the considered frequency band (taken from Table.3 of [Planck Collaboration VII \(2014\)](#)). The **3 redundancy** rows are the estimated average, maximum, and minimum redundancy of observations in one year of operation for $2' \times 2'$ pixels (taken from Table.3 of [Lamarre et al. \(2010\)](#)). The **bandwidth** row is the average bandwidth along the considered frequency band, computed as the FWHM of the bolometers' bandpass (taken from Table.3 of [Planck Collaboration IX \(2014\)](#)). The **total noise** row corresponds to the rms noise at the map level for one second of integration time of a single detector (taken from Table.2 of [Planck Collaboration VI \(2014\)](#)). The **unit conversion** row gives the conversion coefficient for each frequency band to convert the signal from K_{CMB} to $MJy.sr^{-1}$ (taken from Table.6 of [Planck Collaboration IX \(2014\)](#), where the computation is also detailed). For details on the definition of the K_{CMB} unit, see subsection [II.3.2.b](#)).

Frequency band [GHz]	100	143		217		353		545	857
Bolometer type	PSB	PSB	SWB	PSB	SWB	PSB	SWB	SWB	SWB
Number of bolometers	8	8	3	8	4	8	4	3	4
Bolometers id. numbers	1; 2; 3; 4	1; 2; 3; 4	5; 6; 7	5; 6; 7; 8	1; 2; 3; 4	3; 4; 5; 6	1; 2; 7; 8	1; 2; 4	1; 2; 3; 4
FWHM [arcmin]	9.65	7.25		4.99		4.82		4.68	4.33
Avg. redundancy (1 year)	1380	850	1250	400	600	400	600	600	600
Max. redundancy (1 year)	12 000	7700	12 000	3700	5600	3700	5600	5600	5600
Min. redundancy (1 year)	700	420	630	200	300	200	300	300	300
Bandwidth (average) [GHz]	32.9	45.76		64.53		101.4		171.3	245.9
Total noise [$\mu K_{CMB}.Hz^{-\frac{1}{2}}$] (100-353); [$MJy.sr^{-1}.Hz^{-\frac{1}{2}}$] (545-857)	132	65	49	101	66	397	205	0.052	0.056
Unit conversion [$MJy.sr^{-1}.K_{CMB}^{-1}$]	244.1	371.74		483.690		287.450		58.04	2.27

quality of the *Planck* mission products, as well as a cornerstone of the SRoll algorithm and my ^{12}CO and ^{13}CO maps construction, as described in Chapter III, by providing the necessary information to fit and remove systematic effects, and to perform null tests to verify the statistical properties of the data.

II.3 Data structure and processing pre-SRoll

In this section, I will summarize the *Planck* analysis and processing pipeline, by focusing on the processing performed before the intervention of the SRoll mapmaking algorithm. A schematic of the final state of the HFI data analysis and processing pipeline is plotted in Fig.II.10, corresponding to the one applied in the 2018 *Planck* legacy release, where references to sections from the relevant *Planck* legacy publications are also indicated.

I will not detail each step of the data processing, as they can be found in the adapted *Planck* collaboration publications (see Fig.II.10), but I will introduce the essential steps which condition the state and shape of the HFI data, as it is later exploited by the SRoll algorithm and used as its main input. I will first define how the raw data packets transmitted by the satellite are converted into time-ordered information (TOI), and summarize the processing and cleaning steps applied on this TOI. Then I will present how these cleaned TOIs are further compressed before being used as input by SRoll, as well as introduce a few crucial characteristics of this data which will later be exploited by the SRoll solution.

II.3.1 TOI processing pipeline

The bolometers' TOI consists of the temporally ordered sequence of measurements taken by the instrument. It is built by first decompressing the transmitted data packet, then converting it from ADU to Volt readout by a constant factor. The resulting data samples, which were measured at the acquisition frequency of 180.4 Hz, are finally arranged temporally in a file. This file, which at the end of the mission contains the 29 months of observation of its corresponding bolometer, is the TOI. The acquisition frequency of 180.4 Hz leads to a *time* separation of 5.54404ms between 2 samples, which at a rotation speed of 1 rpm leads to a *spatial* separation on the sky of 2.0 arcmin between 2 samples.

Before they can be projected into maps and exploited, these TOIs go through several processing and cleaning procedures, which were improved and refined over time, and evolved with each official *Planck* release.

ADC correction Depending on the pipeline generation, the ADC correction is the first one to appear. This effect, first detected as apparent time variation of the gains in [Planck Collaboration VIII \(2014\)](#), is due to a defect of the ADC displaying non-linearity in the signal digitization, i.e. an irregular distribution of the ADU over the monitored Voltage range. Ideally, the full response of each ADC to input signal should have been mapped on the ground, but this was not carried out since this feature of the ADC was not originally expected to have any detectable effect. Given the complexity to model and correct the full extent of the ADC non-linearity (ADCNL) effect, its correction evolved widely over time. It was first only compensated by fitting the apparent gains' time variation at the level of the mapmaking computation (cf. [Planck Collaboration VIII \(2014\)](#)), a first extensive model of the effect was proposed in the following *Planck* release (in 2015), which was fitted using not only the TOI values, but exploiting ancillary data from ground-based tests as well as additional tests done during the LFI extension of the mission (beyond the 29 months of HFI operation), when the dilution cooler no longer operated (cf. [Planck Collaboration](#)

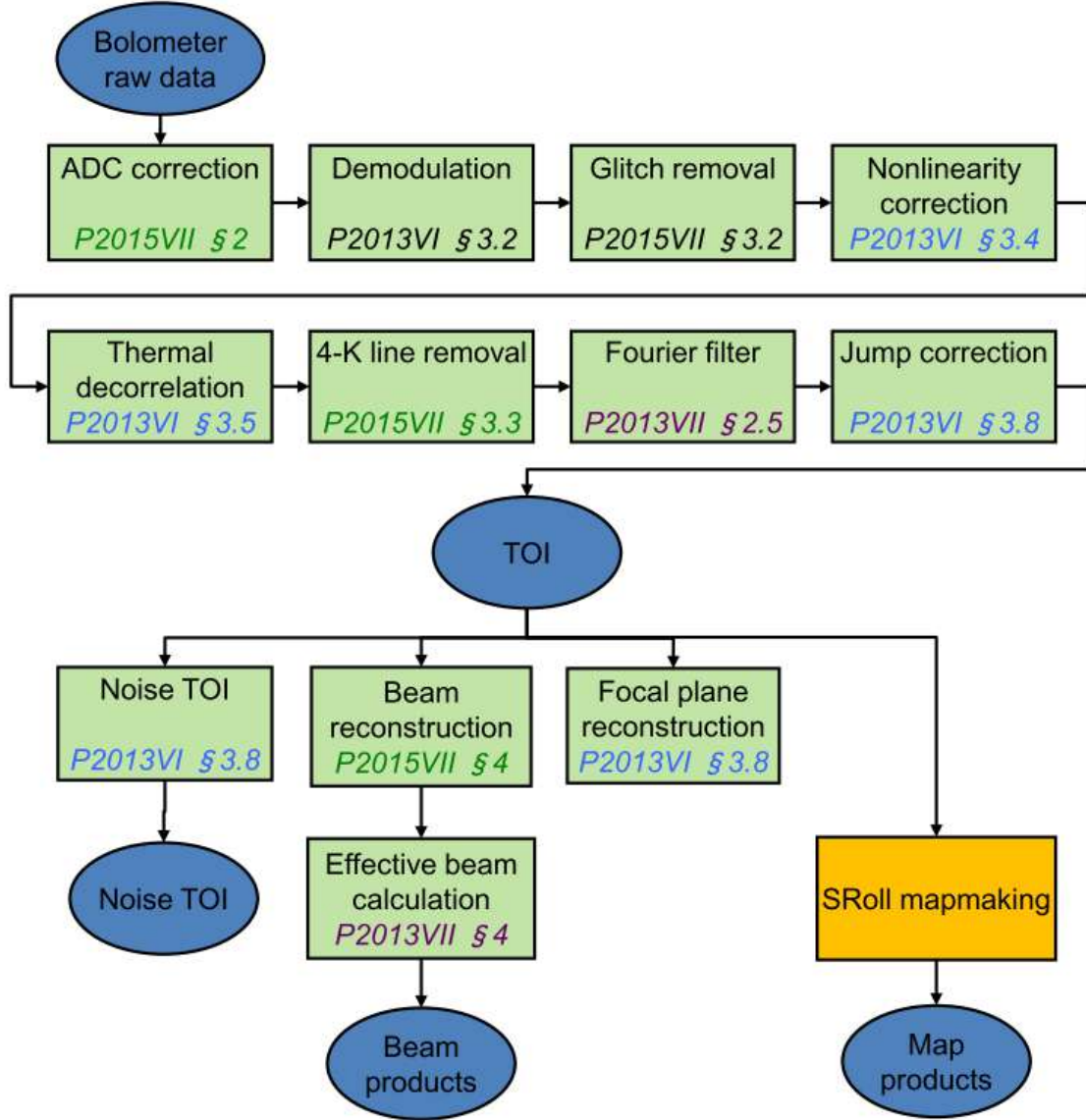


Fig. II.10 Schematic of the HFI data analysis and processing pipeline, with references to sections from the relevant Planck legacy publications at each step. The SRoll mapmaking algorithm, used as a basis for this work, is thoroughly described in Chapter III.

Taken from Planck Collaboration III (2020), Fig.1.

VII (2016)). In the *Planck* release of 2015, the ADC was then corrected at the TOI level, just before their conversion from ADU to Volt. In the following and last official release, an additional fitting of the residuals left from this first ADCNL correction was performed in SRoll, still in the form of a gain variation over time. However, the SRoll2 version of the mapmaking algorithm completely changed the way to correct for the ADCNL, as will be further detailed in subsections III.1.1.b), III.1.2 and III.2.1.c).

Demodulation As previously mentioned, the semi-conductors in the bolometers are biased by an alternating square-wave current, preferred to a direct current in order to put the acquisition electronics $1/f$ noise at high temporal frequencies, producing a noise spectrum that is flat down to very low frequencies (cf. Lamarre et al. (2010)). The modulation of this bias current can be independently tuned for each readout electronic channel, but the optimal frequency is $f_{mod} = f_{acq}/2 = 90.18685\text{Hz}$. The second step of the TOI processing then consists in computing the corresponding demodulation, as detailed in Planck Collaboration VI (2014).

Deglitching & flagging The TOIs are regularly affected by "glitches", essentially caused by cosmic ray hits. They are quite conspicuous in the TOI as can be seen in Fig.II.11, where I reported the plot of 3 raw TOIs from bolometers from the 143GHz and 545GHz bands, as well as one of the 2 blind "dark" bolometers. Three main types of glitches have been reported (cf. Planck Collaboration VII (2016), Planck Collaboration X (2014)):

- Short glitches (with a peaked amplitude distribution), due to the direct impact of a cosmic particle on the grid or the thermometer,
- Long glitches, the dominant population, due to the impact of a cosmic particle on the support grid structure of the bolometer's absorber,
- Slow glitches, with a tail similar to the long ones and showing no fast part. The physical origin of this last population is not yet understood.

The cleaning of these glitches is performed by flagging the regions of the data affected by the main response to the cosmic rays hits, and by subtracting the tails of long and slow glitches by computing a template of those tails from the remaining *unflagged* part of the data (see Planck Collaboration VII (2016) and Planck Collaboration X (2014) for more details on the deglitching procedure).

Response nonlinearity correction The response of a bolometer to the energy deposited by incoming radiations changes depending on the bolometer's configuration and its environmental conditions. The extreme thermal stability of *Planck* is shown by the measured total power level, which is seen as almost constant, with a nonlinearity correction implying a drift in the gain of typically 10^{-3} at most during the mission, for all detectors. The correction has a relative precision better than 10%, so that the gain uncertainty is at the 10^{-4} level. Nonetheless, the timelines are corrected to account for this slightly varying power absorbed by the bolometer coming from the sky load and 100mK bolometer plate temperature fluctuations. This is done by assuming a volt-to-watt conversion in the form of a second-order polynomial applied to the timeline, defined as (cf. Planck Collaboration VI (2014)):

$$P = \frac{v}{S_0} \left(1 + \frac{v}{v_0} \right),$$

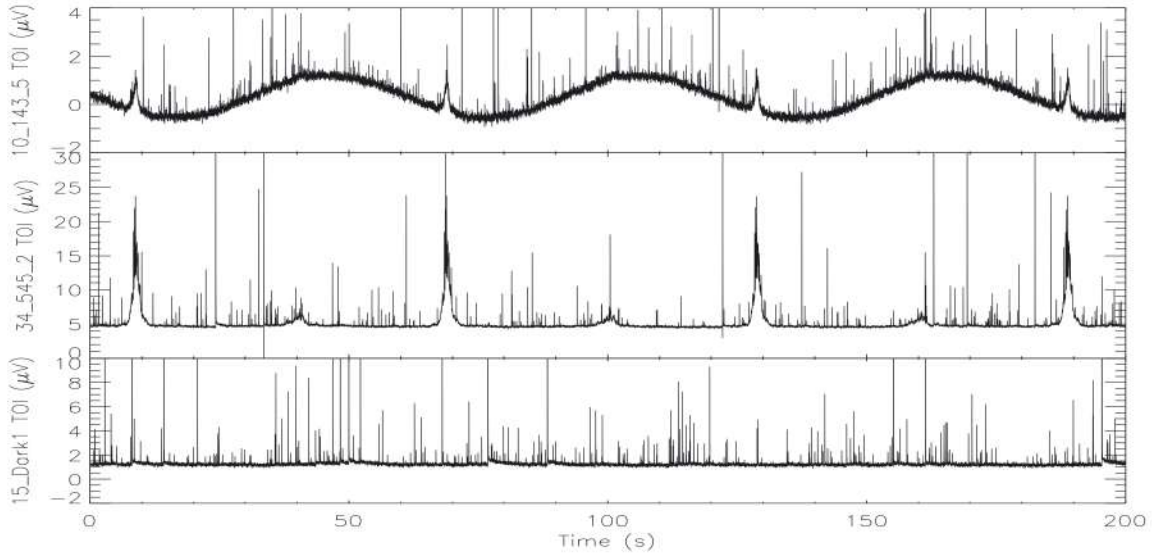


Fig. II.11 Raw TOIs of three bolometers: the 143-5 (**top**), 545-2 (**middle**), and Dark1 (**bottom**), illustrating the typical behaviour of bolometers at 143GHz, 545GHz, and of a blind detector over the course of three rotations of the spacecraft at 1 rpm. At 143GHz, one clearly sees the solar dipole with a 60s period (for more details on the dipole terms in the data, see subsection II.3.2). The 143 and 545GHz bolometers show vividly the two Galactic Plane crossings, also with 60s periodicity. The dark bolometer exhibits a nearly constant baseline together with a population of glitches from cosmic rays similar to those seen in the two upper panels.

Taken from [Planck HFI Core Team \(2011b\)](#), Fig.4.

where P is the power of the incoming radiation, v is the out-of-balance demodulated voltage (i.e., the voltage difference between the square-wave compensating input voltage and the total bolometer voltage), S_0 is the responsivity (typically of order 10^9V.W^{-1}) for the fiducial $v = 0$ voltage, and v_0 is determined for each bolometer from measurements performed before the start of the sky scanning by HFI and LFI.

Thermal drift decorrelation An additional decorrelation step is computed to correct the main temperature fluctuations of the 100mK cooler stage, performed by using the 2 dark bolometers as a proxy for these temperature fluctuations. Note that the total range of the temperature fluctuations of the bolometer plate is less than $80\mu\text{K}$, except for some strong solar flares (cf. [Planck Collaboration VI \(2014\)](#)).

4K cooler line removal The only moving part aboard the satellite is part of the 4K cooler, and is driven at a main frequency of 40 Hz, synchronously with the HFI data acquisition. This cooler impacts the HFI data and induces, by conduction, electromagnetic and microphonic interference in the readout boxes and wires. This was observed in ground-based characterization of the system but could not be corrected within acceptable costs and delays (cf. [Planck Collaboration VI \(2014\)](#)). These interferences from the 4K cooler appear as very narrow lines in the Fourier domain, whereby the name of this effect: the 4K cooler lines. Nine individual frequencies are detectable, 8 of which can be traced to some 4K cooler harmonic, and are located in the signal domain at 10, 20, 30, 40, 50, 60, 70, 80, and 17 Hz (cf. [Planck Collaboration VI \(2014\)](#)). The effect of these lines is subtracted

from the timeline by building a model timeline of this effect based on measuring the Fourier coefficients of these lines. The amplitude of these lines was then observed to increase at the end of the mission (especially in the last year of operation), therefore, rings displaying 4K cooler lines above a certain threshold were additionally flagged as a precaution (cf. [Planck Collaboration VII \(2016\)](#)).

Time response deconvolution The last deconvolution of the TOIs is the main one: the deconvolution of the transfer function associated to the bolometers' *time response*. This time response is a key parameter of the detectors, describing the relation between the incident radiation signal on the instruments and the output of the readout electronics, characterized by a time shift and a detection tail. It is described and corrected for as a complex transfer function in the frequency domain, often called the time transfer function. It is worth noting that the deconvolution of this time transfer function significantly reduces the long tail of the scanning beam (seen in Fig. II.8), it also symmetrizes the time response, which allows to combine surveys obtained by scanning in opposite directions (cf. [Planck Collaboration VII \(2016\)](#)). However, given that the time response amplitude decreases with frequency, the deconvolution boosts the noise at high frequency, which is addressed, in order to avoid unacceptably high noise in the highest temporal frequencies, by applying a phaseless low-pass filter during the deconvolution (see [Planck Collaboration VII \(2014\)](#)). The time transfer function is defined, as a function of the angular frequency ω , as follows:

$$TTF(\omega) = F(\omega)H'(\omega, S_{phase}, \tau_{stray}),$$

where $H'(\omega, S_{phase}, \tau_{stray})$ is the analytic model of the electronics transfer function, which is fully detailed in structure and parameters (notably the 2 S_{phase} and τ_{stray} terms) in [Planck Collaboration VII \(2014\)](#), and $F(\omega)$ is the term associated with the bolometer's time response. The time response term is defined as a sum of five single-pole low-pass functions, each characterized by a time constant τ_i and an associated amplitude a_i , which writes:

$$F(\omega) = \sum_{i=1}^5 \frac{a_i}{1 + i\omega\tau_i},$$

and for which each of the 5 pairs (a_i, τ_i) are measured as described in [Planck Collaboration VII \(2014\)](#) and [Planck Collaboration VII \(2016\)](#), essentially based on a combination of observations from planets, short glitch stacking, and the solar dipole (the solar dipole is fully defined in the next subsection II.3.2).

Jump correction After all the previously introduced TOI processing are applied, an intermediate set of TOIs can be constructed where the resulting signal is *removed* from the deconvolved data. In these *signal removed* TOIs a "jump" effect is observed, either positive or negative, for which no actual explanation was confirmed, although it was suspected that violent cosmic ray hits on the warm electronics were its origin. The jumps are corrected for by subtracting a piecewise constant template from the timeline, while preserving the mean level of the TOI. However, since the exact location of the jump could not be accurately determined, an additional flagging was performed around the recovered positions with 100 samples on each side (cf. [Planck Collaboration VI \(2014\)](#)), leading to a fraction less than 10^{-5} of flagged data.

Sample flagging A final flagging file is built from all origins of sample flags from the previously mentioned correction steps. In addition to the sources of flagging already introduced, the full final flagging file also contains:

- Unstable pointing flags: they correspond to unstable parts of the pointing data which cannot be trusted and lead to a flagging of the associated survey data, typically accounting for three minutes per stable pointing period, amounting to roughly 7% of data (see [Planck Collaboration VI \(2014\)](#) and [Planck Collaboration I \(2014\)](#)),
- Missing or compression error data flags: corresponding to samples displaying either erroneous or missing data in the transmitted packet, discarding a fraction of 10^{-9} of the whole mission (see [Tauber et al. \(2010\)](#)),
- Solar System object flags: they serve to exclude data from defined zones around Mars, Jupiter, Saturn, Uranus, Neptune, and around the 24 asteroids detected at 857GHz (cf. [Planck Collaboration VI \(2014\)](#) and [Planck Collaboration XIV \(2014\)](#)). As the Solar System objects are moving, this flagging must be done in the time domain, but this allows the holes in the data created from this flagging to be filled by information from later observation of the corresponding regions of the sky.

As indicated by the schematic of Fig. II.10, the processed and cleaned TOIs from all the previously summarized steps are ready to be projected on the sky by an algorithm which then outputs fullsky maps from the HFI observations. Such algorithms are the so-called mapmaking algorithms. The one used in the 2018 *Planck* legacy release is SRoll, and will be fully described in Chapter III.

II.3.2 Shaping the data for SRoll

Before the projection of the TOIs into fullsky maps, the mapmaking algorithm SRoll performs additional cleaning steps, as will be detailed in the next chapter. In order to reduce the total weight of the full mission TOIs from the HFI bolometers when handled by the mapmaking algorithm, a first compression of the data is computed. The cleaned TOIs are binned and averaged over the stable pointing periods, and gathered into new data structures called HEALPix Rings (HPR). These HPRs are constructed by binning the TOIs from each stable pointing period into HEALPix pixels, making use of the HEALPix pixelisation scheme of the sphere, presented in Appendix A, at a resolution of $N_{side} = 2048$.

a) The TOI compression into HPRs

The full procedure of projection of one ring into an HPR proceeds as follows:

- (i) The TOI of the ring is browsed a first time, along its time axis,
- (ii) For each sample, the associated pointing data for the considered ring is used to determine which pixel is "hit", i.e. in which pixel the observation sample falls, and the intensity of the sample is **added** to the pixel value,
- (iii) The pixels hit *at least once* in the ring are attributed an HPR index as follows: when a pixel is pointed and hit by a sample, either it was already hit by a previous sample in the ring and is already identified by an index, or the considered hit is the first for this pixel in the considered ring and the pixel is identified by the smallest *unattributed* integer index,
- (iv) In parallel to the *data* HPR, a second complementary HPR is filled: the *hitcount* HPR. For each browsed sample, once the hit pixel is identified and indexed if needed, its value is incremented by 1 in the *hitcount* HPR, resulting in a file keeping a record of the number of samples' hits in each pixel observed by the considered ring,

- (v) Once the entire ring TOI has been browsed, and the encountered pixels have been filled in both the *data* and the *hitcount* HPRs, the signal data is browsed again in the HPR to average the value in each pixel: while the first filling of the *data* HPR **summed** the samples' amplitude into the hit pixel, the second browsing divides the value in each pixel of the *data* HPR by its hitcount, which is stored as the value of the pixel of same index in the associated *hitcount* HPR. This results in a *data* HPR in which the value of each pixel is the *average* amplitude value over all samples falling in said pixel.

Actually, 3 more HPRs are produced for each ring: by binning and averaging the pointing data along the ring, the HPRs of the θ , φ , and ϕ coordinates are also built for each pixel seen in the ring, with θ and φ being the average coordinates on the sky along samples within each pixel, and ϕ the average polarization angle of the considered bolometer along samples within each pixel.

This procedure significantly compresses the data, where one full ring over a stable pointing period is reduced to 5 values per pixel seen by the considered ring. The result of such compression is displayed in Fig.II.12, where I plotted 2 HPRs from the data collected by the HFI bolometer 100-1a. The first part of the HPR resembles closely the shape of the TOI on one rotation: this is because most samples from the first rotation of the ring encounter the hit pixels for the first time. Therefore, in this first range of indices, 2 pixels of consecutive indices are almost always *neighbouring pixels on the sky*. This pixel range varies between HPRs as it depends on the ring orientation and path on the sky map: in Fig.II.12 this range corresponds to pixel indices below ~ 7500 for the ring number 10730, and to pixel indices below ~ 8500 for the ring number 255. However, in the subsequent rotations from the ring, most pixels hit have already been encountered once. Consequently, for pixel indexes above this first range, pixels of consecutive indexes *need not* be neighbours on the sky, and can be separated by any angular distance in the sky map. This leads to the quick amplitude variation observed for indexes greater than $\gtrsim 14000$ in both HPRs of Fig.II.12. Finally, the HPRs always end with zero-valued pixels above a certain index threshold, which again varies from one HPR to the other. This is due to the fact that, given the HPR construction defined above, the total number of pixels hit *at least once* depends on the path of the ring on the sky. Also, this total number of pixels hit *at least once* in any ring **cannot** be known before the first browsing of the ring TOI is completed. To circumvent this limitation, the total number of indexes within an HPR is fixed in advance at a value of $N_{pixels} = 27664$. At the end of the first full browsing of the ring TOI, all indexes which **were not associated** to any pixel hit *at least once* remains zero-valued.

As introduced above by the varying shape of the HPRs depending of the ring path on the sky, the samples binning into pixels is imperfect in that it is unavoidably irregular on the sky and in time, i.e. there is an uneven distribution of hits within pixels. This uneven distribution results in not only an irregular number of samples per pixels (i.e. an anisotropic distribution of hitcounts on the sky), but also an uneven spatial distribution of samples *within* each pixel. This is shown in Fig.II.13, where the path of the scanning rings as well as the associated samples are displayed over their binning into pixels. The full effect of this uneven pixel sampling is discussed in Planck Collaboration VII (2014).

It should also be noted that **all timeline data handled by SRoll** is in the form of HPRs. Some templates used in its data model are constructed at the timeline level, sometimes computed from TOIs, but such templates are **always** compressed into HPRs before being used as input for SRoll.

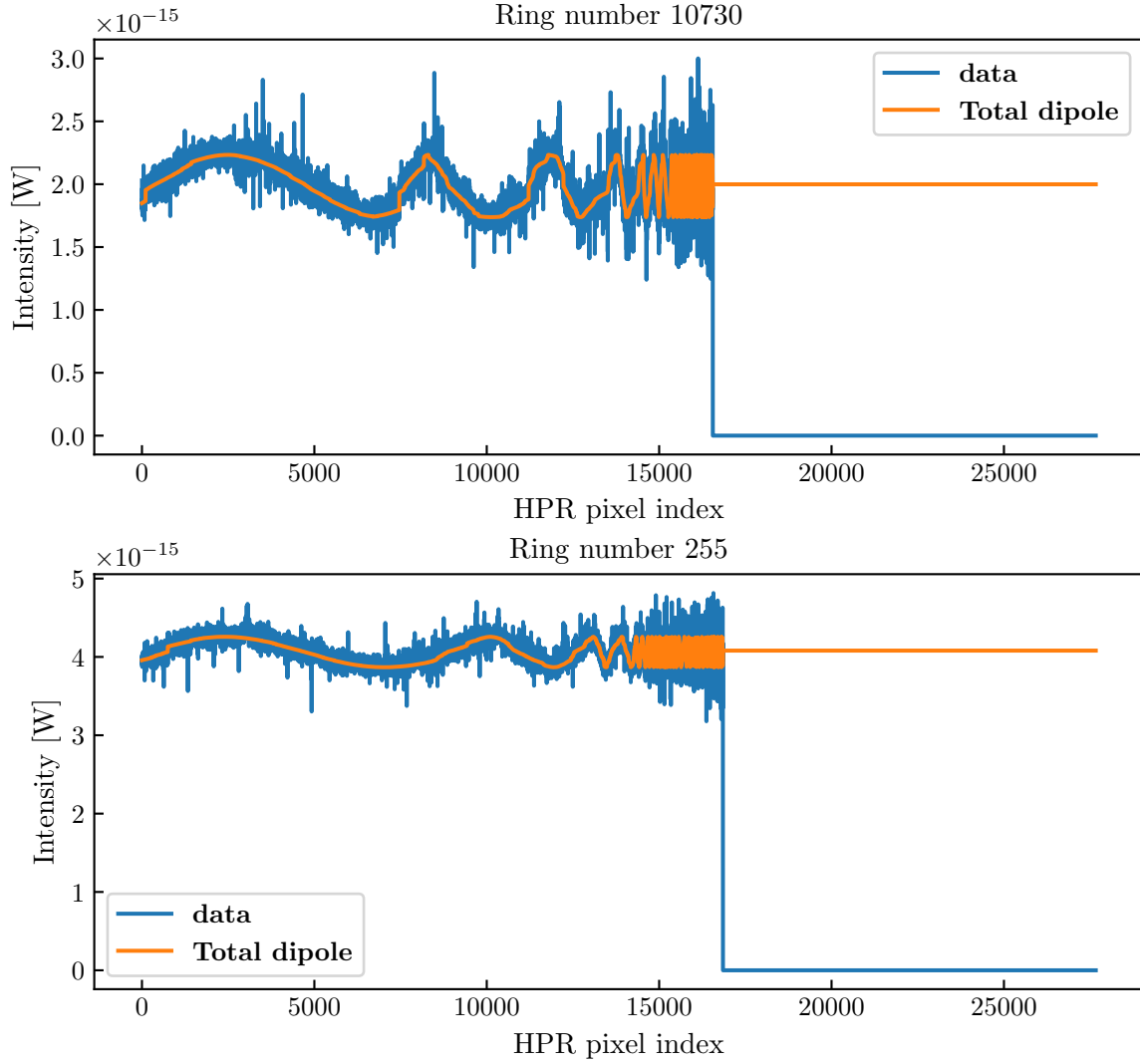


Fig. II.12 Plot of the total dipole (defined in Eq.II.2) and the data from the 100-1a PSB of HFI's 100GHz frequency band. They are plotted for the HPRs number 10730 (**top**), and 255 (**bottom**) from the *Planck* mission.

The *Planck* subsurveys In order to qualify methods applied on the *Planck* data, or the data itself, it is useful to define what I often refer to as *subsurveys*. These subsurveys are subsets of the full *Planck* mission data, built differently depending on the effect to be explored. One such survey is the odd-rings and even-rings surveys, which are respectively defined as the set of all odd-numbered and even-numbered rings from the science mission data HPRs. But the subsurvey type I will use most in this work, and on which is based the detection method of CO at high latitude and low column density, as will be exhaustively detailed in Section V.3, is the Half-Mission (HM) subsurvey: it is defined by the division in two of the whole science mission set of rings. The resulting subsurveys are called HM1 and HM2, respectively comprised of the first and second half of rings from the full mission. The main advantage of these 2 subsurveys is that they are of same length, affected by the same types of noises, but with uncorrelated noise realizations.

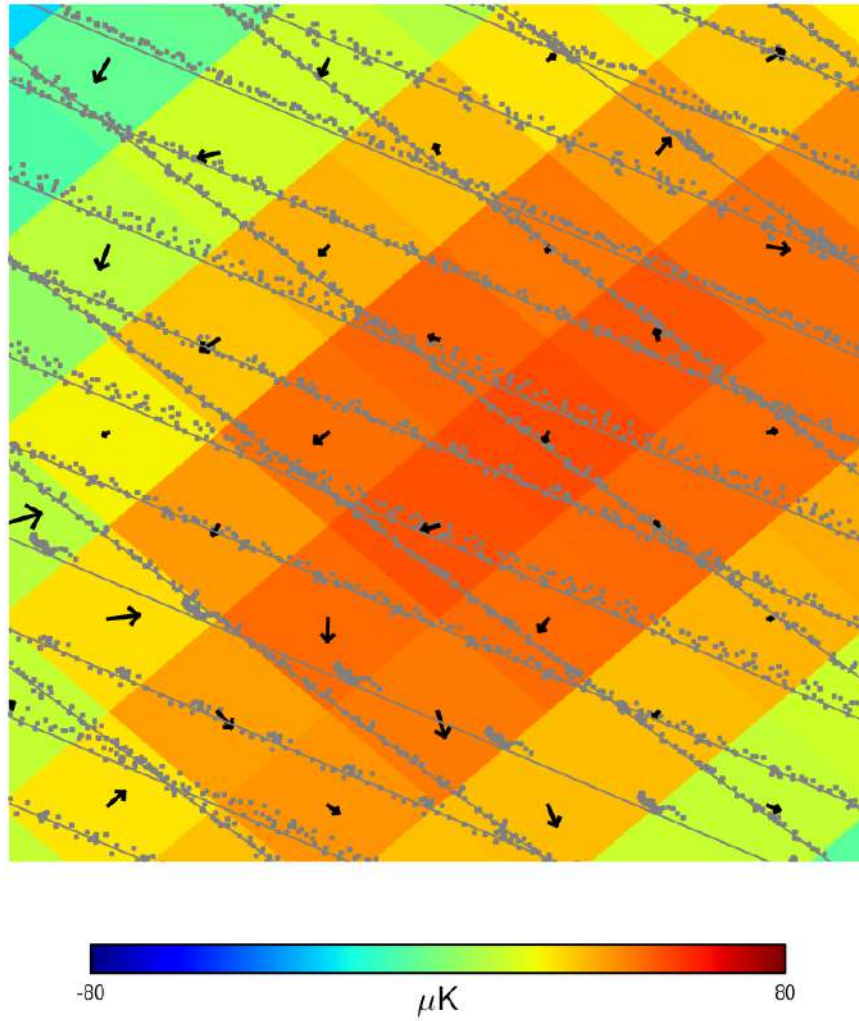


Fig. II.13 Illustration of TOI samples near the Galactic plane (**grey dots**), over-plotted on a simulated CMB realization which has been convolved with a Gaussian $7'$ FWHM beam and pixelized at $N_{side} = 2048$. Associated scanning rings (**grey lines**) as well as centres of mass for the hit distribution (**black arrows**) are also plotted.

Taken from *Planck Collaboration VII (2014)*, Fig.F.1.

b) Calibration and the dipoles

The HFI *Planck* data are filled, in all bolometers at all frequencies, by 2 dipole terms, both originating from a distinct Doppler effect on the signal. These dipole terms are essential, as they are everywhere in the data, well known, and of significant and *regular* intensity, making them perfect candidates for the photometric calibration of the data. More precisely, these 2 dipoles correspond to the decomposition of the Doppler effect rising from the full motion of the satellite with respect to the CMB frame into 2 distinct contributions. I describe in the paragraphs below this total Doppler effect and its decomposition.

Total dipole expression The Doppler effect rising from the full motion of the satellite with respect to the CMB frame, imprinted in the signal at time t , can be expressed as follows

(cf. Planck Collaboration VIII (2016); Peebles & Wilkinson (1968)):

$$T_{Doppler}(t, \hat{\mathbf{u}}_t) = \frac{T_{CMB}}{\gamma_t(1 - \beta_t \cdot \hat{\mathbf{u}}_t)}, \quad (\text{II.1})$$

where $T_{Doppler}$ is the CMB temperature as observed shifted by the total Doppler effect, $\hat{\mathbf{u}}_t$ is the unit vector along the line of sight, $\beta_t = \mathbf{v}_t/c$ with $\mathbf{v}_t$ the satellite velocity at time t with respect to the CMB frame, and $\gamma_t = (1 - \beta_t^2)^{-1/2}$ with $\beta_t = \|\beta_t\|$. The expansion of the above equation in β_t gives:

$$\begin{aligned} T_{Doppler}(t, \hat{\mathbf{u}}_t) &= T_{CMB} \left(\left(1 + \frac{\beta_t^2}{2} + \mathcal{O}(\beta_t^4) \right) \left(1 + \beta_t \cdot \hat{\mathbf{u}}_t + (\beta_t \cdot \hat{\mathbf{u}}_t)^2 + \mathcal{O}(\beta_t^3) \right) \right) \\ &= T_{CMB} \left(1 + \beta_t \cdot \hat{\mathbf{u}}_t + \frac{\beta_t^2}{2} + (\beta_t \cdot \hat{\mathbf{u}}_t)^2 + \mathcal{O}(\beta_t^3) \right) \\ &\simeq T_{CMB} \left(1 + \beta^{sol} \cdot \hat{\mathbf{u}}_t + \beta_t^{orb} \cdot \hat{\mathbf{u}}_t \right), \end{aligned} \quad (\text{II.2})$$

where $\beta^{sol} = \mathbf{v}^{sol}/c$ and $\beta_t^{orb} = \mathbf{v}_t^{orb}/c$ with $\mathbf{v}^{sol}$ and $\mathbf{v}_t^{orb}$ respectively the Sun's velocity with respect to the CMB frame, and the satellite's velocity with respect to the Sun. In Eq. II.2 above, the terms $\beta_t \cdot \hat{\mathbf{u}}_t$, $\beta^{sol} \cdot \hat{\mathbf{u}}_t$, and $\beta_t^{orb} \cdot \hat{\mathbf{u}}_t$ are then respectively called the total, solar, and orbital dipoles. The **total dipole** is then the dipole part of the Doppler effect induced by the full motion of the satellite with respect to the CMB frame. The total dipole computed from the satellite's pointing data and velocity is plotted in Fig. II.12 for the HPRs number 10730 and 255. The $\frac{\beta_t^2}{2} + (\beta_t \cdot \hat{\mathbf{u}}_t)^2$ term is called the total quadrupole. Originally, the data was calibrated on the solar dipole, but since the 2015 *Planck* release, it is calibrated on the only time-varying part of the total dipole, which is the orbital dipole, by taking as reference value for the CMB monopole temperature T_{CMB} the measurement performed by Fixsen (2009): $T_{CMB} = 2.72548 \pm 0.00057 K$.

The orbital dipole The bolometer instrument is sensitive to variation of incoming power from the collected sky radiation, to which the registered signal is proportional. Therefore, to calibrate the bolometer, we need a strong source in the sky which is both very well known, and with high Signal-to-Noise Ratio (SNR) in the data. Since the *Planck* release of 2015, in the case of the lower frequency bands of HFI (100-143-217-353GHz bands), this source is the dipole part of the Doppler effect induced on the signal by the movement of the satellite with respect to the Sun at a speed of about $\sim 30 \text{ km.s}^{-1}$, referred to as the **orbital dipole**.

This dipole is almost insensitive to the beam profile, apart from the FSL which are corrected for in the mapmaking computation before the calibration, and also weakly affected by the pointing errors, which are very low anyway, leading to a pointing drift of the order of the arc-second per pointing period in the co-scan and cross-scan directions (respectively directions parallel and orthogonal to the direction of the scanning strategy) common to all detectors (Planck Collaboration VIII (2016), Planck Collaboration VII (2014)). This error on the pointing is sufficiently small to be negligible compared to the other errors on the bolometers' beams.

The orbital dipole *almost* does not project on the sky but only at the level of the ring: indeed, the satellite's spin axis changes following the satellite's revolution around the sun, ensuring that it always remains anti-Solar (spin axis is always along the Sun-satellite direction). This implies that the ring orientation rotates throughout the mission such that a ring is scanned with orbital dipole velocity of same direction and opposite sign 6 months

later. Once averaged over the whole mission, this dipole is almost invisible on the projected sky as it almost averages to zero over one year of mission. It does not *completely* vanish, as the scanning strategy does not *rigorously* cover the exact same region of the sky within 2 successive 6 months periods, and mainly because the total duration of the mission is not an integer number of years. Its effect is corrected for anyway in SROLL at the cleaning step, just before projecting the maps, as defined in the next chapter by Eq.III.27.

This dipole is also very accurately known: since the *Planck* satellite is spinning at 1 rpm, it leads to the orbital dipole being scanned about ~ 60 times during each stable pointing period. Also, the satellite velocity is known at the level of 10^{-4}km.s^{-1} (Planck Collaboration VIII (2016)), and the orbital dipole has high SNR thanks to its full-sky coverage. It is an intense signal, with a quasi-constant amplitude of $271\mu\text{K}_{\text{CMB}}$ (cf. Planck Collaboration III (2020)), but not intense enough to induce non-linearities in the detector's response chain. As described by Eq.II.1 below, the orbital dipole signal is equivalent to an apparent shift of the black-body temperature of the CMB radiation. Consequently, calibrating the power deposited in the bolometers using this orbital dipole results in its expression in sky temperature units, defined as an apparent shift of the CMB black-body temperature, whereby the data unit name: K_{CMB} .

In Planck Collaboration IX (2014), many useful unit conversion coefficients were computed for easy conversion between useful and often employed units. I reported in Table.II.1 the conversion coefficients for each frequency bands to convert the signal from K_{CMB} sky temperature unit into MJy.sr^{-1} .

The solar dipole This aforementioned *orbital dipole* signal should not be confused with the other dipole, mentioned in the previous sections, defined as the dipole induced by the Doppler effect caused by the motion of the Sun with respect to the CMB frame, and called the **solar dipole**. The *solar dipole* is the strongest CMB signal and displays tremendous SNR. It projects fully on the sky and can be considered stationary on the duration of the *Planck* mission. It is therefore present in the frequency maps as an $\ell = 1$ component, and impacts the components of higher ℓ values only marginally due to anisotropies stemming from second-order relativistic corrections. These $\ell > 1$ anisotropies are proportional to $\beta^\ell = (\frac{v_{\text{sat}}}{c})^\ell$, with v_{sat} the satellite velocity with respect to the CMB frame, and c the celerity of light (cf. Planck Collaboration VIII (2016)). I plotted in Fig.II.14 a map of the *solar dipole* signal.

In the work of Delouis, J.-M. et al. (2021), relying on the *solar dipole* direction being the same for all detectors, they managed to improve the bolometers zero level accuracy by factors 2 to 3 with respect to the values obtained in all previous papers. More details about their method and results are given in later subsection III.1.2. Their final measurement of the *solar dipole* amplitude is $3361.90 \pm 0.36\mu\text{K}_{\text{CMB}}$, with direction $263.959^\circ \pm 0.017^\circ$ in longitude and $48.260^\circ \pm 0.007^\circ$ in latitude. These values are the ones used in Fig.II.14.

The case of the 545GHz and 857GHz bands In the *Planck* legacy pipeline, the 545 and 857 GHz bands were calibrated using neither the orbital nor the solar dipole, but by using planets as the source signal for the calibration (cf. Planck Collaboration VIII (2016), Planck Collaboration III (2020)). The reason, also discussed in Planck Collaboration IX (2014), is twofold: the CMB blackbody emission is a lot weaker at these frequencies, and its spectral radiance decreases rapidly with the frequency.

This can be seen in Fig.II.15, where I represent the average bandpasses of all the HFI frequency bands, as well as the CMB blackbody emission spectral radiance, and the thermal dust modified black-body (MBB) emission spectral radiance. The thermal dust emission is

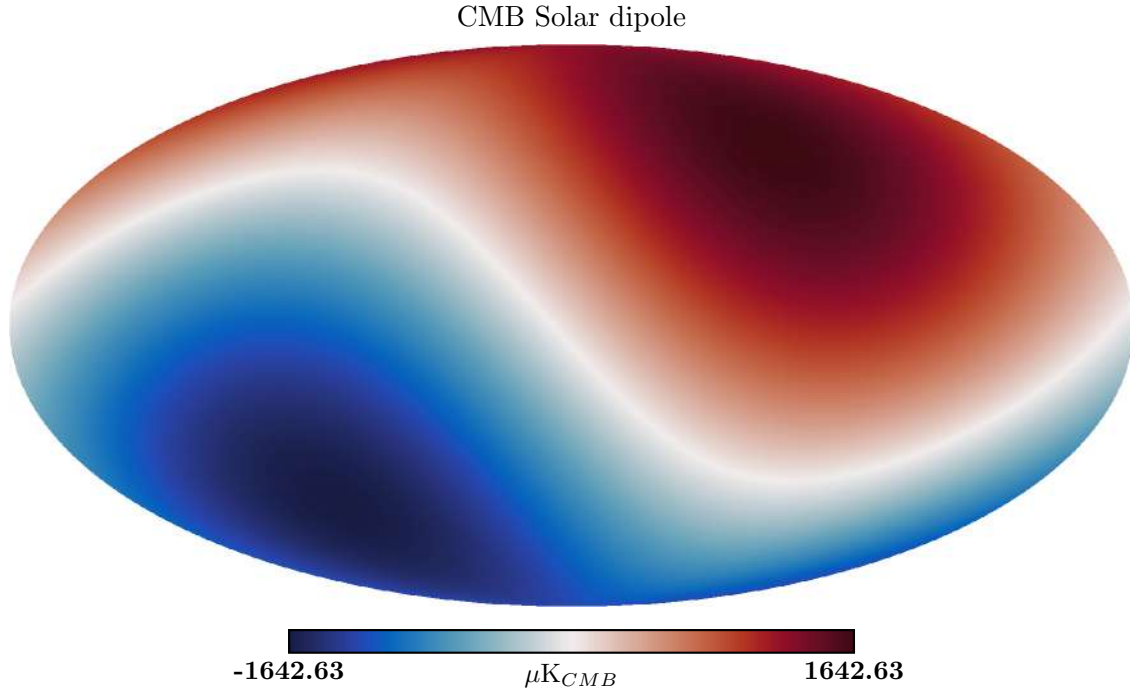


Fig. II.14 Map of the solar dipole signal on the sky, as determined in the work of [Delouis, J.-M. et al. \(2021\)](#): amplitude of $3361.90 \pm 0.36 \mu\text{K}_{CMB}$; direction of $263.959^\circ \pm 0.017^\circ$ in longitude and $48.260^\circ \pm 0.007^\circ$ in latitude.

computed as a modified black-body emission (cf. [Planck Collaboration Int. XIV \(2014\)](#)) with a temperature and spectral index identical to those taken in the *Commander* component separation procedure from the 2018 *Planck* legacy release (cf. [Planck Collaboration IV \(2020\)](#), Appendix A), i.e. $T_{dust} = 19.5\text{K}$ and $\beta_{dust} = 1.55$, where the reference frequency was taken as $\nu_{ref} = 857\text{GHz}$. The plotted spectral radiance for the thermal dust is therefore computed as :

$$I_\nu = \tau_{\nu_{ref}} \left(\frac{\nu}{\nu_{ref}} \right)^{\beta_{dust}} B(\nu, T_{dust}), \quad (\text{II.3})$$

where B is the Planck function, and $\tau_{\nu_{ref}}$ is the dust optical depth at reference frequency ν_{ref} . In the full general case, $\tau_{\nu_{ref}}$ would correspond to the sky map of the dust emission at the reference frequency ν_{ref} , but for the plot of Fig. II.15, I used an arbitrary value of $\tau_{\nu_{ref}} = 1$.

The issue spawning from this steep decrease of the CMB radiance at higher frequencies is that calibrating on the dipole at said frequencies would then tilt the contribution of the signal towards the lower frequencies of these bands (see Fig.14 of [Planck Collaboration IX \(2014\)](#) for more details on this tilt). This is why the 545 and 857 GHz HFI frequency bands were always calibrated on planets' signal, and also why this photometric calibration was later unchanged in the SRoll procedure detailed in the next chapter.

The pre-SRoll photometric calibration of the 545GHz and 857GHz bands was performed using the signal from Uranus and Neptune, measured several times during the *Planck* mission. This signal is modelled using the *HERSCHEL* calibrator models: ESA2 for Uranus and ESA3 for Neptune (cf. [Moreno \(2010\)](#), [Planck Collaboration VIII \(2014\)](#)), integrated along each of the 545GHz and 857GHz bolometers bandpasses. For more details, refer to [Planck Collaboration VIII \(2016\)](#).

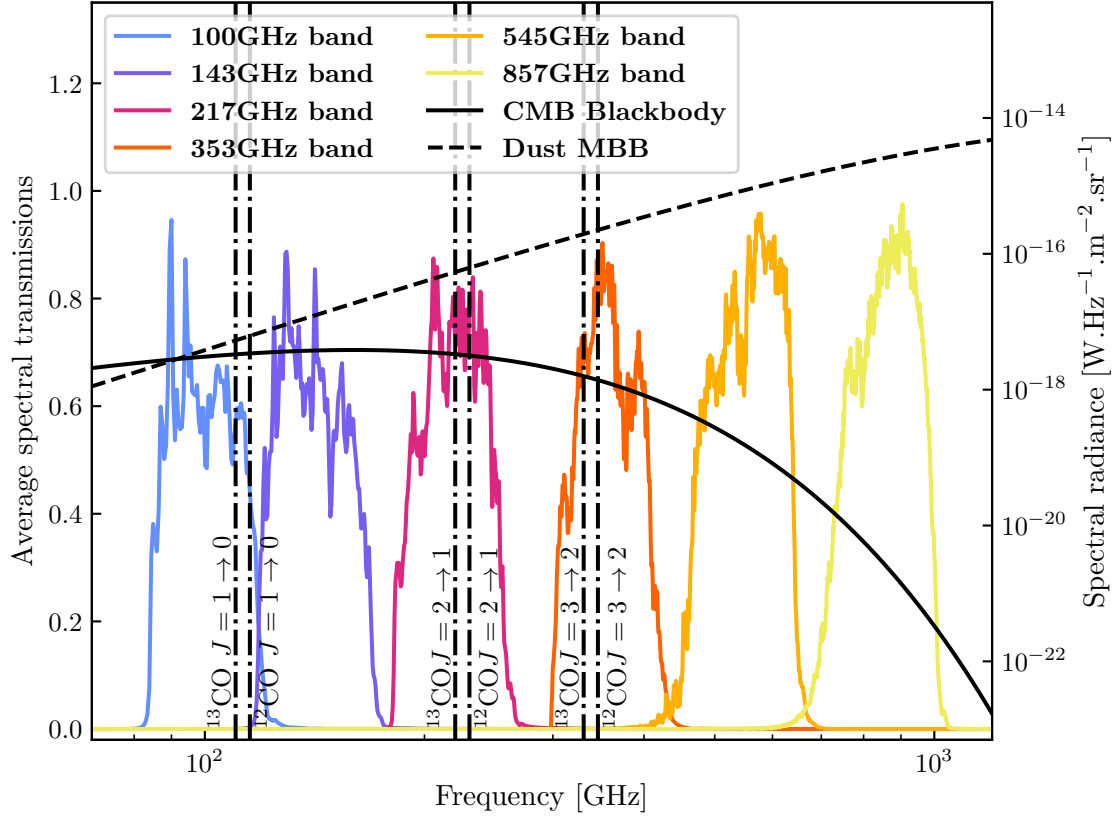


Fig. II.15 Plot of the average bandpasses of all the HFI frequency bands. I also display the CMB blackbody and thermal dust MBB spectral radiances, as well as the frequencies of the 3 first ^{12}CO and ^{13}CO rotational lines. CMB temperature is taken as $T_{\text{CMB}} = 2.72548\text{K}$ (cf. [Fixsen \(2009\)](#)). Dust temperature, spectral index and reference frequency are $T_{\text{dust}} = 19.5\text{K}$, $\beta_{\text{dust}} = 1.55$, and $\nu_{\text{ref}} = 857\text{GHz}$ (cf. [Planck Collaboration IV \(2020\)](#)). The dust MBB was plotted by taking an arbitrary value of $\tau_{\nu_{\text{ref}}} = 1$ in Eq. II.3.

II.4 CO maps from the *Planck* mission

Among the numerous products resulting from the *Planck* mission, the one most closely related to my work is the set of fullsky maps of the CO emission, constructed from the diffuse component separation applied on the HFI frequency maps. My work is thus revisiting the CO map building from *Planck* data, using SRoll2.1 frequency maps (see Section IV.1), and aims to improve the quality of those CO maps, especially in the high latitude and low column density regions of the sky, by developing a new method of construction, as well as by introducing the additional separation of the ^{12}CO and ^{13}CO isotopologues, which are **not** separated in the *Planck* collaboration products.

I present here an overall summary of the CO maps provided by the *Planck* legacy release, as these maps will be extensively used in the scope of my work as a reference for various characterization of the structure and quality of my later new ^{12}CO and ^{13}CO maps. The separation of the CO emission in the *Planck* data lead to the production of 3 types of CO maps, applied to produce maps of the 3 $J = 1 \rightarrow 0$, $J = 2 \rightarrow 1$, and $J = 3 \rightarrow 2$ lines. The full description of the CO maps construction in *Planck* is given in [Planck Collaboration XIII](#)

(2014). Note that the latest *Planck* CO maps correspond to the 2015 release, described in [Planck Collaboration X \(2016\)](#), where the maps were updated with the data produced from the latest state of the analysis and processing pipeline, but where the methods of separation are the same as those defined in the work of [Planck Collaboration XIII \(2014\)](#). For this reason, whenever I reference the *Planck* CO maps in this manuscript, I am always speaking of the maps produced in the 2015 release, yet I always reference the methods from [Planck Collaboration XIII \(2014\)](#).

I will first introduce the CO conversion coefficients and the way they are determined, as they are at the base of the CO emission separation. I will then summarize the 3 types of multi-algorithm separation schemes applied on the various *Planck* frequency maps, leading to the production of 3 types of CO maps.

II.4.1 CO conversion coefficient determination

The CO emission is generally expressed in velocity integrated brightness temperature unit K.km.s^{-1} (see subsection [I.4.2](#)). However, the signal collected in the *Planck* data is calibrated in sky temperature units, i.e. in K_{CMB} , and so we need to determine coefficients to convert the CO emission between these 2 units.

Thankfully, the CO spectral lines are extremely narrow compared to the HFI bolometers bandpasses, and can be approximated by a delta function at their respective transition frequencies (cf. [Planck Collaboration XIII \(2014\)](#)). This renders the CO emission highly sensitive to the wiggles visible in Figs. [II.3](#) and [II.5](#), and which were discussed in subsection [II.1.2.a](#)). This means that each of the HFI bolometers containing CO emission is characterized by a distinct response to this emission, which in turn spawns notable differences of CO conversion coefficients between bolometers even within the same frequency band. This difference in response between bolometers is at the base of the CO separation, exploiting the fact that the frequency maps containing the sky emission summed over all incoming sources observed the *same* fullsky CO emission but with *different* bolometer response to said emission.

The most straightforward way to determine these CO conversion coefficients is to compute them from the ground-based measurements of the bolometers spectral transmission, which can be expressed as (cf. [Planck Collaboration XIII \(2014\)](#)):

$$F_b^{CO, BP} = \frac{\int I_\nu^{CO} H_\nu^b d\nu}{\int I_\nu^{CMB} H_\nu^b d\nu}, \quad (\text{II.4})$$

where H_ν^b is the spectral transmission (also called bandpass) of bolometer b at frequency ν , and I_ν^{CO} and I_ν^{CMB} are respectively the intensities of the CO and the CMB. However, in [Planck Collaboration XIII \(2014\)](#), an improved method was employed for the determination of the CO conversion coefficients: they were fitted on the sky signal by using template maps of the emission.

Indeed, computing the conversion coefficients on the bandpasses present 2 major downsides: the uncertainty on the bandpass measurement, which then propagates to the coefficients computation, and the fact that the CO maps are here computed *without* separating the ^{12}CO and ^{13}CO isotopologues, which emit 2 distinct spectral lines at distinct frequencies, and are spatially correlated on the sky but with varying ratio of intensities. The workaround chosen to mitigate these issues was to estimate the CO conversion coefficients directly from sky measurements of well-known molecular clouds. To this end, assuming that these molecular cloud regions can be modelled as a linear combination of the CO and dust emission weighed by their respective conversion coefficient, and neglecting other astrophysical components (i.e. CMB, free-free, synchrotron, etc...), they computed a linear fit on each

bolometer map M_b using templates of the CO and dust emissions, which can be expressed as (cf. [Planck Collaboration XIII \(2014\)](#)):

$$M_b = F_b^{CO, fit} P^{CO} + F_b^{dust, fit} P^{dust}, \quad (\text{II.5})$$

where $F_b^{CO, fit}$ and $F_b^{dust, fit}$ are respectively the fitted CO and dust conversion coefficients, P^{CO} is the CO emission template map taken as the $^{12}\text{CO } J = 1 \rightarrow 0$ survey map produced by [Dame et al. \(2001\)](#), and P^{dust} is the dust emission template map taken as the 545GHz frequency map. While they assumed that the dust conversion coefficients are not affected by CO contamination significantly, i.e. their final dust conversion coefficients fitted on the sky are thus $F_b^{dust, sky} = F_b^{dust, fit}$, the CO conversion coefficients are expected to be affected by dust contamination because of the spatial correlation between the CO and dust emissions. They corrected for this contamination by computing the final CO conversion coefficients fitted on the sky as (cf. [Planck Collaboration XIII \(2014\)](#)):

$$F_b^{CO, sky} = \alpha F_b^{CO, fit} + \beta,$$

where α and β are fitted parameters (see [Planck Collaboration XIII \(2014\)](#) for more details). Additionally, in Eq.II.5 above, they combined the maps of PSB pairs into a single map, and a single CO conversion coefficient is computed for the pair of detectors, in order to avoid contamination from dust polarization emission.

The final determination of the CO conversion coefficients resulted in greatly reduced uncertainty compared to the one from their determination using the bolometers bandpasses. Their respective values can be found in the Table.2 of [Planck Collaboration XIII \(2014\)](#), and I reported their values in Figs.II.16 and II.17, respectively for the $J = 1 \rightarrow 0$ and $J = 2 \rightarrow 1$ CO lines. Note that in these figures the sky fitted conversion coefficients only appear for the ^{12}CO , caused by the absence of separation of the ^{12}CO and ^{13}CO isotopologues in the sky fit described above. I plotted, as was displayed in the reference figure, the CO conversion coefficients averaged over each PSB pairs, and individually for SWBs, and normalized by the average CO conversion coefficient along the considered frequency (either 100GHz or 217GHz) for both types of computations. These CO coefficients will later be useful for my work, as I can compare them to the equivalent CO coefficients determined by SRoll2.1 (see later Figs.IV.16 and IV.17), which are later used in my computation of the ^{12}CO and ^{13}CO maps.

II.4.2 Component separation

The CO maps from *Planck* were built using 3 different separation methods, resulting in 3 types of CO maps, incidentally called Type 1, Type 2, and Type 3 CO maps. Each computation method is based on a different model of the signal from the sky and exploits a different set of frequency maps, each method linked to its respective algorithm developed around its own sky emission model hypotheses.

I will summarize here the specificities of each computation method, and the main differences it implies between the resulting CO maps of different Type.

a) Type 1

The first approach, leading to the production of the Type 1 *Planck* CO maps, is based on the exploitation of a single frequency band, and is computed by the MILCA algorithm (Modified Internal Linear Combination Algorithm, see [Hurier et al. \(2013\)](#)). This method only uses the response differences between bolometers from a *single* frequency band: the

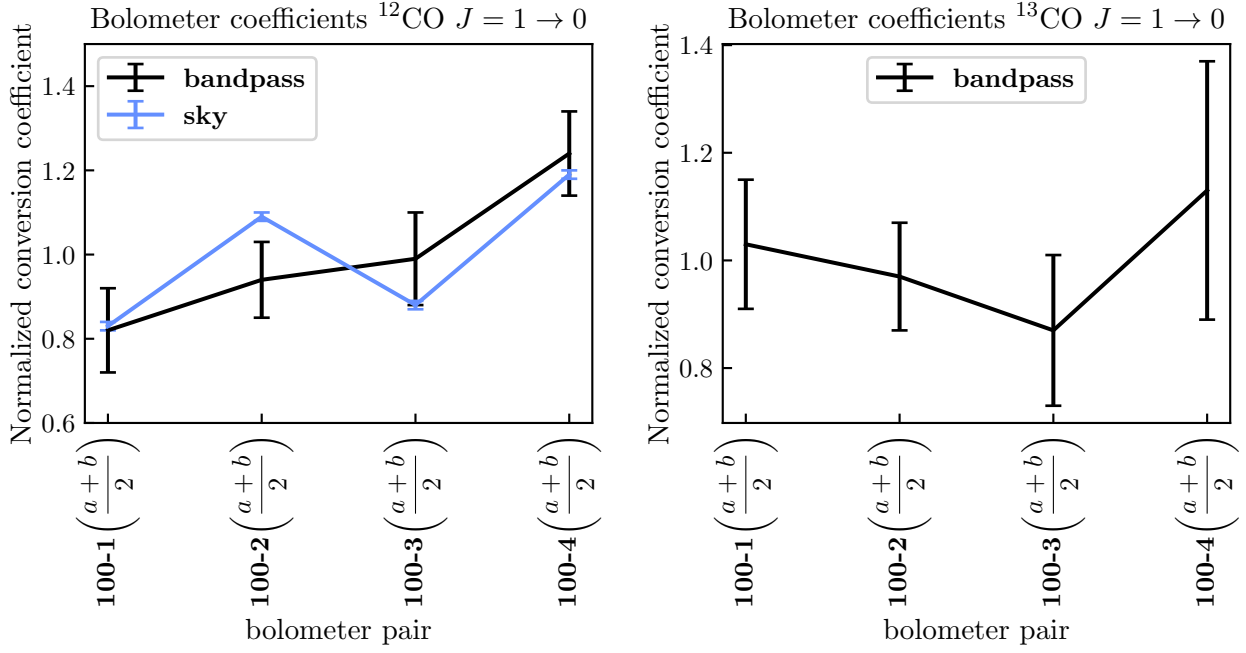


Fig. II.16 CO conversion coefficients for the HFI 100GHz band, i.e. the $J = 1 \rightarrow 0$ CO line. Coefficients from PSBs are averaged over the pair, and all coefficients are normalized by the corresponding average $\overline{F^{CO^b}}$ over the full frequency band. Comparison between the determination from the bandpass and the fit on sky signal. **Left:** ^{12}CO . **Right:** ^{13}CO . Adapted from fig.2 from [Planck Collaboration XIII \(2014\)](#).

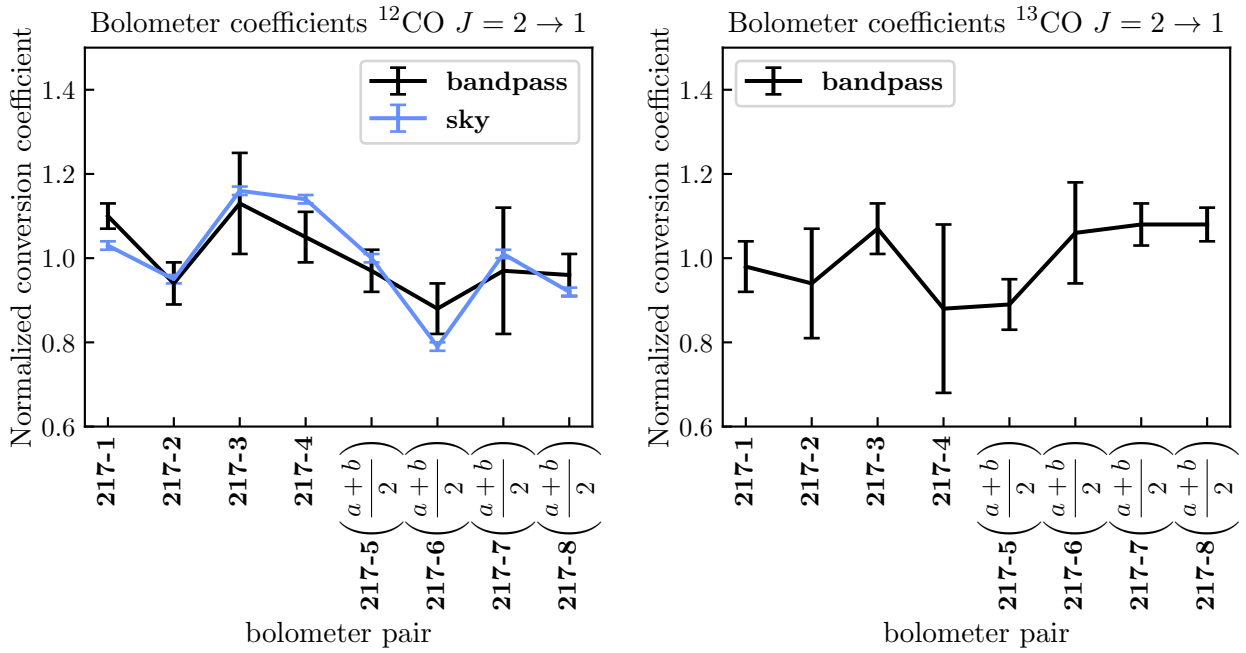


Fig. II.17 CO conversion coefficients for the HFI 217GHz band, i.e. the $J = 2 \rightarrow 1$ CO line. Coefficients from PSBs are averaged over the pair, and all coefficients are normalized by the corresponding average $\overline{F^{CO^b}}$ over the full frequency band. Comparison between the determination from the bandpass and the fit on sky signal. **Left:** ^{12}CO . **Right:** ^{13}CO . Adapted from fig.2 from [Planck Collaboration XIII \(2014\)](#).

100GHz band for the $J = 1 \rightarrow 0$ CO rotational line, the 217GHz band for the $J = 2 \rightarrow 1$ CO rotational line, and the 353GHz band for the $J = 3 \rightarrow 2$ CO rotational line.

In these conditions, the frequency maps $M_{\nu_{band},b}$ for each bolometer b from a frequency band ν_{band} can be modelled as a linear combination of the main sky source, i.e. the CMB, dust, and CO emission, which writes (cf. [Planck Collaboration XIII \(2014\)](#)):

$$M_{\nu_{band},b} = I^{CMB} + F_{\nu_{band},b}^{dust, sky} I_{\nu_{band}}^{dust} + F_{\nu_{band},b}^{CO, sky} I_{\nu_{band}}^{CO} + noise.$$

Assuming that the dust emission does not vary across bolometers (i.e. $F_{\nu_{band},b}^{dust, sky}$ does not depend on b), the difference between two bolometer maps gives directly an estimate of the CO emission. In this case, based on the same idea but still using component separation techniques to regularize the system and account for uncertainties, they exploit all differences between bolometer maps to extract the CO emission.

The main advantages of this approach are:

- The resulting maps are at a beam corresponding to the average scanning beam along the considered frequency, i.e. at the native angular resolution of HFI (see [Table II.1](#)),
- It avoids the risk of contaminants from other frequencies or biases introduced by additional hypotheses on the sky emission shapes and SED.

However, since this method uses a small number of individual bolometer (bolometer pair) maps, this results in a lower SNR (cf. [Planck Collaboration XIII \(2014\)](#)).

I displayed in the top row of [Fig. II.18](#) the Type 1 CO maps of the $J = 1 \rightarrow 0$ and $J = 2 \rightarrow 1$ rotational lines.

b) Type 2

The second approach, leading to the production of the Type 2 *Planck* CO maps, is based on the exploitation of multiple frequency bands, but still mapping only **one** CO rotational line. It is computed by the Ruler algorithm (see [Planck Collaboration XII \(2014\)](#)). This method uses the data from multiple frequencies but only **one** of them containing CO emission: either the 100GHz band for the $J = 1 \rightarrow 0$ CO rotational line, or the 217GHz band for the $J = 2 \rightarrow 1$ CO rotational line.

Here, the frequency maps from other frequency bands are used to better identify and separate the other astrophysical sources from the CO: in addition to the frequency band containing the CO emission, the 70GHz band from LFI, as well as the HFI 143GHz and 353GHz bands are used. In these condition, it is assumed that the CO $J = 3 \rightarrow 2$ rotational line is negligible in order to use the 353GHz band to identify other components, typically the dust. This is why a Type 2 CO $J = 3 \rightarrow 2$ line map **cannot** be constructed using this approach. For both mapped lines, the requirement is to extract CO while at the same time fitting for the CMB, dust, and free-free emissions (cf. [Planck Collaboration XIII \(2014\)](#)).

The main advantage of this approach is a great improvement of the maps' SNR while still mapping a *single* CO rotational line. However, since this method uses multiple frequency bands and needs to model the different emissions (i.e. the SED) of the other astrophysical sources, this method is more subject to contamination, systematic errors and biases.

I displayed in the bottom row of [Fig. II.18](#) both Type 2 CO maps.

c) Type 3

The third and last approach, leading to the production of the Type 3 *Planck* CO map, is based on the exploitation of multiple frequency bands, **combining all 3 first** CO rotational

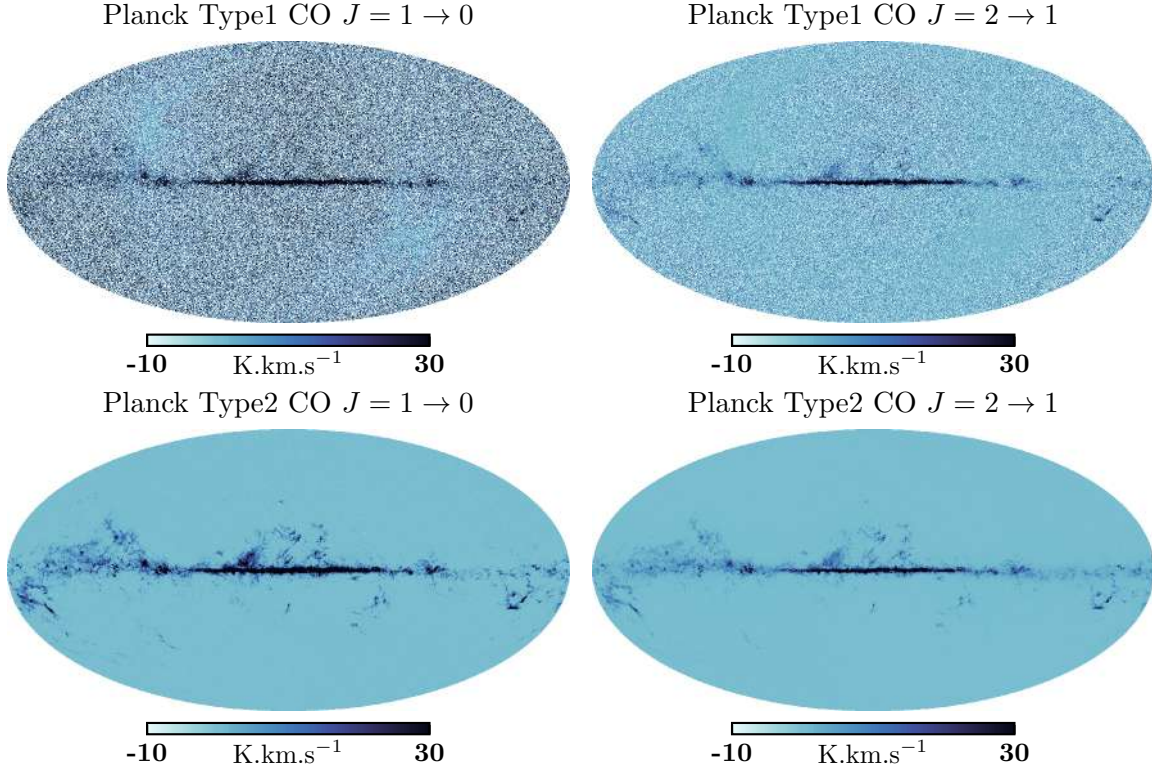


Fig. II.18 *Planck* CO fullsky maps from the 2015 release (*Planck Collaboration X (2016)*). **Top:** Type 1 maps; **Bottom:** Type 2 maps. **Left:** $J = 1 \rightarrow 0$ CO rotational line; **Right:** $J = 2 \rightarrow 1$ CO rotational line.

lines to build a composite CO map of the 3 $J = 1 \rightarrow 0$, $J = 2 \rightarrow 1$, and $J = 3 \rightarrow 2$ lines. It is computed by the Commander algorithm (see *Planck Collaboration XII (2014)*). This method uses the data from all 3 LFI frequency bands as well as the first 4 100-143-217-353GHz HFI bands at the same time, assuming model emission from the other foreground components, and also assuming uniform line ratios $\frac{CO(2 \rightarrow 1)}{CO(1 \rightarrow 0)}$ and $\frac{CO(3 \rightarrow 2)}{CO(1 \rightarrow 0)}$ for the CO emission. This implies that the CO emission has the *same spatial distribution* for all rotational lines.

This method achieves the best SNR, but is the most susceptible to contamination, systematic errors and biases. For this reason, and since my ^{12}CO and ^{13}CO maps' computation method is far too different from this one, I decided not to use this Type 3 map for comparison to qualify my resulting maps, i.e. I do not use the *Planck* Type 3 map in this work.

II.4.3 ^{13}CO contamination in the *Planck* CO maps

As previously mentioned, it should be noted that the CO maps produced for the *Planck* legacy release did not separate the ^{12}CO and ^{13}CO isotopologues. Therefore, their CO maps are expected to contain some ^{13}CO signal. The evaluation of the degree of contamination of the *Planck* CO maps by ^{13}CO signal was computed in *Planck Collaboration XIII (2014)*. Notably, they computed the multiplicative factor of the ^{13}CO signal (in K.km.s^{-1}) present in the *Planck* CO maps, due to the HFI bandpass at the ^{13}CO lines frequency. This can be written as:

$$M_{line}^{Type\ i} = M_{line}^{12CO} + X_{line}^{Type\ i} \times M_{line}^{13CO}, \quad (\text{II.6})$$

where $M_{line}^{Type\ i}$ is the *Planck* CO map of Type $i \in \{1, 2\}$ for the line $line \in \{J = 1 \rightarrow 0, J = 2 \rightarrow 1\}$, M_{line}^{12CO} and M_{line}^{13CO} are respectively the ^{12}CO and ^{13}CO emission maps of line $line$

(in K.km.s⁻¹), and $X_{line}^{Type\ i}$ is the aforementioned multiplicative factor of the ¹³CO signal of Type i and line $line$.

The following values can then be found in [Planck Collaboration XIII \(2014\)](#): $X_{J=1\rightarrow0}^{Type1} = 0.53$, $X_{J=1\rightarrow0}^{Type2} = 1.14$, $X_{J=2\rightarrow1}^{Type1} = 0.01$, and $X_{J=2\rightarrow1}^{Type2} = 0.82$. I will use them extensively when comparing my maps to the Type 1 and Type 2 *Planck* CO maps, as well as when comparing them to state-of-the-art external CO surveys (see Section [V.1](#)).

Chapter III

SRoll mapmaking algorithm

Though my PhD work aims at the construction of new ^{12}CO and ^{13}CO maps from the *Planck* data, the method I devised for said construction, later detailed in Chapter IV, is entirely based on the exploitation of the outputs of the *Planck* legacy mapmaking algorithm: SRoll (see [Planck Collaboration Int. XLVI \(2016\)](#), [Planck Collaboration III \(2020\)](#), and [Delouis et al. \(2019\)](#)).

This chapter aims then to present the SRoll mapmaking algorithm. I start with the nature and goal of such a mapmaking algorithm, which is the last step of the *Planck* data analysis pipeline to produce frequency maps, as was presented in Section II.3. After describing its generic mode of operation, I discuss how it came to be developed to extend and improve the *Planck* data analysis, what it aims to correct, and how its various versions evolved from one generation of the algorithm to the other. Then I detail the structure of the algorithm and how it operates in the case of the SRoll2 version, version on which a few modifications were added to produce the ^{12}CO and ^{13}CO maps resulting from this work, as presented in the next chapter. Finally, I introduce the simulations, taken from the official *Planck* simulations and slightly modified for the characterization of SRoll and its output.

This chapter aims to introduce the SRoll algorithm in detail, or at least extensively enough to give the reader a complete view of all aspects of the SRoll algorithm which are crucial in the scope of this work. Note that this chapter, and especially Section III.2, will be fairly technical, and probably the most strenuous part of this manuscript. I tried to make it as clear and pleasant as possible, and in doing so I also chose to make some elements of my presentation repetitive. Be advised that the elements, terms, or concepts on which the reader may feel that I ponder redundantly are *key* to the overall understanding of this work and the associated results.

III.1 Basic nature, objective, and history

The SRoll algorithm is the last step in the *Planck* data processing pipeline, as can be seen on Fig. II.10. This algorithm was first used and introduced in [Planck Collaboration III \(2020\)](#), with the initial central goal of improving the polarization part of the signal, and was then improved and/or modified in its use at multiple occasions [Delouis et al. \(2019\)](#); [Pagano et al. \(2020\)](#); [Delouis, J.-M. et al. \(2021\)](#). The present work is based on the outputs of a modified version of this algorithm, specially adapted to our central goal : the production of new and improved fullsky ^{12}CO and ^{13}CO emission maps.

This section aims to introduce the general aspects of the SRoll mapmaking algorithm : its main structure and objectives, as well as how it operates. The main philosophy of SRoll,

as well as its main characteristics common to all generations are presented in subsection III.1.1. Then the successive generations of the algorithm, performed before or parallel to this work are presented in subsection III.1.2.

III.1.1 Philosophy of the mapmaking algorithm

The goal of the SRoll algorithm is basically to *process* the observational data of a signal which has been observed redundantly (multiple times and/or by multiple similar instruments) to correct for the instrumental systematics affecting it. The idea is to use a model of the expected instrumental systematics, then exploit the redundancy of the observations to fit these systematics in the model in order to remove them from the data, and finally output the sought cleaned signal.

Generic description of SRoll

The general structure of the SRoll data model, later fully detailed in Eq. III.8 for the SRoll2 version of the algorithm, can be summarized as follows:

$$M = Px + \mathcal{N} + n, \quad (\text{III.1})$$

where M is one of the observations* input to SRoll, x is the sought signal*, P is a projection matrix, $\mathcal{N}$ is the model accounting for the instrumental systematics impacting the signal in the observation M , and n is a white noise. $\mathcal{N}$ therefore contains all the parameters to be fitted by SRoll. The projection matrix P only serves to describe the projection of the sought signal x into the shape of the specific observation data M , and it can take any form as long as it is *known*. Since the observation M is a scalar, of dimension 1×1 , it forces $\mathcal{N}$ and n to also be of same scalar dimension, and if the signal x is a vector of dimension $N_x \times 1$, then P is of dimension $1 \times N_x$.

To express the use of the full extent of data redundancy available, we define the following *collections*: $\mathbb{M} = (M_1, M_2, M_3, \dots)$ the collection of observation data used as input, $\mathbb{N} = (\mathcal{N}_1, \mathcal{N}_2, \mathcal{N}_3, \dots)$ the collection of the modelled instrumental systematics corresponding to all the observations of the signal x , $\mathbb{P} = (P_1, P_2, P_3, \dots)$ the collection of the known projectors P corresponding to the projection of the signal x onto each of its individual observation and $\mathbf{n} = (n_1, n_2, n_3, \dots)$ the collection of white noises affecting each observation. Using the above equation (Eq. (III.1)), we can write:

$$\begin{aligned} P(\mathbb{P}^\top \mathbb{P})^{-1} \mathbb{P}^\top \mathbb{M} &= P(\mathbb{P}^\top \mathbb{P})^{-1} \mathbb{P}^\top \mathbb{P} x + P(\mathbb{P}^\top \mathbb{P})^{-1} \mathbb{P}^\top \mathbb{N} + \hat{n} \\ &= Px + P(\mathbb{P}^\top \mathbb{P})^{-1} \mathbb{P}^\top \mathbb{N} + \hat{n}, \end{aligned} \quad (\text{III.2})$$

with $\mathbb{P}^\top$ being the transpose of the matrix $\mathbb{P}$, and $\hat{n}$ a *coloured* noise of covariance matrix $(\mathbb{P}^\top \mathbb{P})^{-1}$, which corresponds to $\hat{n} = P(\mathbb{P}^\top \mathbb{P})^{-1} \mathbb{P}^\top \mathbf{n}$.

From equations (III.1) and (III.2), we can obtain a relation between the quantities $R = M - P(\mathbb{P}^\top \mathbb{P})^{-1} \mathbb{P}^\top \mathbb{M}$ and $\mathcal{R} = \mathcal{N} - P(\mathbb{P}^\top \mathbb{P})^{-1} \mathbb{P}^\top \mathbb{N}$, respectively called the *data* vector

*We are placing ourselves in the most generic case of application of SRoll. In this setting, the signal x being redundantly observed by many measurements M can be anything, from the elevation map of an ocean's surface captured from space, to the integrated total power radiating from a star, or a spectroscopic survey of a supernova remnant. M is defined as a scalar (though it may depend on other parameters, such as time, position, etc...), and x can be of any dimension, from a scalar to a multi-dimensional matrix.

Table III.1: Dimensions of all quantities appearing in the equations representing the generic description of SRoll.

Quantities	Notation
$M, \mathcal{N}, n, \hat{n}, R, \mathcal{R}$	scalar (1×1)
$x, \tilde{x}, n_x$	$N_x \times 1$
P	$1 \times N_x$
$\mathbb{M}, \mathbb{N}, \mathbf{n}$	$N_{hit} \times 1$
$\mathbb{P}$	$N_{hit} \times N_x$
$\mathbb{P}^\top$	$N_x \times N_{hit}$
$\mathbb{P}^\top \mathbb{P}$	$N_x \times N_x$

residual and the *model* vector residual, which writes as follows:

$$M - P(\mathbb{P}^\top \mathbb{P})^{-1} \mathbb{P}^\top \mathbb{M} = \mathcal{N} - P(\mathbb{P}^\top \mathbb{P})^{-1} \mathbb{P}^\top \mathbb{N} + n - \hat{n} \Leftrightarrow R = \mathcal{R} + n - \hat{n}. \quad (\text{III.3})$$

This relation is sufficient to define the backbone of SRoll’s minimization: it states that the difference between one observation of the signal and the same observation averaged over the whole collection of observations, i.e. $M - P(\mathbb{P}^\top \mathbb{P})^{-1} \mathbb{P}^\top \mathbb{M}$, should be equal to the difference between the corresponding realisation of the instrumental systematics and the same realisation averaged over the whole collection of observations, i.e. $\mathcal{N} - P(\mathbb{P}^\top \mathbb{P})^{-1} \mathbb{P}^\top \mathbb{N}$, up to the level of noise in the data. This is fully developed for the SRoll2 version in later Eqs.III.21, III.23, and III.24. Therefore, the algorithm minimizes the quadratic difference between R and $\mathcal{R}$ for the whole collection of observations M , by fitting all the parameters defined in $\mathcal{N}$.

Finally, from this determination of $\mathcal{N}$, SRoll can evaluate the signal x using the relation obtained from Eq.(III.2):

$$\tilde{x} = (\mathbb{P}^\top \mathbb{P})^{-1} \mathbb{P}^\top (\mathbb{M} - \mathbb{N}), \quad (\text{III.4})$$

where $\tilde{x}$ is the output evaluation of the signal x , impacted by noise and errors, which can therefore also be written:

$$\tilde{x} = x + n_x,$$

with $n_x = (\mathbb{P}^\top \mathbb{P})^{-1} \mathbb{P}^\top \mathbf{n} + err_{residual}$ being a noise comprised of the sum of the noise $(\mathbb{P}^\top \mathbb{P})^{-1} \mathbb{P}^\top \mathbf{n}$ and the residual $err_{residual}$ stemming from $(\mathbb{P}^\top \mathbb{P})^{-1} \mathbb{P}^\top (\mathbb{M} - \mathbb{N})$ due to any error in the instrumental systematics model $\mathcal{N}$. The reconstruction of the *cleaned* signal defined by above Eq.III.4 is expressed in details for the SRoll2 version in Eqs.III.27 and III.28.

Taking the total number of considered observations as N_{hit} , the respective dimensions of all the terms from the equations above are given in Table.III.1.

Closure condition requirement Given the nature of the minimization computed by SRoll, as defined by Eq.III.3, it bears emphasis that the parameters determined by SRoll are only fitted on *data differences*, thereby allowing the algorithm to only determine the parameters *relative distribution* from the available observations, and **not** the *absolute* reference value from which this distribution is defined. Indeed, let the set of $\{\alpha_k\}_{k \in \llbracket 1, N_\alpha \rrbracket}$ define a set of parameters fitted by SRoll, appearing in the instrumental systematics model $\mathcal{N}$, and depending on an index k (this index k depends on the observation, and can span any

instrumental characteristic set, e.g. it can span the set of instruments redundantly observing the signal x with varying responses). Then, following the minimization defined in Eq.III.3, SRoll fits the following differences: $\alpha_k - \alpha_{ref}$, with α_{ref} a reference for the whole set of α_k , which depends on the structure of the collection $\mathbb{M}$, therefore function of a linear combination of the α_k , but of **unknown absolute** value. This definition of the minimization, by fitting *differences*, provides us with N_α equations for $N_\alpha + 1$ unknowns (the α_k and α_{ref}), but since α_{ref} is function of a linear combination of the α_k , this means that SRoll provides us with $N_\alpha - 1$ *independent* equations for N_α unknowns. This leaves us with 2 possibilities:

1. Find an exterior source of information allowing us to determine the value of α_{ref} ,
2. Make an *arbitrary* hypothesis on this value of α_{ref} .

In any case, an additional equation, setting the value of α_{ref} , must be added in SRoll to reach N_α *independent* equations for N_α unknowns and allow the minimization to be computed.

It bears emphasis that when no *absolute* calibrator for the reference value is available, i.e. second case defined above, the closure condition defined to set the value of the reference is purely *arbitrary* in the sense that this closure condition only defines the **average distance between the reference value and the α_k** . This numerically defines the value of the reference, but **without knowing** its *absolute* value. As will be demonstrated in subsection III.2.2, since the closure condition defines the *average distance between the reference value and the α_k* , setting this distance to zero equates to setting the reference value as the average value over all the α_k , but still *without knowing* its *absolute* value.

The definition of this *closure condition*, defining the taken reference value for a set of parameters to be fitted by SRoll, will be detailed in subsection III.2.2 in the case of the SRoll2 version of the algorithm, and redefined in subsection IV.1.3.c) for the new SRoll2.1 version constructed in the scope of this manuscript and geared towards the production of new ^{12}CO and ^{13}CO maps.

In the context of this work, SRoll is applied on the *Planck* mission, where M corresponds to the TOIs measured by the considered bolometers and pre-processed as described in Section II.3, as well as compressed into HPRs. I describe in the present section the main elements of instrumental systematics that SRoll was developed to correct, introducing their respective nature and the way they impact the data. The full data model and minimization procedure, where the structure and content of the term $\mathcal{N}$ is detailed in the case of the *Planck* mission, are then presented in Section III.2.

As *Planck*'s mapmaking algorithm, SRoll uses as its main input the HPRs of the full *Planck* mission data signal and projects this compressed time signal on the sky to output fullsky frequency maps, one per bolometer, using the HEALPix pixelisation scheme of the sphere Gorski et al. (2005). SRoll was developed to reduce a set of remaining systematic errors impacting the polarized signal (especially at large angular scales) in the HFI frequency maps from the 100, 143, 217, and 353 GHz bands. Fitting and correcting these remaining systematic errors required some additional data analysis steps, constituting the core of the SRoll algorithm.

To correct the targetted systematic errors, SRoll exploits the redundancy in the data, stemming from the same sky being observed multiple times by different bolometers with varying orientations. Using this redundancy, SRoll extracts knowledge from the data to

improve the instruments' characterization compared to the pre-launch ground-based measurements of said characterization. This information is extracted by fitting key parameters of its data model, such as the detectors response to different emissions of varied SED. This fitting is based on the comparison of each individual observation of a pixel to the average over all the observations of that pixel by the HFI instrument. Indeed, a single bolometer observes each pixel multiple times with varying angle (changing the reception of the polarized fraction of signal power in the case of PSBs), and each pixel is also measured redundantly by several bolometers along a frequency band. The details of the SRoll data model and the minimization it computes are described in Section III.2 for the second generation of its implementation, called SRoll2, and introduced first in the work of Delouis et al. (2019). Only this version of the algorithm will be detailed in depth as it is the one I expanded and modified to achieve the objectives we set in this work.

In the following subsections III.1.1.a), b), c), d), e) I present the aforementioned systematic errors corrected by SRoll. In each case, I discuss the nature of the error and how it impacts the data as well as the generic way to correct it. The specifics of the fitting and cleaning steps will be later detailed in subsections III.2.2 & III.2.3 in the case of the SRoll2 version of the algorithm.

In order to fully exploit the polarization in the HFI data, it was paramount to reduce the effect of signal intensity-to-polarization leakage, i.e. non-polarized intensity signal leaking in the polarized part of the final frequency maps. This leakage has 3 main sources:

- ◊ The *photometric calibration* mismatch between detectors, discussed in subsection III.1.1.a),
- ◊ The *bandpass* mismatch between detectors, discussed in subsection III.1.1.c),
- ◊ The relative error on the bolometers' monopole determination leading to monopole mismatches between bolometers.

However, the bolometers monopoles cannot be extracted from *Planck* data alone as the HFI detectors are only sensitive to differential incoming power from the sky. In our work, these monopoles are those determined in the 2015 *Planck* release, as they were unchanged in the 2018 *Planck* release and also used in the work of Delouis et al. (2019). Nevertheless, the error on the monopoles very weakly impacts the construction of the ^{12}CO and ^{13}CO emission maps, as will be detailed in subsection IV.3.3. Therefore, achieving a reduction of the intensity-to-polarization leakage required to improve the determination and correction of the *photometric calibration* and *bandpass* mismatch between detectors.

Furthermore, just as the 3 aforementioned mismatches between bolometers (calibration, bandpass, and monopole) induce intensity-to-polarization leakage, 2 other types of leakage can result from errors on the bolometers' characterisation: a polarization-to-intensity and a cross-polarization leakage, the latter referring to signal leaking from one polarization Stokes parameter to the other (i.e. from Q to U or from U to Q, see Appendix.E for more details on the Stokes parameters). These 2 types of leakage can result from an error in the determination of the bolometers' polarization efficiency, as well as the bolometers' orientation, also called bolometers' polarization angle (cf. Rosset et al. (2010)). This is why, in the SRoll2 version of the mapmaking algorithm, a correction is fitted for these 2 parameters (bolometers' polarization efficiency and orientation).

a) Absolute photometric calibration from the orbital dipole

One of the key parameters determined by SRoll for each bolometer is the absolute response of the bolometer chain, generally referred to as the absolute **gain** of said bolometer. The re-

sponse of the bolometer chain corresponds to the detection, transmission, amplification, and conversion of all radiation collected from the sky and registered in the *Planck* HFI database, as described in subsection II.1.3. This *gain* comprises the absolute photometric calibration of the bolometer in sky temperature units, called K_{CMB} (see subsection II.3.2.b)). The mismatch of this calibration between bolometers is one of the main origins of the leakage of signal from intensity to polarization, induced on the *whole* signal. In addition, the gain determination is degenerate with the non-linear effect of the ADC non-linearity (ADCNL) (discussed below), forcing these 2 determinations to be performed in separate loops in the algorithm as detailed in subsection III.2.1.c).

In the 2018 *Planck* legacy release, the photometric calibration of the signal is fitted with the gain determination within the SRoll mapmaking data model. This calibration is fitted on the time dependant signal of the orbital dipole, as detailed in subsection II.3.2.b).

For reasons in part presented in subsection II.3.2.b), the 545GHz and 857GHz frequency bands were not treated by SRoll, and are processed as was previously done in the *Planck* legacy pipeline (cf. [Planck Collaboration VIII \(2016\)](#), [Planck Collaboration III \(2020\)](#)). More precisely, a calibration of the 545GHz band using the orbital dipole was attempted using the SRoll2 version, but the resulting accuracy (relative error of $\sim 5\%$) was worse than the one obtained from the photometric calibration on planets (relative error of $\sim 2\%$).

b) Analogue-to-digital converter non-linearity (ADCNL)

As first mentioned in subsection II.1.3, the ADCs display a non-linearity of their ADU distribution along the range of the analogue voltage to be digitized. This is visible in Fig.III.1, where I reported the errors on the transition code positions measured on a spare ADC chip around the mid-scale, which is the most populated area by the voltage analogue signal. The aforementioned transition code error corresponds to the error on the ADC response, for each ADU, with respect to an *ideal* perfectly linear ADC. We can see that a major non-linearity is displayed at the mid-scale, as well as errors of about 1 ADU visibly occurring regularly every 64 steps, which can be either positive or negative (for another visualization of the ADCNL, see Fig.2 from [Planck Collaboration VII \(2016\)](#)).

In order to precisely correct all the data for this effect, accurate measurements of the ADCNL along the full ADU range of all ADCs', together with a good model for the bolometer raw signal (including systematics) is required (cf. [Planck Collaboration VIII \(2014\)](#)). This was not carried out on the ground, since this effect was not originally expected to be impactful, and the in flight mapping of the ADC response is limited.

The first correction of this effect, in the first module of the data processing pipeline described in subsection II.3.1 was not sufficient, and was by far the dominant residual systematic effect in the frequency maps. In the *Planck* 2018 legacy release, the residual effect was additionally fitted as time variations of the bolometers' gain in SRoll. This improved correction decreased the residuals of the ADCNL effect in the frequency maps, but this effect remained the dominant residual systematic. The first ADCNL correction in the data processing pre-SRoll was then discarded in the second implementation of the mapmaking algorithm, SRoll2, in which this systematic effect is fully fitted as a full nonlinear correction per detector and per ring. This fit of the ADCNL effect in SRoll2 is based on a *bold* hypothesis: the idea is that after correcting all other effect accounted for in the SRoll2 data model, any remaining residual effect on the discrepancy between bolometers signal is **entirely caused** by the ADCNL. The ADCNL is then iteratively fitted, and the gains (now fixed in time) in the data model are expected to converge towards a stable value *only* if the aforementioned hypothesis is valid. Details for this iterative fitting procedure

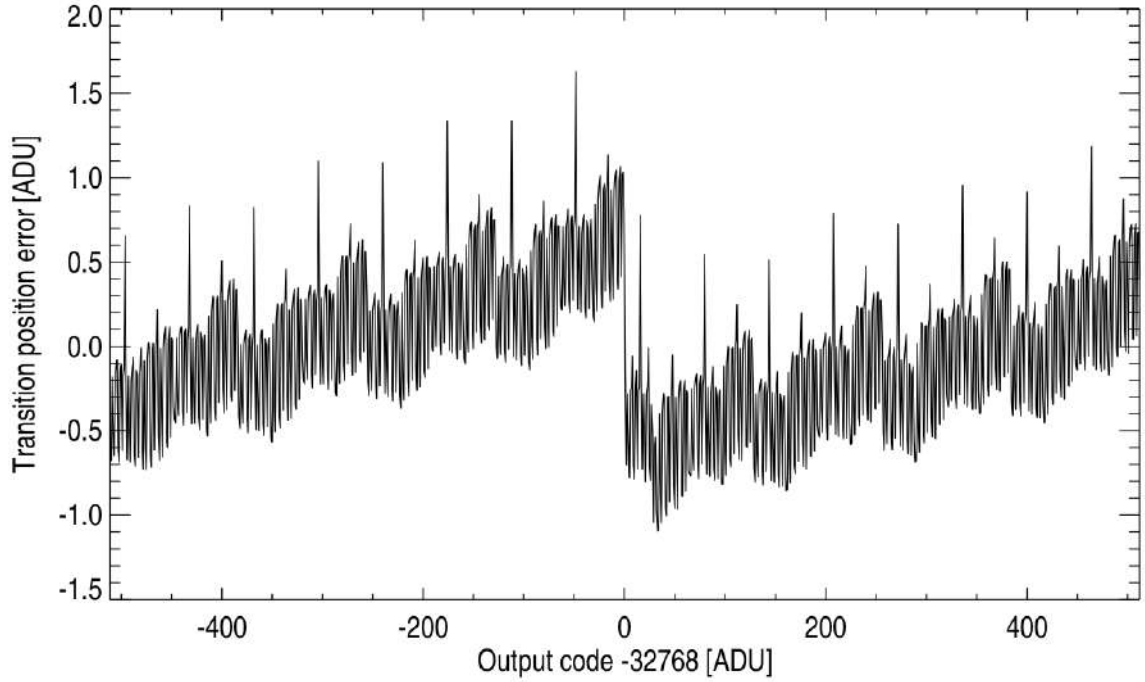


Fig. III.1 Error on transition code positions measured, on the ground, on one spare ADC chip around the ADC mid-scale. The largest error occurs at the sign transition, but errors of about 1 ADU also occur regularly every 64 steps. Taken from [Planck Collaboration VIII \(2014\)](#), Fig.2.

in SRoll2 are presented in subsection III.2.1.c). It proved to be the case, which allowed a radical improvement of the ADCNL effect in the data, as seen in later subsection III.2.4.

c) Foreground bandpass mismatch

A second main source of signal leakage from intensity to polarization is the *bandpass* mismatch between bolometers. This causes an intensity-to-polarization leakage essentially comprised of the foreground emission components (dominated by the thermal dust and CO emission at the 100-143-217-353 GHz bands), whose emission spectrum differs from that of the CMB, on which the signal of *each* bolometer is calibrated. This bandpass mismatch is visible on Fig. II.3, II.4, and II.5 presented in subsection II.1.2.a). It is shown further in Figs. III.2 and III.3, where I plotted the bandpass mismatches between the bolometers respectively from the 100GHz and 217GHz bands, by computing the spectral transmission of each bolometer divided by the average transmission along the considered frequency band, i.e. respectively plotting $H_\nu^b/\overline{H}_\nu^{100}$ and $H_\nu^b/\overline{H}_\nu^{217}$.

At any HFI Frequency, the physical sources composing the total signal received by any bolometer b can be described as the sum of the CMB anisotropies, the total dipole, and the foreground emissions, namely: the dust (dominated by the thermal dust), CO lines (absent only from the 143 GHz band), free-free, and synchrotron emissions. This total signal from bolometer b , and its decomposition into the sum of its contributions, can then be written:

$$S_b^{tot} = S^{CMB} + D^{sol} + D^{orb} + \sum_f L_b^f S^f,$$

where S^{CMB} is the signal of the CMB anisotropies, S^f is the signal of the foreground f

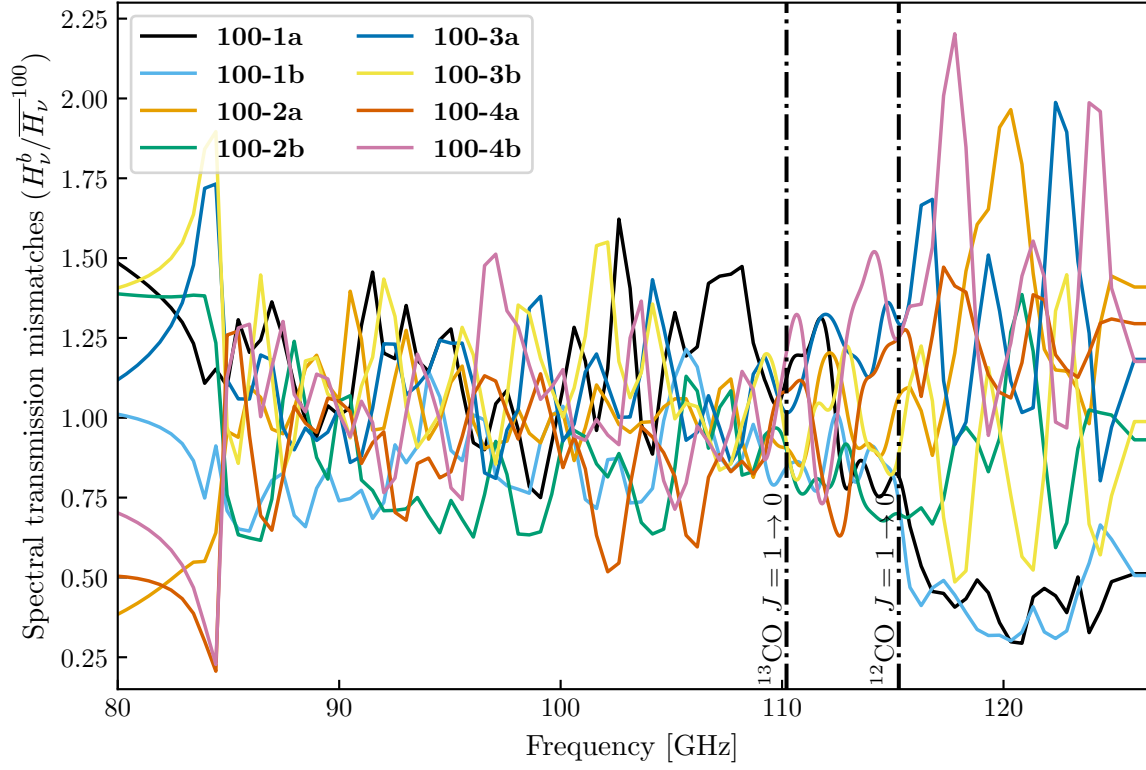


Fig. III.2 Bandpass mismatches between bolometers from the 100GHz band. It is computed as the ratio between each bolometer's spectral transmission and the average transmission along the band. The positions of the ^{12}CO and ^{13}CO lines are also represented.

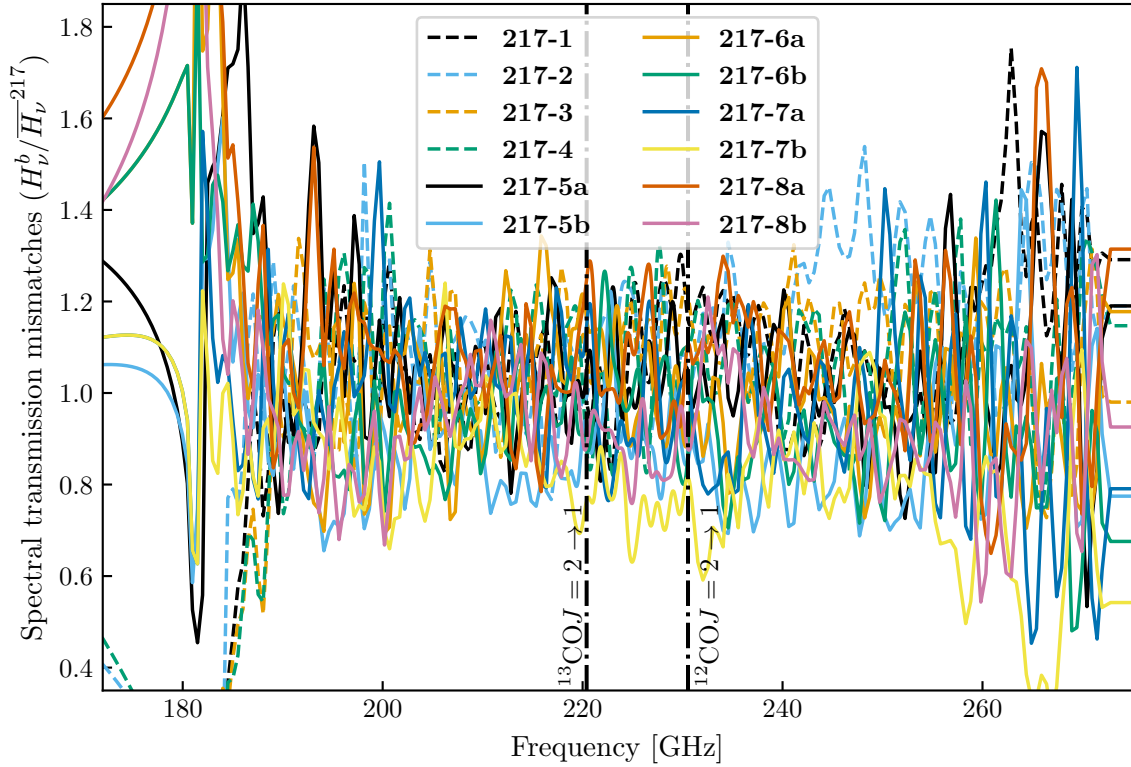


Fig. III.3 Bandpass mismatches between bolometers from the 217GHz band. It is computed as the ratio between each bolometer's spectral transmission and the average transmission along the band. The positions of the ^{12}CO and ^{13}CO lines are also represented.

emission with $f \in \{\text{dust}, {}^{12}\text{CO}, {}^{13}\text{CO}, \text{free-free}, \text{synchrotron}\}$, D^{sol} and D^{orb} are the solar and orbital dipoles, and L_b^f is the bolometer response coefficient to the foreground f emission. Since the intensity of a signal detected by a bolometer is expressed in units of $[\text{K}_{\text{CMB}}]$, that means that S_b^{tot} is also in units of $[\text{K}_{\text{CMB}}]$. Therefore, using the equation above, we obtain that for a foreground emission signal expressed in units of $[\text{X}]$, the associated bolometer response coefficient's unit would be $[\frac{\text{K}_{\text{CMB}}}{\text{X}}]$, e.g. for the ${}^{12}\text{CO}$ emission usually expressed in brightness temperature units of $[\text{K.km.s}^{-1}]$, the unit of the bolometer response coefficient $L_b^{{}^{12}\text{CO}}$ is $[\frac{\text{K}_{\text{CMB}}}{\text{K.km.s}^{-1}}]$.

From this definition, the value of the bolometer response coefficient to the foreground f emission, from any of the 100-143-217-353GHz frequency bands, can then be expressed as a generalisation of Eq.II.4, leading to (cf. Planck Collaboration XIII (2014), Planck Collaboration IX (2014)):

$$L_b^f = \frac{\int I_\nu^f H_\nu^b d\nu}{\int I_\nu^{\text{CMB}} H_\nu^b d\nu}, \quad (\text{III.5})$$

where H_ν^b is the spectral transmission of bolometer b , and I_ν^f is the intensity of the foreground f signal at frequency ν (cf. Fig.II.15). The division by the term $\int I_\nu^{\text{CMB}} H_\nu^b d\nu$ stems from the data having been calibrated in units of K_{CMB} . The foreground bandpass mismatch for any foreground f therefore is characterized by a difference between the L_b^f for each value of the index b , stemming from the integrated differences between the H_ν^b , as can be seen on Figures II.3, II.4, and II.5.

The *realignment* procedure SRoll fits these foregrounds bandpass mismatches for all main foregrounds: the thermal dust, CO (a single CO template at first in SRoll1, then the ${}^{12}\text{CO}$ and ${}^{13}\text{CO}$ are separated in following versions, cf. subsection III.1.2), free-free, and synchrotron emissions. It then corrects for this effect for each foreground by bringing all bolometer responses to the same response along the frequency band, before projecting the input HPRs into fullsky frequency maps. This means that in the resulting frequency maps, for each foreground, all bolometers in the considered frequency band have been brought to the same foreground response coefficient L_{ref}^f , which is the reference value for the L_b^f , defined by its corresponding *closure condition*, as discussed above. I call this procedure of bringing all bolometers' response coefficients to the same average value: ***realignment***.

Since SRoll only fits the foreground bandpass **mismatches**, it only works on *differences* between bolometers. A significant consequence of this way of extracting knowledge on the HFI instruments from the data is, as previously mentioned, that for these parameters the *average value* is **not accessible** (which is equivalent to say that the *absolute* value of the parameter reference is **not accessible**). Therefore, what SRoll can determine in the case of the foreground emission response coefficients' determination is not directly the values of the L_b^f , but that of the $\delta L_b^f = L_b^f - L_{\text{ref}}^f$. I will show later that the *closure condition* taken for the δL_b^f (or conversely the L_b^f) is defined so that the response difference coefficients (RDC) are fitted with respect to the foreground bandpass average, i.e. such that $\delta L_b^f = L_b^f - \overline{L^f}^{N_{\text{bolo}}}$, with $\overline{L^f}^{N_{\text{bolo}}} = \frac{1}{N_{\text{bolo}}} \sum_b L_b^f$, where N_{bolo} is the number of bolometers in the band (see Eq.III.27

below). The value of the average $\overline{L_b^f}$ is not directly accessible by SRoll and would require additional information in the form of a global calibrator for the considered foreground f .

A schematic serving to illustrate the *realignment* procedure is plotted in Fig.III.4. It displays an arbitrary distribution of 5 response coefficients L_b^f to a foreground f for each bolometer b . The left plot displays the general case, with an *arbitrary* reference value L_{ref}^f – used for the determination of the *difference* response coefficients δL_b^f – of which the *absolute* value,

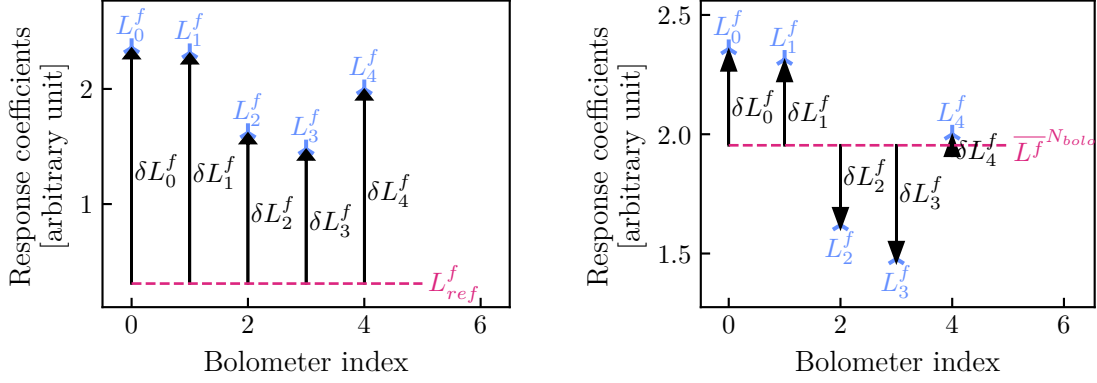


Fig. III.4 Schematic of an arbitrary distribution of 5 response coefficients $L_{b \in [1,5]}^f$ to a foreground f , displayed in blue. The same response coefficients distribution is displayed in both plots. The values of the difference response coefficients, i.e. δL_b^f , and only displayed quantity accessible to SRoll, are represented as the length of the black arrows. The reference value L_{ref}^f (**not** accessible to SRoll), with respect to which the values of the δL_b^f are defined, is displayed by the red dashed line. Left: generic case, with an arbitrary setting of the reference L_{ref}^f through an arbitrary closure condition; Right: typical SRoll2 case, where the reference value is set as the average value of the $L_{b \in [1,5]}^f$, i.e. $L_{ref}^f = \overline{L}^f^{N_{bolo}=5}$.

although displayed in the plot, can **not** be determined by SRoll. The only values available to SRoll are the *difference* response coefficients, i.e. the δL_b^f . Without an absolute calibrator for the reference value, neither the value of L_{ref}^f nor the L_b^f can be determined by SRoll. This is not an issue since the goal of the *realignment* consists in deleting any bandpass *mismatch* between the bolometers, which is achieved by bringing all the bolometers to the same response L_{ref}^f . To do so, the value of L_{ref}^f need not be known, and just by subtracting from the signal of each bolometer the effect of its respective *difference* response coefficients δL_b^f , the *realignment* objective is fulfilled. In Fig. III.4, the right plot displays the same response coefficients distribution, but where the reference value has been defined, through its corresponding *closure condition*, as the average value $\overline{L}^f^{N_{bolo}=5}$, as is performed in SRoll2 (cf. Section III.2). The complete expression of the *realignment* procedure in the case of SRoll2 is defined later in Eq. III.27.

It is to note that SRoll uses templates to fit the values of the δL_b^f . They come in the form of maps of the considered foreground emissions which are necessary to the fitting process. These template maps are detailed in subsection III.2.1.a) and do **not change throughout any SRoll run**. This unchanging nature of the input foreground template maps is true for SRoll1, SRoll2, as well as for the work discussed in the present manuscript, i.e. later SRoll2.1. The impact of choice of these foreground templates and of their quality is discussed in subsection III.2.4.

d) Ring offset

Though it was not detected in any of the ground-based measurements of the HFI characterisation, a special component of the detector's noise was detected from the very first

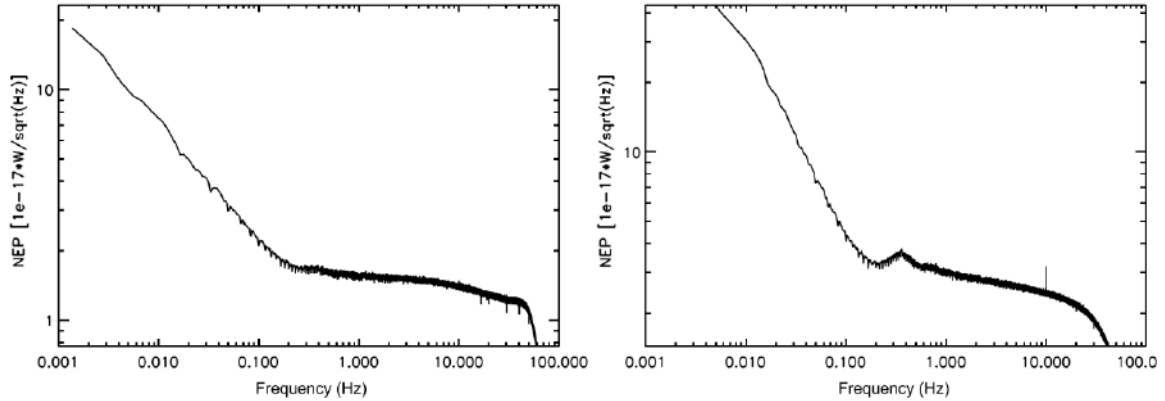


Fig. III.5 Typical power spectrum amplitude of bolometers 143-5 (**left**) and 545-2 (**right**). Power spectrum density of valid samples, after an average ring (the sky signal) has been subtracted from the TOI, and stacked for 200 rings. Here, the instrument time response was not deconvolved from the data. Taken from the bottom row of Fig.14 of [Planck HFI Core Team \(2011a\)](#).

Planck results, as described in [Planck HFI Core Team \(2011a\)](#) and visible in their Fig.14. Indeed, the power spectrum of the bolometers' noise displays, at low frequency ($f \leq 0.2\text{Hz}$), a component following a $1/f^\alpha$ law with $\alpha \simeq 1$ (cf. Fig.14 from [Planck HFI Core Team \(2011a\)](#), Fig.2 from [Planck Collaboration Int. XLVI \(2016\)](#), Fig.32 of [Planck Collaboration III \(2020\)](#), Figs.12 & 13 from [Planck Collaboration X \(2014\)](#)), whereby its given name: **the $1/f$ noise**. Above the knee value of $f \sim 0.2\text{Hz}$, the noise is mainly white. It was observed that the frequency of the knee almost does not depend on the noise level, and is the same for all HFI detectors (cf. [Planck Collaboration Int. XLVI \(2016\)](#) and [Planck Collaboration III \(2020\)](#)). The $1/f$ noise was also confirmed to be Gaussian (cf. Fig.3 of [Planck Collaboration Int. XLVI \(2016\)](#)). The $1/f$ noise can be seen as was first encountered in Fig.III.5, which is taken from Fig.14 of [Planck HFI Core Team \(2011a\)](#).

In all versions of SRoll, this $1/f$ noise component is fitted as **one offset value** per stable pointing period. The full definition and integration in the data model are later detailed in Eq.III.8. The data is then cleaned within SRoll by removing these fitted offsets, as detailed in Eq.III.27 below. However, this cleaning only affects the effects of the noise on the maps above $\ell > 100$, and the lower scale parts of $\ell \leq 100$, dominated by the effect of undetected glitches, is not corrected using this method (see subsection III.2.4).

e) Additional empirical transfer function

Even after the correction brought by the time response deconvolution applied in the *pre*-SRoll timeline data processing, as presented in subsection II.3.1, the long tails of the bolometer beams seen in Fig.II.8 are reduced but not entirely deleted. This means that the long time parts of the bolometers' time response are not correctly cleaned in the TOIs. This incorrect correction of the long time part of the time response also resulted in stripes in the maps called the "zebra patterns", as seen in Fig.III.6, which is a plot of the difference between odd and even 6-months surveys, i.e. difference maps built from the sky observed in opposite direction. In the 2018 *Planck* legacy release, these patterns were first identified but not corrected, but they are well corrected in SRoll2, as seen in Fig.III.6.

This is corrected in SRoll by fitting additional empirical transfer functions built from the signal TOIs in Fourier domain. More precisely, the empirical time-transfer functions

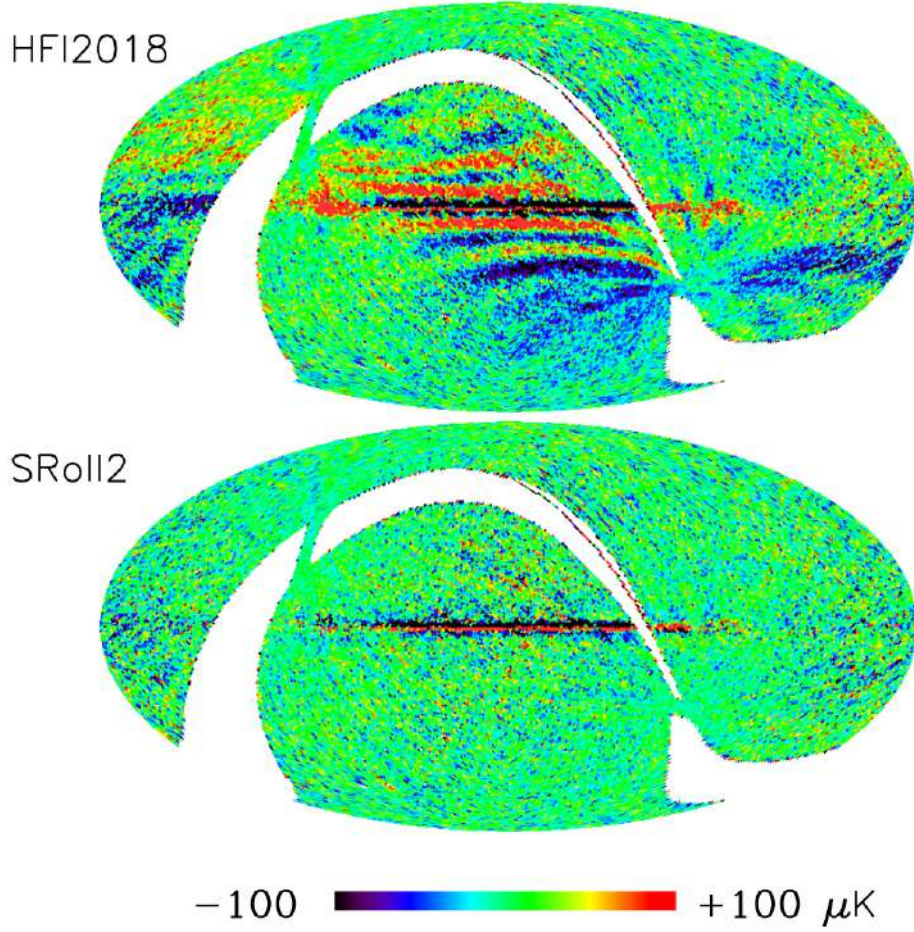


Fig. III.6 Intensity maps of the odd-even 6-months survey difference at 353GHz from HFI2018 (**top**) and SRoll2 (**bottom**) after changing the definition of the empirical time-transfer functions for the 353GHz band (see Eq.III.6).

are defined as partial sums of the first terms of the signal decomposition into Fourier series, excluding the average value. More precisely, the Fourier series decomposition of the TOI (not yet projected into HPR) of each bolometer is fitted as follows:

$$\forall t \in [0, T_{\tau}^{rot}], \text{TOI}_{b,\tau}(t) = \sum_{k=1}^{15} (a_k \cos(k\omega_{\tau}t) + b_k \sin(k\omega_{\tau}t)),$$

where $\text{TOI}_{b,\tau}$ is the signal of the rotation τ from the TOI of bolometer b , t is the time going from 0 s to $T_{\tau}^{rot} \simeq 60$ s with T_{τ}^{rot} the duration of the rotation which depends on τ (because of small variations of the rotation speed along the mission), a_k and b_k are fitted parameters, and $\omega_{\tau} = \frac{2\pi}{T_{\tau}^{rot}}$. Then, the empirical time-transfer functions are computed as HPRs projected

from the following TOIs:

$$\begin{aligned} \text{TOI}_{b,\tau}^1(t) &= a_k \cos(\omega_\tau t + \frac{\pi}{2}) + b_k \sin(\omega_\tau t + \frac{\pi}{2}), \\ \text{TOI}_{b,\tau}^2(t) &= \sum_{k=2}^3 \left(a_k \cos(k\omega_\tau t + \frac{\pi}{2}) + b_k \sin(k\omega_\tau t + \frac{\pi}{2}) \right), \\ \text{TOI}_{b,\tau}^3(t) &= \sum_{k=4}^7 \left(a_k \cos(k\omega_\tau t + \frac{\pi}{2}) + b_k \sin(k\omega_\tau t + \frac{\pi}{2}) \right), \\ \text{TOI}_{b,\tau}^4(t) &= \sum_{k=8}^{15} \left(a_k \cos(k\omega_\tau t + \frac{\pi}{2}) + b_k \sin(k\omega_\tau t + \frac{\pi}{2}) \right). \end{aligned}$$

The HPRs of the 4 empirical time-transfer functions $V_{b,r,p}^h$ are then computed as the HPRs projected from the $\text{TOI}_{b,\tau}^h$.

Note that for the PSBs, the empirical time-transfer functions are fitted on the TOI of the *other* bolometer from the pair, i.e. the template $V_{b,r,p}^h$ used for the bolometer a is fitted on the TOI of the bolometer b , and conversely the template $V_{a,r,p}^h$ used for the bolometer b is fitted on the TOI of the bolometer a . This allows to decorrelate the noise from the template and data when using the fitted $V_{b,r,p}^h$ as input templates in SRoll.

Additionally, SRoll2 used another definition of the empirical time-transfer functions for bolometers from the *353GHz band*. Indeed, along this frequency, the empirical time-transfer functions are defined as:

$$\begin{aligned} \text{TOI}_{b,\tau}^1(t) &= \sum_{k=1}^{15} \frac{1}{1 + (k\omega_\tau \tau_5)^2} \left(a_k \cos(k\omega_\tau t + \frac{\pi}{2}) + b_k \sin(k\omega_\tau t + \frac{\pi}{2}) \right), \\ \text{TOI}_{b,\tau}^2(t) &= \sum_{k=1}^{15} \frac{1}{1 + (k\omega_\tau \tau_{30})^2} \left(a_k \cos(k\omega_\tau t + \frac{\pi}{2}) + b_k \sin(k\omega_\tau t + \frac{\pi}{2}) \right), \end{aligned} \tag{III.6}$$

where $\tau_5 = 5$ s and $\tau_{30} = 30$ s. The HPRs of these 2 empirical time-transfer functions $V_{b,r,p}^h$ are then computed as the HPRs projected from the $\text{TOI}_{b,\tau}^h$. Here, the aforementioned inversion between PSBs within a pair is also applied.

f) The need for a *mask*

The fitting of SRoll's data model parameters is computed on a sky partially masked. This *mask*, given as input, excludes from the exploited sky data regions which might decrease the quality of the parameter fit from 2 main effects ([Planck Collaboration III \(2020\)](#), [Planck Collaboration Int. XLVI \(2016\)](#)):

- Time-variable emission sources,
- Strong signal gradient, typically present in the galactic plane as well as some molecular cloud cores.

These masks, built for each frequency (100-143-217-353 GHz) treated by SRoll, therefore mask all of the brightest point sources, known time-varying emission sources, and the galactic plane.

The strong gradient regions degrade the quality of fit because of pixelation of the sky: strong gradients are not well resolved by the pixel grid, and all subpixel variations are lost. Removing the pixels with such strong gradients from the minimization computation

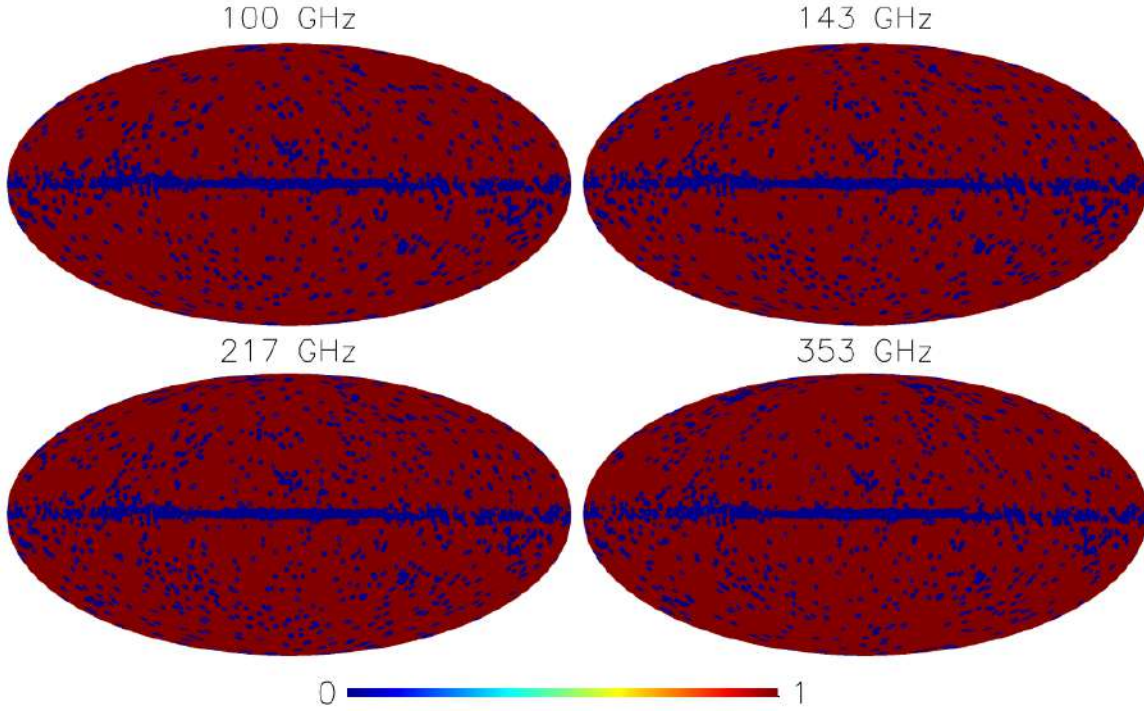


Fig. III.7 Masks used during the SRoll mapmaking solution. The fraction of the sky used is $f_{sky} = 86.2\%$ at 100 GHz, $f_{sky} = 85.6\%$ at 143 GHz, $f_{sky} = 84.6\%$ at 217 GHz, and $f_{sky} = 86.2\%$ at 353 GHz.

Taken from Fig.B.4 of *Planck Collaboration Int. XLVI (2016)*.

preserves the quality of fit while maintaining a sufficient sky coverage. Indeed, a good sky coverage is needed to achieve a proper determination of all parameters fitted by SRoll. This sky coverage is ensured, as the masks used by SRoll retain a sky fraction of $f_{sky} = 86.2\%$ at 100 GHz, $f_{sky} = 85.6\%$ at 143 GHz, $f_{sky} = 84.6\%$ at 217 GHz, and $f_{sky} = 86.2\%$ at 353 GHz. These masks are represented on Fig.III.7.

III.1.2 History of evolutions

SRoll was originally developed as a global mapmaking solution, concentrating the data compression into HPRs, fitting and cleaning the systematic instrumental effects observed at the end of the *Planck* data processing pipeline, and finally projects the cleaned intensity and *polarization* maps, as was first introduced and defined in *Planck Collaboration Int. XLVI (2016)*. This implementation of the mapmaking algorithm, referred to in this work as SRoll1, was then employed for the *Planck* 2018 legacy release (see *Planck Collaboration III (2020)*).

SRoll1 already included:

- ✓ The bolometer response chain calibration on the orbital dipole,
- ✓ The foreground bandpass mismatch determination and correction,
- ✓ The offset per ring, to take into account the $1/f$ noise,
- ✓ Empirical transfer functions for 4 bands of spin frequency harmonics.

In that version of the algorithm, the gain was solved for as a *time-dependant* parameter to account for the effect of the ADCNL. Also, the imperfect templates of the empirical transfer function in SRoll1 were leaving "zebra" patterns in the 353GHz band maps, as observed in [Planck Collaboration III \(2020\)](#) and seen in Fig.III.6, which was not yet corrected.

Later, the work of [Delouis et al. \(2019\)](#) provided us with an improved version of the SRoll mapmaking algorithm, called SRoll2. This is the version presented in detail in the present chapter as it is the one I started from and slightly modified in the scope of my work. This new version of the algorithm mainly brought the following additions and improvements:

- ✦ An improved model of the ADCNL effect, discarding the time variation of the gains, never observed in any of the ground-based measurements, and instead fitting a stable gain for the whole mission, still calibrated on the orbital dipole, and completed by an additional step in the algorithm specifically built and dedicated to the fitting of the ADCNL effect,
- ✦ A new and improved empirical transfer function model at 353GHz (see subsection III.1.1.e)), accounting for and correcting the zebra patterns that were previously observed in [Planck Collaboration III \(2020\)](#), as shown in Fig.16 of [Delouis et al. \(2019\)](#),
- ✦ Improved input template maps of the foreground emissions, leading to a better bolometers' bandpass mismatch fitting and correction,
- ✦ The addition of the sky signal into the data model used to fit the gain (it corresponds to the term $M_{b,r,p}^{eval}$ in Eq.III.24), leading to drastically improved convergence efficiency of the algorithm, as shown by Fig.2 of [Delouis et al. \(2019\)](#),
- ✦ A fit of a correction term for both the bolometers' polarization efficiency and polarization angle orientation.

This new version brought a better cleaning of the already accounted systematic effects, leading to reduced residuals of these effects, as well as the correction of additional terms, providing the current best quality of data cleaning for the *Planck* HFI data, as succinctly qualified in subsection III.2.4.

The central modification in the SRoll2 version from SRoll1 is the radical change in the way it fits and corrects the ADCNL. Instead of only fitting the residual effects left from the incomplete ADCNL correction carried out *before* SRoll in the data processing pipeline (see subsection II.3.1) as was done in SRoll1, SRoll2 *discards* this first correction and instead completely fits the ADCNL within its model. The whole iterative fit rests upon the hypothesis that any residual, left after the fitting and correction of *all the other systematics* accounted for in its data model, is due to the ADCNL. The ADCNL model is then iteratively fitted on this residual, as further described in subsection III.2.1.c). Given the strong hypothesis it rests upon, there was no assurance that the hereby defined SRoll2 version would actually converge, but the said hypothesis was confirmed by the very stable convergence of SRoll2 eventually observed.

The next version of this algorithm, called SRoll2.1, and adapted from the SRoll2 version, is the one I devised and used in the scope of my PhD work, with the goal of producing better fullsky ^{12}CO and ^{13}CO maps of the $J = 1 \rightarrow 0$ and $J = 2 \rightarrow 1$ lines. This new version, and all the associated results are presented in the next Chapter IV.

Another alternate version of the SRoll2 algorithm, called SRoll2.2, was built and used in the work of [Delouis, J.-M. et al. \(2021\)](#). Their work aimed at the improvement of the solar dipole amplitude and direction determination. Hence, relying on the Solar dipole direction

being the same for all detectors, they managed to improve the bolometers zero-level accuracy by factors of 2 to 3 with respect to the values obtained in all previous papers by extrapolation to zero level of the 21cm H I intensity. In the same work, they also introduced a new thermal dust emission model taking into account its spatial variation of SED. The SED variations in the new model are fitted at the same time as the bolometers zero levels, and therefore also rely on the minimization of the Solar dipole direction difference between frequency bands. By using the 545GHz band as reference dust model, it is propagated to the other frequency for which the SED variation is fitted as an additive correction map. Furthermore, [Delouis, J.-M. et al. \(2021\)](#) investigated the optimum sky fraction on which to fit the Solar dipole against the bias on the parameters induced by the galactic disk, which needs to be masked (i.e. the bias increases with the sky fraction conserved after masking), and the uncertainties produced by the necessary filling of this masked region by a constrained CMB (this time decreasing with an increase of the sky fraction conserved after masking). Their work also used a smoothed version of the SRoll2 template, but taken at full resolution, modification that I wanted to also include in the SRoll2.1 version, but did not have the time to fully implement, due to it needing to correct for the beam difference effect, as will be further discussed in subsection IV.3.4.

While the improved determination of the bolometers signal zero level from [Delouis, J.-M. et al. \(2021\)](#) was not included in my work, it is to note that such monopole values in the signal only impact negligibly the map construction method used in my work, as demonstrated later in subsection IV.3.3.

III.2 The SRoll2 version

This work is based on the use of a modified version of SRoll. This modified version starts from the SRoll2 version of the algorithm as presented in [Delouis et al. \(2019\)](#), and only slightly modifies its use and some of its inputs to better adapt to our main goal, while keeping most of the functions and inputs unchanged.

This section details the SRoll algorithm as it is in its second generation: SRoll2. It serves the purpose of detailing the nature and origin of all inputs and outputs of the algorithm, as well as the structure of a complete SRoll2 run. The SRoll2 data model and fitting procedure are also presented to the extent of what is needed to fully grasp any step of the procedure and results presented in the scope of this work.

Indices notations From here onward, I will use the following indices notations:

- b stands for the bolometer index, up to a total number of N_{bolo} bolometers (this total number depends on the SRoll2 run), with typical values of N_{bolo} being:
 - $N_{bolo} = 8$ at 100GHz (8 PSBs),
 - $N_{bolo} = 11$ at 143GHz (8 PSBs + 3 SWBs),
 - $N_{bolo} = 12$ at 217GHz (8 PSBs + 4 SWBs),
 - $N_{bolo} = 12$ at 353GHz (8 PSBs + 4 SWBs),
- p stands for the sky pixel index, up to $N_{pix} = 12 \times N_{side} \times N_{side}$, with N_{side} the HealPix resolution of the output frequency maps, always built at the output resolution of $N_{side} = 2048$, i.e. $N_{pix} = 50331648$,

- r stands for the stable pointing period (also called ring), up to $N_{ring} = 25811$, starting at ring $r = 240$ and ending at ring $r = 26050$ as the rings at the beginning and end of the mission are discarded for their low quality,
- f stands for the foreground emission and spans the ensemble of total size $N_{comp} = 5$, defined as:

$$\mathfrak{F} = \{Dust, Free-free, Synchrotron, {}^{12}CO, {}^{13}CO\},$$

- h stands for the bin of satellite spin frequency harmonics, up to N_{harm} , with bins built as: $h = 1$ for the spin frequency (0.017 Hz), $h = 2$ for the harmonics 2 and 3 ([0.033, 0.050] Hz), $h = 3$ for the harmonics 4 to 7 ([0.067, 0.117] Hz), and $h = 4$ for harmonics 8 to 15 ([0.133, 0.250] Hz) (cf. Delouis et al. (2019), Planck Collaboration Int. XLVI (2016), and see subsection III.1.1.e) for more details), **except** for the 353GHz band, where it only spans the set $\{1, 2\}$ as defined in subsection III.1.1.e),
- i stands for the SRoll2 iteration number (see subsection III.2.1.c) for more details on the looping structure of SRoll2).

While the sets spanned by these indices may vary depending on the SRoll version used, or the considered SRoll run, I will keep these notations throughout this manuscript in an attempt for both consistency and clarity.

An additional notation used in the description of the SRoll2 algorithm (and also SRoll2.1 in the next chapter), is that a quantity X is noted $\tilde{X}$ when it refers to a quantity determined by SRoll2 and affected by noise and error.

Also, for the sake of clarity, I will mostly respect the notations from the original publication when discussing the workings of SRoll2 and when referencing its outputs, in this section as well as in the rest of this manuscript.

 Note that one of the notations used in the original paper Delouis et al. (2019), which I keep in this manuscript *when referencing or discussing the SRoll mapmaking algorithm*, can be somewhat misleading: the capital letter M corresponds to *bolometer signal* data and does **not** correspond to maps, while the capital letter S corresponds to *maps* of various emissions. A suggested mnemonic to keep this notation in mind while reading about the SRoll algorithm is to think of the letter M as standing for the word *model*, and to think of the letter S as standing for *sky*. In the next chapters, I will return to more conventional notations when discussing other matters, e.g. I will use M for my constructed CO maps, but I will keep these notations when explicitly discussing the SRoll algorithm.

III.2.1 Structure of algorithm

I start by introducing all the elements required for a SRoll run, the nature and sources of all its necessary inputs, whether or not they explicitly appear in the later data model and minimization equations. I list all of SRoll's outputs, whether or not they are used in the CO maps computation procedure. I then present the main skeleton of the SRoll mapmaking algorithm, its general structure made of successive and complementary looping steps.

a) Inputs

I list here all inputs necessary to compute a SRoll2 run. Note that all the input templates introduced in this subsection are **fixed** throughout any given SRoll2 run. All these various data and parameters are summarized in Table.III.2, where the first column lists the

names and/or nature of said inputs, the second column gives the notation used for the corresponding quantity in case it appears explicitly in either the data model (cf. III.2.2) or the minimization computation (cf. III.2.3), and the third column gives the origin of each considered quantity. Each input appearing in Table III.2 is then further defined and detailed in nature and origin in this subsection.

Some of these inputs, while necessary at some point in the SRoll2 run, may not need to appear in the equations detailed in later sections, and therefore do not have any corresponding *notation*. For full clarity and exhaustivity, I chose nonetheless to mention these inputs. However, given that they do not need to appear again later in this work, any input without any *notation* appearing in the second column of Table III.2 can be disregarded by the reader without risking any detrimental consequence on their understanding of the rest of this manuscript, **with the exception of one** such input: the *Mask*. This input does not appear explicitly in the SRoll equations written in this manuscript, yet plays an essential role in the SRoll2 mapmaking algorithm, and an even more crucial one in the later adaptation of SRoll2 described in the next chapter, for the computation of the sought fullsky ^{12}CO and ^{13}CO maps.

- **HPRs of signal from bolometers** This is the main input of SRoll2 and corresponds to the cleaned TOI of all bolometers used in the run projected into HPRs (typically all bolometers of *one* frequency band), as described in subsection II.3.2. The full TOI from each bolometer has been cleaned following *Planck*'s data analysis pipeline described in Section II.3, **with the exception of** the ADCNL correction since it is fitted and corrected for within SRoll2. Each TOI has also been compressed into HPRs, as it is the case with all *timeline* data input to SRoll2.

- **Parameters:**

Here I detail the parameter values needed as input for a SRoll2 run. Some of them are additionally fitted by SRoll2, but only corrections of said parameters are fitted, therefore starting values are given as input to ensure good and efficient convergence of the minimization algorithm.

- **Initial calibration** The initial values for the absolute response of the bolometer chain. These values are taken from the *Planck* 2015 legacy release (cf. [Planck Collaboration VIII \(2016\)](#)). SRoll2 then fits corrections for this parameters on each bolometer.

- **Monopoles** The monopoles on the signal of each bolometer, i.e. the non-zero average value of incoming power received by the bolometer chain. This value cannot be determined by the HFI instrument since the bolometers are only sensitive to a variation of incoming power from the sky. The monopoles used in SRoll2 are those determined in the *Planck* 2015 legacy release (cf. [Planck Collaboration VIII \(2016\)](#)).

- **Noise-equivalent Power (NEP)** The NEP is the instrument specific noise rms level (integrated over 1 second) expressed in units of power equivalent to the incoming signal: in $\text{W}\cdot\text{Hz}^{-\frac{1}{2}}$ (cf. [Planck HFI Core Team \(2011b\)](#)). This input is taken from the *Planck* 2013 legacy release determination (cf. [Planck Collaboration VI \(2014\)](#)). It is used to compute the later term $\sigma_{b,r,p}^2$.

Table III.2: Nature and source of all inputs for the SRoll2 mapmaking algorithm. If the input is explicitly mentioned in the data model or convergence computation of SRoll2, then the corresponding notation is also given.

Quantity	Notation	Source
TOI signal from bolometers, compressed into HPR	$M_{b,r,p}^{tot}$	<i>Planck</i> full TOI processing pipeline, ex-empted from its ADCNL correction
Parameters: Initial calibration Monopoles NEP Initial polarization efficiency	$1/g_{b,i=0}$ ρ_b	<i>Planck</i> 2015 release determination <i>Planck</i> 2015 release determination <i>Planck</i> 2013 release determination Ground-based measurements
Mask		Built for SRoll1, and used for the <i>Planck</i> 2018 legacy release
Templates: Foreground emissions: Dust $^{12}\text{CO} / ^{13}\text{CO}$ Free-free Synchrotron FSL Sky signal map template Polarization corrections templates Empirical time-transfer function template	C_p^f C_p^{Dust} $C_p^{^{12}\text{CO}} / C_p^{^{13}\text{CO}}$ C_p^{FF} C_p^{Sync} $F_{b,r,p}^{tot}$ $M_{b,r,p}^{eval}$ $\left. \frac{\partial(gM)}{\partial\rho} \right _{b,r,p}, \left. \frac{\partial(gM)}{\partial\phi} \right _{\phi_{b,r,p}}$ $V_{b,r,p}^h$	<i>Commander</i> separation from <i>Planck</i> 2015 release G08 Taurus survey Goldsmith et al. (2008) <i>Commander</i> separation from <i>Planck</i> 2015 release <i>Commander</i> separation from <i>Planck</i> 2015 release <i>Planck</i> 2015 release determination 1° smoothed <i>Planck</i> 2018 release HFI frequency maps, containing the <i>total</i> dipole, $N_{side} = 2048$ <i>Planck</i> 2018 release HFI frequency maps, downsampled to a HealPix resolution of $N_{side} = 128$ Template HPRs fitted from the data TOIs, see subsection III.1.1.e)
Pointing		<i>Planck</i> 2013 release determination
Total dipole HPR	$D_{r,p}^{tot}$	<i>orbital</i> dipole computed from pointing data assuming a time-invariant amplitude <i>solar</i> dipole from <i>Planck</i> 2018 release with constant direction ($l = 264.021^\circ$, $b = 48.253^\circ$) and amplitude ($3362.08\mu\text{K}_{\text{CMB}}$), cf. Planck Collaboration III (2020)
Flags of discarded rings		<i>Planck</i> 2015 data flagging Planck Collaboration VII (2016)
Hitcount	$\mathfrak{h}\mathfrak{c}_{b,r,p}$	Compression procedure of TOI data into HPRs
ADU distribution model		Built from sparse raw data gap filled using a simplified sky model

– **Initial polarization efficiency** The initial value of the polarization efficiency for each PSB used in the run. This value is taken from the ground-based measurements performed pre-launch (cf. Rosset et al. (2010)), and then fitted for correction in SRoll2.

• **Mask** The sky mask used in SRoll2’s minimization, as discussed in subsection III.1.1.f) above. It was built first with SRoll1, and used in the work of Planck Collaboration Int. XLVI (2016), then also used to build the frequency maps from SRoll1 in the *Planck* legacy release 2018, cf. Planck Collaboration III (2020). They correspond to the masks displayed in Fig. III.7.

• **Foreground templates:**

The *Planck* data model used in the SRoll2 algorithm uses spatial templates of the foreground emissions to determine the bandpass colour-correction coefficients corresponding to each foreground. These templates are sky maps also following the HEALPix pixelisation scheme (Gorski et al. (2005)), and are fixed inputs which never change during a given SRoll2 run.

These templates are taken at a fairly low resolution of $N_{side} = 128$ for 2 main reasons:

- To avoid introducing a significant amount of correlated noise from low intensity regions in the maps, and more precisely at high latitude,
- To avoid effects from beam differences between bolometers.

Indeed, as can be seen from subsection II.1.2.b), each bolometer has a different beam shape and size. Taking pixels larger than the size of the bolometer beams for the foregrounds templates erases the potential beam difference effects. This effect will be discussed further in subsection IV.3.4.

Some other instrumental systematic effects are modelled using templates built either as sky maps, or at the timeline level, depending of the modelled instrumental systematic.

Template orthogonality issue These foreground template maps need to be sufficiently orthogonal, i.e. of sufficiently low spatial correlation, to the CMB as well as to each other in order for a correct separation of the various signal sources by SRoll2. This was evaluated by using simulations in Planck Collaboration III (2020). In this work, aiming at the production of all-sky ^{12}CO and ^{13}CO maps, this need for orthogonality between the foreground emissions is especially delicate in the case of the separation between the ^{12}CO , ^{13}CO , and thermal dust foregrounds, which dominate the foreground intensity signal and happen to be significantly spatially correlated. However, we reckon that the Taurus region covered by the CO templates, as well as the higher sky galactic latitudes, possess enough surface with very low CO intensity while retaining significant dust signal. These regions of low spatial correlation between dust and CO are enough to separate these foregrounds, as can be seen from the low residues in the maps (see subsection III.2.4, and the results from Chapter IV). As for the separation of ^{12}CO and ^{13}CO , this is made possible by the fact that the emission of the less abundant ^{13}CO saturates at higher column density than the ^{12}CO , and therefore traces the regions of higher density. This abundance difference between ^{12}CO and ^{13}CO leads to regions of low but non-zero ^{12}CO intensity with no detectable presence of ^{13}CO , as well as regions of saturated and constant ^{12}CO emission with ^{13}CO variations, introducing some spatial orthogonality between the 2 isotopologues (cf. Lequeux et al. (2002)). The level of orthogonality between the ^{12}CO , ^{13}CO , and thermal dust foreground templates in the Taurus region is shown in Fig. III.8. In this figure I display the maps of the same region for these 3 foregrounds, as well as the longitude profile of these maps at

-14° of galactic latitude. Though we can clearly see the spatial correlation between these foregrounds, we can also see regions of higher orthogonality between them, especially for the orthogonality between dust and CO which is very high on the edges of the molecular cloud, i.e. from 175° to 180° of longitude in the cut of Fig.III.8. On such molecular cloud edges, the CO emission (^{12}CO and ^{13}CO) drops to zero abruptly while the dust intensity only slowly decreases, breaking the spatial correlation between dust and CO emissions.

- **Dust** Fullsky template of the thermal dust emission, in intensity and polarization. Template maps are taken from the *Commander* separation from *Planck* 2015 release, cf. Planck Collaboration X (2016). This template is plotted in the next chapter in Fig.IV.7. As will be discussed in subsection IV.1.3.a), the polarization components of this template will be changed for the sake of the ^{12}CO and ^{13}CO maps production. This template is then plotted in its final state, as it was used in the scope of my thesis work.

- $^{12}\text{CO} / ^{13}\text{CO}$ Sky template of the ^{12}CO and ^{13}CO emission. This template is taken from a large radio survey of the Taurus region covering 100 square degrees, centred on the position 172.9° of galactic longitude and -14.8° of galactic latitude, cf. Goldsmith et al. (2008). This survey observed the $J = 1 \rightarrow 0$ rotational line for both ^{12}CO and ^{13}CO with high spatial (uniform grid of $20''$ spacing, beams of FWHM $45''$ for the ^{12}CO and $47''$ for the ^{13}CO) and spectral resolutions (channel widths of 0.26km.s^{-1} for the ^{12}CO and 0.27km.s^{-1} for the ^{13}CO). The maps are integrated in velocity, resulting in template maps in units of K.km.s^{-1} . In this manuscript, this survey will often be referred to as the G08 survey. The maps, projected in HEALPix, are shown in Fig.III.9, at both a high resolution of $N_{\text{side}} = 2048$ (i.e. pixel angular size of $\sim 1'.7$, cf. Table.A.1), and the resolution used for the SROLL2 foreground templates, i.e. $N_{\text{side}} = 128$. Once projected in HEALPix, the Taurus region observed by this survey covers 0.24% of the sky. The sky coverage of this survey can be seen in Fig.III.10. It is to note that this survey is currently the largest radio astronomy survey which observed both ^{12}CO and ^{13}CO with high spatial (and spectral) resolutions.

This template is an intensity map, which means that the ^{12}CO and ^{13}CO templates are taken as non-polarized, given the expected low polarization fraction (at most of the order of $\sim 1\%$) of the CO lines, as was discussed in subsection I.4.2.

This template differs from the SROLL1 version, employed in the *Planck* 2018 legacy release, which used the CO map produced from the *Commander* separation from *Planck* 2015 release, which did not separate the ^{12}CO and ^{13}CO isotopologues.

- **Free-free** Fullsky template of the free-free emission. The template map is taken from the *Commander* separation from *Planck* 2015 release, cf. Planck Collaboration X (2016). This foreground template has only one component map since it is non-polarized (Rybicki & Lightman (1979)). The corresponding map is displayed in Fig.III.11.

- **Synchrotron** Fullsky template of the synchrotron emission, in intensity and polarization. Template maps are taken from the *Commander* separation from *Planck* 2015 release, cf. Planck Collaboration X (2016). The corresponding maps are displayed in Fig.III.12.

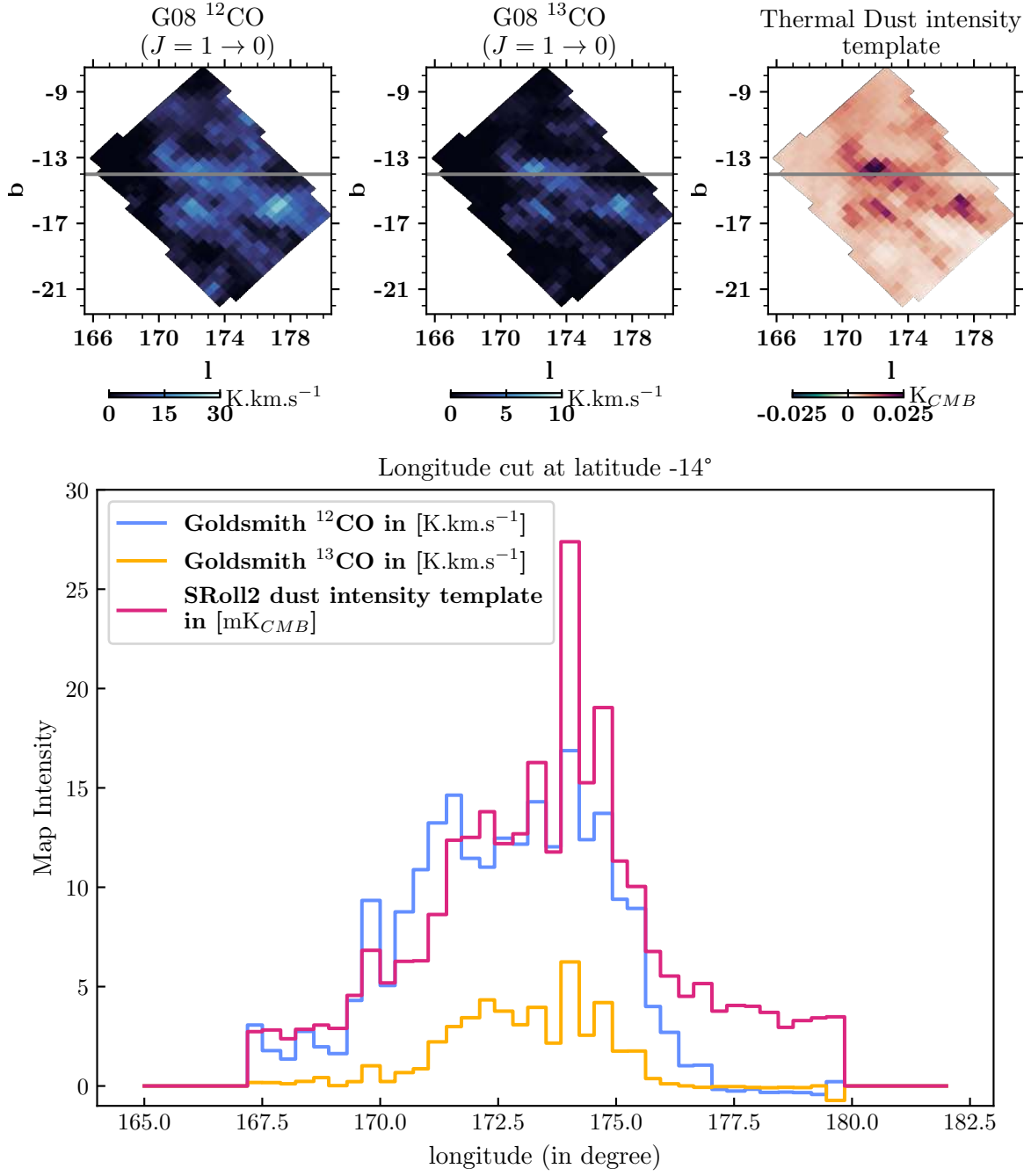


Fig. III.8 Longitude profile of the ^{12}CO (**Upper left** in [K.km.s $^{-1}$]), ^{13}CO (**Upper middle** in [K.km.s $^{-1}$]), and thermal dust intensity (**Upper right** in [K $_{\text{CMB}}$]) SRoll2 foreground templates. The longitude cuts, plotted in the **lower** graph, are performed at -14° of galactic latitude. The line of the longitude cut is represented in grey in the upper maps of the 3 foregrounds. Note that the signal in the dust template map was kept only in the Taurus region as surveyed in Goldsmith et al. (2008), i.e. where signal is present in the SRoll2 ^{12}CO and ^{13}CO templates.

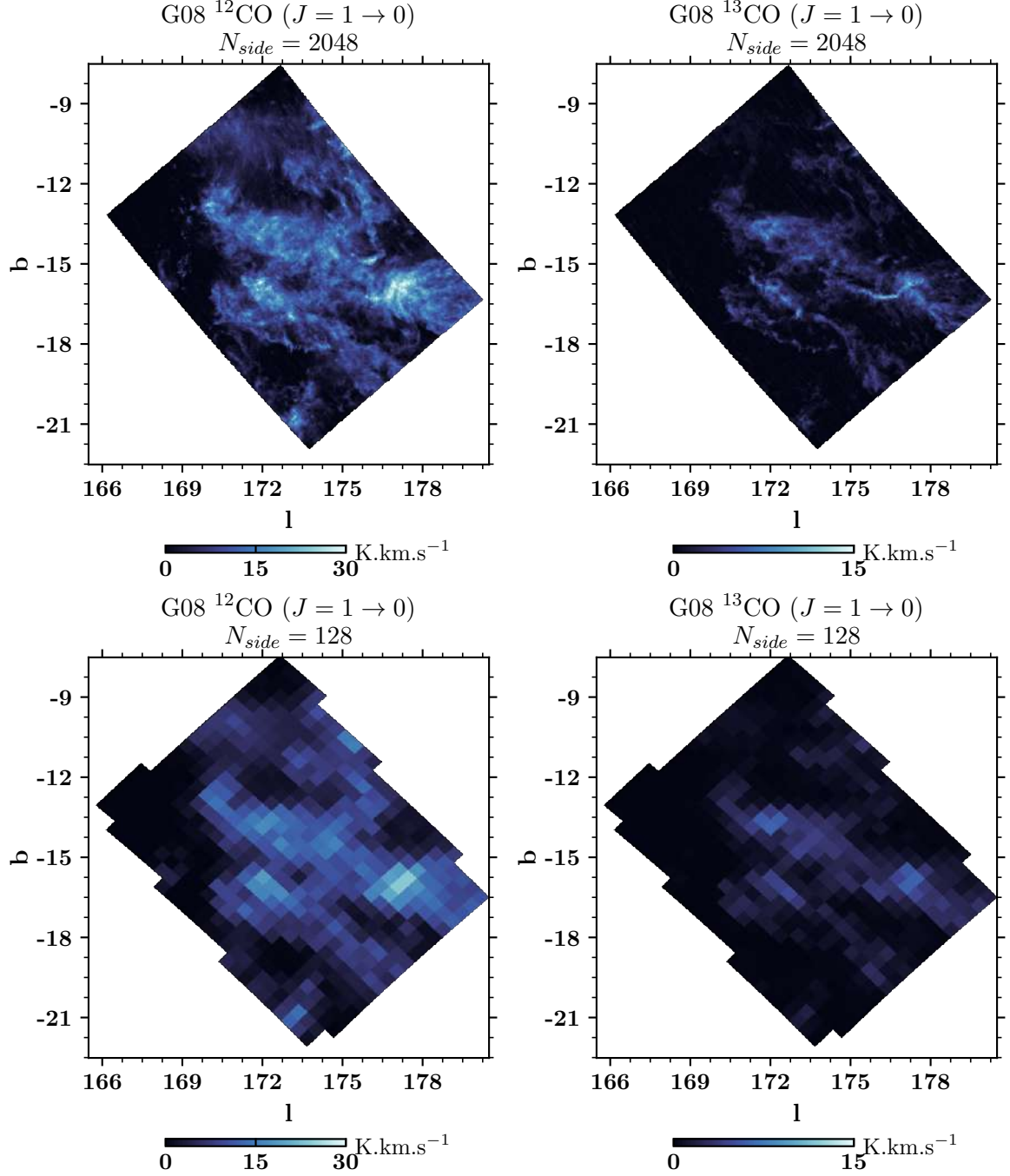


Fig. III.9 **Left:** ^{12}CO ; **Right:** ^{13}CO . Integrated CO maps of the $J = 1 \rightarrow 0$ rotational line emission from [Goldsmith et al. \(2008\)](#). The maps are projected in HEALPix with a resolution of: **Top:** $N_{side} = 2048$; **Bottom:** $N_{side} = 128$ corresponding to the templates used in SROLL2.

G08 ^{12}CO ($J = 1 \rightarrow 0$) $N_{\text{side}} = 2048$

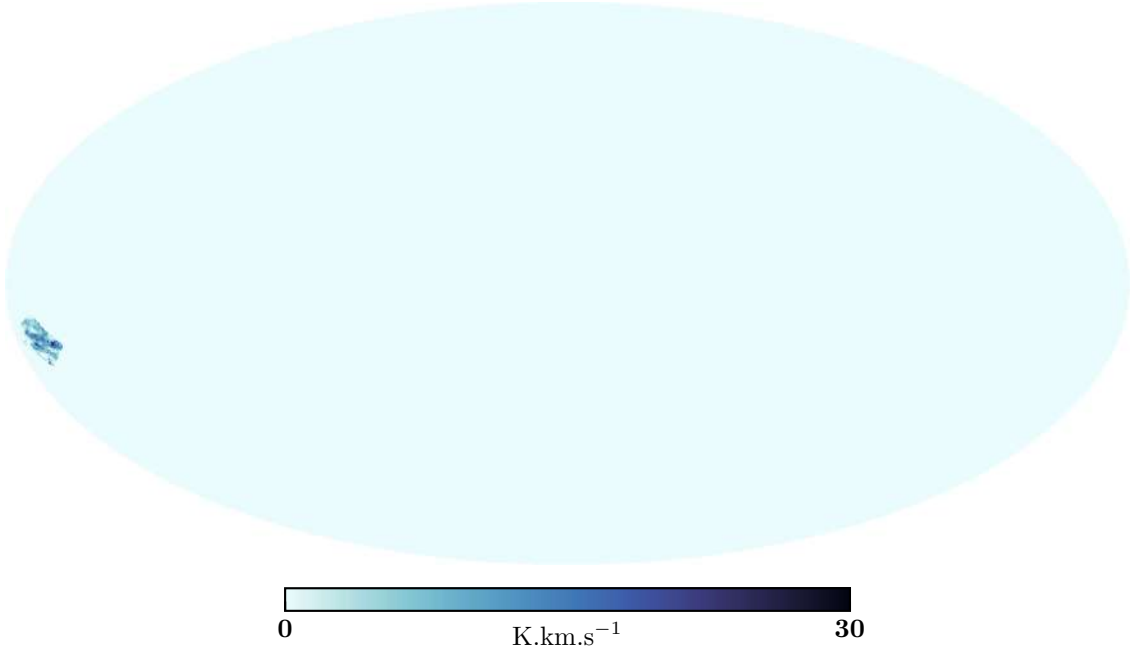


Fig. III.10 ^{12}CO integrated full-sky map of the $J = 1 \rightarrow 0$ rotational line emission from [Goldsmith et al. \(2008\)](#). The map is projected in HEALPix with a resolution of $N_{\text{side}} = 2048$. We can see the surveyed Taurus region position on the sky, centred on the position 172.9° of galactic longitude and -14.8° of galactic latitude, as well as its total sky fraction coverage of 0.24%.

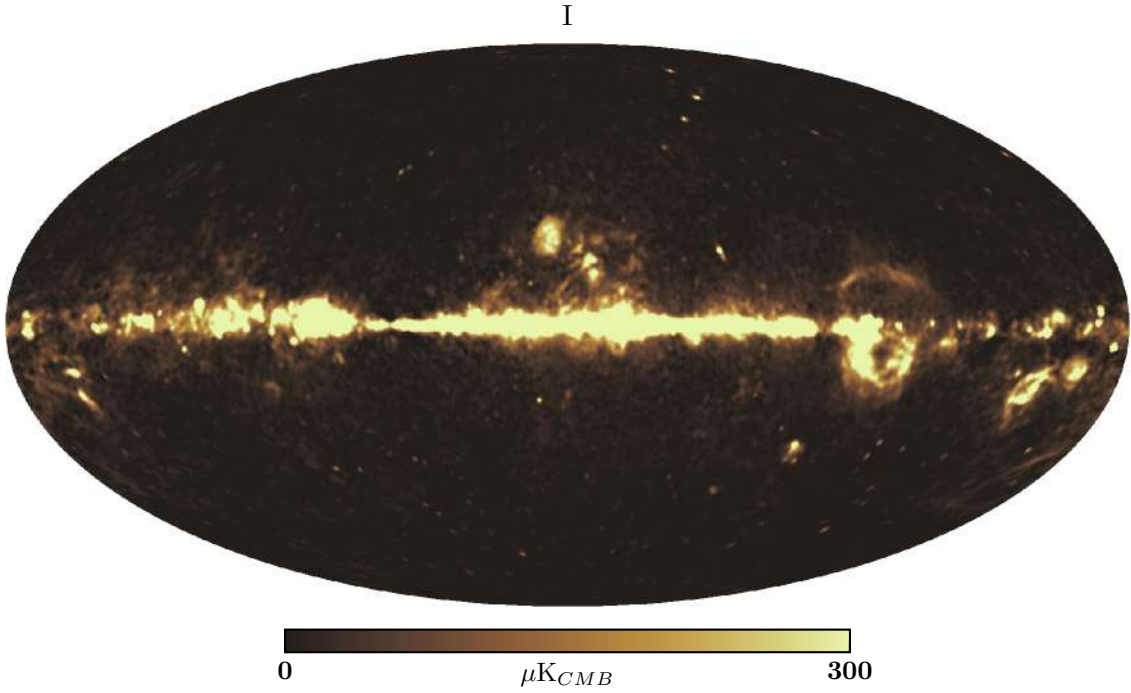


Fig. III.11 SRoll2 template of the free-free foreground emission. The resolution is that of the SRoll2 templates: $N_{\text{side}} = 128$. The map is taken from the Commander separation from Planck 2015 release, cf. [Planck Collaboration X \(2016\)](#).

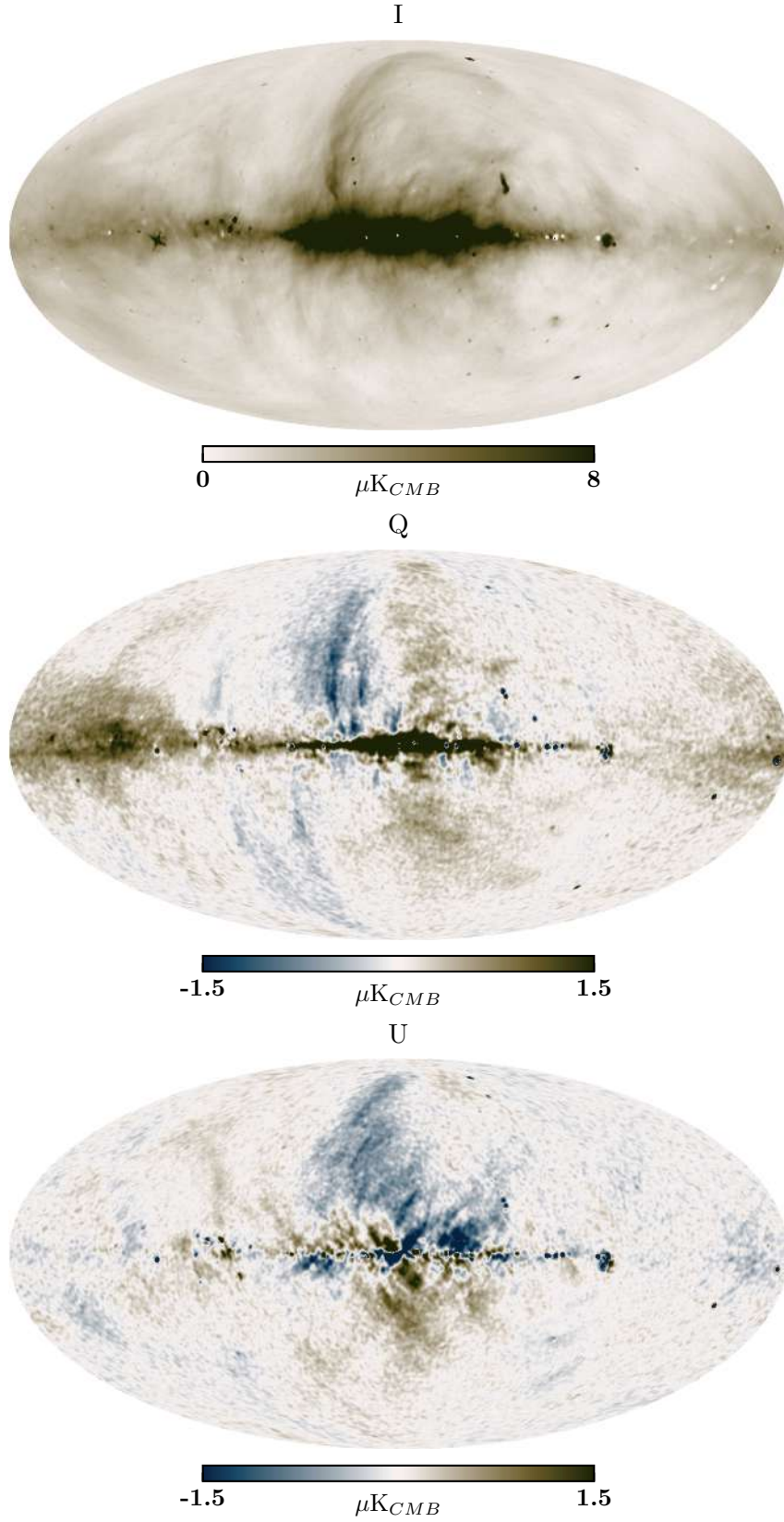


Fig. III.12 *S*Roll2 template of the synchrotron foreground emission (taken from the *Commander* separation from *Planck* 2015 release, cf. [Planck Collaboration X \(2016\)](#)). The resolution is that of the *S*Roll2 templates: $N_{\text{side}} = 128$. Top: *I* Stokes intensity component; Middle: *Q* Stokes polarization component; Bottom: *U* Stokes polarization component.

- **FSL template** The template of the FSL produced by each bolometer used in the considered run, built at the timeline level (input as HPR). This template was produced in the *Planck* data analysis pipeline, and its computation was performed in the *Planck* release of 2015 (cf. [Planck Collaboration VII \(2016\)](#), [Planck Collaboration Int. XLVI \(2016\)](#)), though it was only accounted for since the legacy maps in the release of 2018, i.e. since the use of SRoll as the mapmaking algorithm.

- **Sky signal model** Sky signal used to fit the bolometers' chain response (cf. Eq.III.24). Taken as the 1° smoothed HFI2018 frequency maps, containing the *total* dipole.

- **Polarization corrections templates** The templates used to fit the corrections of the polarization efficiency and the bolometers polarization angle. Since these corrections are defined as the first order of the Taylor expansion of the signal with respect to each respective parameter, it means we can write the following equality:

$$g_b M_{b,r,p}^{tot}(\rho_b + \delta\rho_b, \phi_{b,r,p} + \delta\phi_{b,r,p}) \simeq g_b M_{b,r,p}^{tot}(\rho_b, \phi_{b,r,p}) + \left. \frac{\partial(gM)}{\partial\rho} \right|_{b,r,p} \delta\rho_b + \left. \frac{\partial(gM)}{\partial\phi} \right|_{\phi_{b,r,p}} \delta\phi_{b,r,p},$$

where $M_{b,r,p}^{tot}$ is the bolometer signal, ρ_b and $\phi_{b,r,p}$ are the input values of the polarization efficiency and angle, $\delta\rho_b$ and $\delta\phi_{b,r,p}$ are the respective corrections of the polarization efficiency and the bolometers polarization angle to be determined by SRoll2, $\left. \frac{\partial(gM)}{\partial\rho} \right|_{b,r,p}$ is the derivative of the signal with respect to the polarization efficiency taken at the input value ρ_b , and $\left. \frac{\partial(gM)}{\partial\phi} \right|_{\phi_{b,r,p}}$ is the derivative of the signal with respect to the bolometer polarization angle taken at the input value $\phi_{b,r,p}$. Given the above definitions, we can then express the 2 input templates by the 2 following equations:

$$\begin{aligned} \left. \frac{\partial(gM)}{\partial\rho} \right|_{b,r,p} &= \cos(2\phi_{b,r,p}) Q_p^{eval} + \sin(2\phi_{b,r,p}) U_p^{eval}, \\ \left. \frac{\partial(gM)}{\partial\phi} \right|_{\phi_{b,r,p}} &= 2\rho_b \left(-\sin(2\phi_{b,r,p}) Q_p^{eval} + \cos(2\phi_{b,r,p}) U_p^{eval} \right) \end{aligned} \quad (\text{III.7})$$

where Q^{eval} and U^{eval} are the polarized components of the sky maps, and therefore correspond to the Q and U Stokes components of $M_{b,r,p}^{eval}$, but taken, in the case of SRoll2, at the HEALPix resolution $N_{side} = 128$.

- **Empirical time-transfer function template** Template HPRs of the empirical time-transfer functions, defined as partial sums of the first terms of the signal decomposition into Fourier series, excluding the average value, as defined in subsection III.1.1.e).

- **Pointing data** Onboard satellite position and orientation tracking throughout the whole mission (cf. [Planck Collaboration VII \(2014\)](#)).

- **Total dipole** HPR of the *total* dipole over the whole *Planck* mission.

The *orbital* dipole is computed from the pointing data and trajectory of the satellite, sent over the whole mission.

The *solar* dipole is the one determined in the *Planck* 2018 legacy release, with constant direction ($l = 264.021^\circ$, $b = 48.253^\circ$) and amplitude ($3362.08 \mu\text{K}_{\text{CMB}}$) (cf. [Planck Collaboration III \(2020\)](#)).

Table III.3: List of all outputs for the SRoll2 mapmaking algorithm. A quantity X is often noted $\tilde{X}$ to emphasize when it refers to a quantity determined by SRoll2 and affected by noise and error uncertainty

Quantity	Notation
Cleaned and projected frequency maps , from the full data and subsurveys	$\tilde{S}_{p,i=N_{iter}}^{out}$ & $\tilde{S}_{b,p,i=N_{iter}}^{out}$
Maps of the precision matrix components	$II_{b,p}^{out}, IQ_{b,p}^{out}, IU_{b,p}^{out}, QQ_{b,p}^{out}, QU_{b,p}^{out}, UU_{b,p}^{out}$
Calibrated response	$1/\tilde{g}_{b,i=N_{iter}}$
Ring offsets	$\tilde{O}_{b,r,i=N_{iter}}$
Polarization efficiency correction	$\tilde{\delta\rho}_{b,i=N_{iter}}$
Detector polarization angle correction	$\tilde{\delta\phi}_{b,r,p,i=N_{iter}}$
Foregrounds colour-correction coefficient	$\tilde{\delta L}_{b,i=N_{iter}}^f$
Amplitude of empirical time-transfer function with long time constants	$\tilde{\gamma}_{b,i=N_{iter}}^h$
ADCNL splines (time + ADU)	$\mathfrak{N}_{b,r,i=N_{iter}}$

- **Flags of discarded rings** List of rings flagged and discarded, to be ignored in the SRoll2 minimization computation. This flagging was determined during the *Planck* data analysis pipeline (cf. [Planck Collaboration VII \(2016\)](#)).

- **Hitcount** HPR of the hitcount in each pixel throughout the whole mission. This hitcount corresponds to the number of samples for each ring falling in each pixel, i.e. number of sky incoming power measurements falling in the considered pixel from ring r . This hitcount HPR is automatically produced when the TOI data is compressed into HPRs, as described in Sect. II.3.

- **ADU distribution model** A model of the *expected* ADU distribution to determine the digital conversion of the incoming power. This is built from sparse sampling from the raw data, gap-filled using a simplified sky model made of simple sum of the *Planck* 2018 frequency maps and the total dipole. This input is used to fit the ADCNL splines.

b) Outputs

I list here all outputs computed and provided at the end of a SRoll2 run. All these various data and parameters are summarized in Table. III.3, where the first column lists the names and/or nature of said outputs, and the second column gives the notation used for the corresponding quantity in any of its later mention. Each output appearing in Table. III.3 is then further defined and detailed in nature and/or use in this subsection.

Frequency maps ($\tilde{S}_{p,i=N_{iter}}^{out}$ & $\tilde{S}_{b,p,i=N_{iter}}^{out}$) The *cleaned* frequency map built from an average over all bolometers after correction of all fitted instrumental systematics, as described later by Eq. III.28, as well as the *cleaned* sky map *per bolometer* also corrected for all fitted instrumental systematics, as described later by Eq. III.29.

Precision matrix components maps ($II_{b,p}^{out}, IQ_{b,p}^{out}, IU_{b,p}^{out}, QQ_{b,p}^{out}, QU_{b,p}^{out}, UU_{b,p}^{out}$) The maps of the 6 precision matrices (i.e. inverses of the correlation matrices) components for each bolometer. These maps are used in the next chapter to compute new full-sky ^{12}CO and ^{13}CO emission maps. Their definition and computation are given by Eq.III.30 below.

Calibrated response ($1/g_{b,i=N_{iter}}$) Final fitted bolometer chain responses, holding the information for the whole bolometer chain gain and its absolute photometric calibration.

Ring offsets ($\tilde{O}_{b,r,i=N_{iter}}$) The set of fitted offsets (one per ring) along the whole *Planck* mission data, used to model and correct the $1/f$ noise in the TOI.

Polarization efficiency & detector angle corrections ($\tilde{\delta\rho}_{b,i=N_{iter}}$ & $\tilde{\delta\phi}_{b,r,p,i=N_{iter}}$) The corrections of polarization efficiency and detector polarization angle fitted as first order of the Taylor expansion of the signal with respect to each respective parameter.

Foreground colour-correction coefficients ($\tilde{\delta L}_{b,i=N_{iter}}^f$) The bolometer response *difference* coefficients, also called color-correction bandpass mismatch coefficients, with respect to the reference value defined by the corresponding *closure condition* of the SRoll2 run, for any foreground emissions f . These are the coefficients I use in the next chapter to compute new full-sky ^{12}CO and ^{13}CO maps. It is shown, in the later *closure condition* definition for this parameter (cf. subsection III.2.2), that these response *difference* coefficients are fitted with respect to the average bolometers' foreground response over all bolometers used in the run, i.e. $\forall f \forall b \delta L_b^f = L_b^f - \bar{L}_b^f$.

Amplitude of empirical time-transfer function ($\tilde{\gamma}_{b,i=N_{iter}}^h$) Fitted factors' value of the empirical time-transfer function coefficients for long time constants, for the 4 harmonics bins fitted by SRoll2.

ADCNL splines ($\aleph_{b,r,i=N_{iter}}$) Full spline function parameters of the SRoll2 ADCNL model, fitted in both *time* and *ADU* domains. See Delouis et al. (2019) for more details on this model as well as its fitting computation.

c) Looping structure

The SRoll2 algorithm is an iterative scheme built from two computational blocks called **Step#1** and **Step#2**. The structure of a SRoll2 run is depicted in Fig.III.13, showing the sequence of these 2 steps iterative successions to produce the main algorithm's outputs.

Step#1 The first step is the main one. It consists in a hefty χ^2 minimization (defined below by Eq.III.25), producing an estimate for all the mapmaking parameters from its data model **except** those of the ADCNL effect correction, of which the last determination is subtracted from the signal at the beginning of **Step#1**. Taking the notations introduced above, **Step#1** provides an estimate for: $1/g_{b,i}$, $\tilde{O}_{b,r,i}$, $\tilde{\delta\rho}_{b,i}$, $\tilde{\delta\phi}_{b,r,p,i}$, $\tilde{\delta L}_{b,i}^f$, and $\tilde{\gamma}_{b,i}^h$. From all these parameters, it can then compute polarized frequency maps and sky maps per bolometer, i.e. $\tilde{S}_{p,i}^{out}$ and $\tilde{S}_{b,p,i}^{out}$, respectively determined as defined in equations III.28 and III.29. These maps are then used as a new input for the computation performed in **Step#2**.

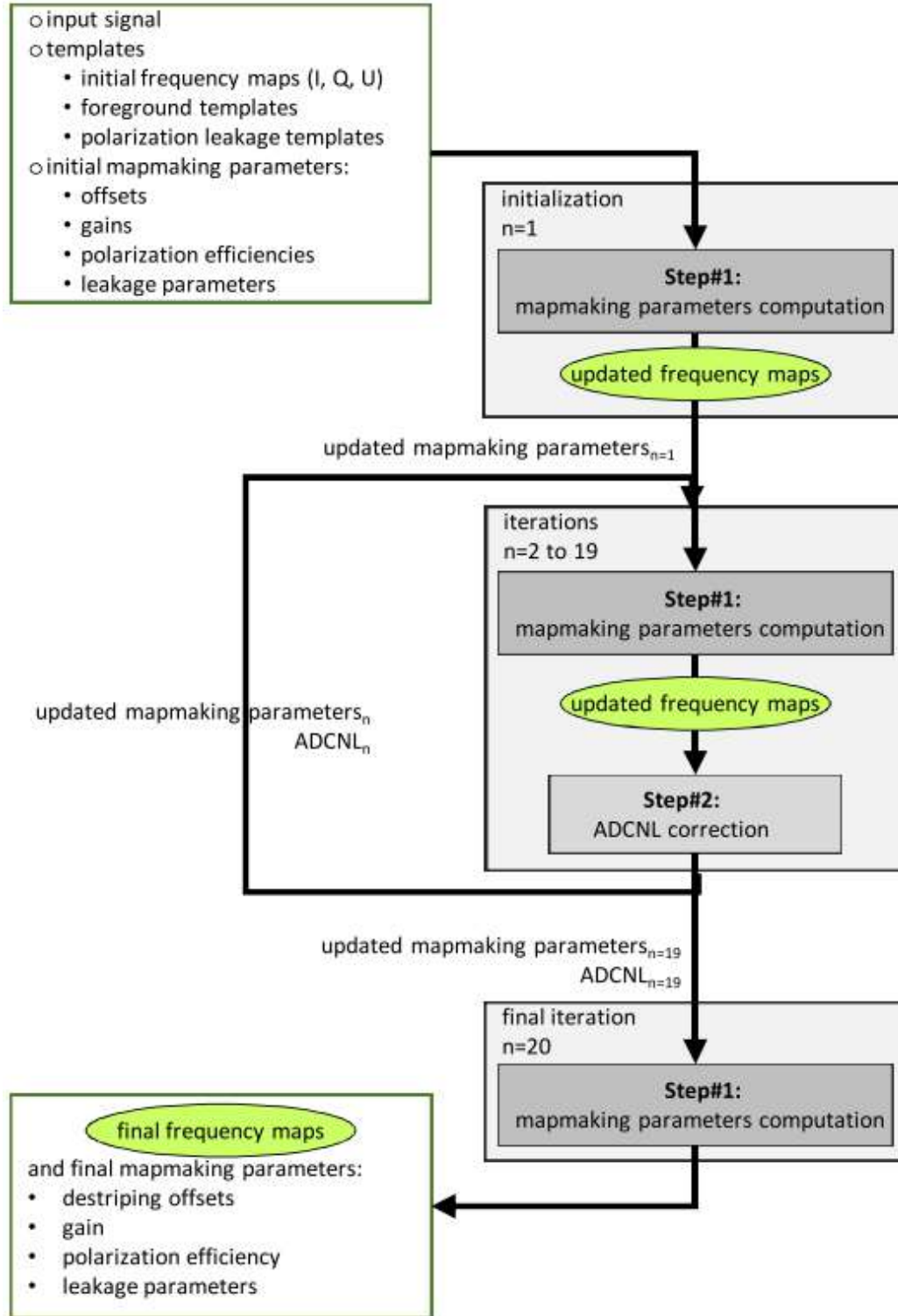


Fig. III.13 Schematic of a standard SROLL2 run structure. It consists in the succession of an initial **Step#1**, fitting all model parameters **except** those of the ADCNL effect, and then building cleaned sky maps, followed by $N_{iter} = 20$ iteration of the central block [**Step#1**+**Step#2**] where **Step#2** fits only the parameters of the ADCNL effect model, and concluded by a final **Step#1** producing all the SROLL2 run outputs.

Taken from [Delouis et al. \(2019\)](#) Fig.1.

Step#2 This second step is entirely dedicated to the determination of the ADCNL systematic effect, by fitting its model on any residual effect remaining after **Step#1**. The hypothesis on which this model is based is to consider any such residual effect remaining after **Step#1** to be entirely caused by the ADCNL. The ADCNL model, built from spline functions spanning both *time* and *ADU* dimensions, is then fitted on the aforementioned residual. Once the weights of the ADCNL spline functions are estimated (i.e. the $\aleph_{b,r,i}$), which I will not detail in this manuscript (see Delouis et al. (2019) for details), they can be subtracted from the signal for the computation of the next **Step#1**, as described by Eq.III.15. The fit performed at this step is made possible by the fact that each bolometer observes the same sky pixel within 2 different ADU locations, because of their response differences.

Steps separation This separation of **Step#1** and **Step#2** is **mandatory** since the effect of the ADCNL and the bolometer chain response are degenerate, whereby the impossibility to compute them at the same time. The algorithm must then first fit, in **Step#1**, its model on the data corrected for the latest estimation of the ADCNL effect, i.e. subtracted of the latest determination of $\aleph_{b,r,i}$. In **Step#2**, a new estimate of the $\aleph_{b,r}$ can thus be determined, which can then be subtracted from the data for the next **Step#1** computation, and so on and so forth, until both the responses $1/g_{b,i}$ and the ADCNL terms $\aleph_{b,r,i}$ have converged. What we call an *iteration* corresponds to one succession of **Step#1** and **Step#2** and their respective computation, as indicated by the central block in the schematic of Fig.III.13.

One can note from SRoll2's depiction in Fig.III.13, that 2 additional computations of **Step#1** are performed in a standard SRoll2 run: one before the first SRoll2 *iteration*, and one at the end of the last *iteration*, i.e. when $i = N_{iter}$. Each additional **Step#1** computation has its own reason of being:

- **First initialization Step#1** ► This first independent computation of **Step#1** is performed before the first complete SRoll2 *iteration* to ensure a correct convergence of the mapmaking model parameters determination before computing the first ADCNL determination. It was observed, during the SRoll2 version development, that without the addition of this first initialization **Step#1**, the ADCNL determination tended to significantly oscillate for the first few SRoll2 iterations. By correcting this oscillation of the ADCNL determination, this additional **Step#1** improves the overall algorithm convergence efficiency;
- **Last finalization Step#1** ► This last independent computation of **Step#1** is performed to produce the final estimate of all the SRoll2 mapmaking model parameters and cleaned sky maps. This last computation is performed using the last and stabilized determination of the ADCNL effect spline functions, i.e. the $\aleph_{b,r,i=N_{iter}}$, which are subtracted from the data signal to produce the final outputs.

N_{iter} determination The choice for the total number of iterations N_{iter} is driven by two antagonistic incentives:

- ☞ On the one hand, the higher the value of N_{iter} , the better the SRoll2 solution convergence, whereby the higher the final precision of the estimated parameters (at least until $N_{iter} \sim 55$, as shown by Fig.4 of Delouis et al. (2019)),
- ☞ On the other hand, the higher the value of N_{iter} , the higher the computational load of a run, which is a costly and limited resource provided by the NERSC.

Consequently, the middle ground value decided upon was the following: $N_{iter} = 20$. This allows a satisfactory convergence of the algorithm while retaining an acceptable computational load for a complete run. The standard SRoll2 run structure, which remains fixed throughout this whole manuscript, therefore consists of one initialization **Step#1**, followed by $N_{iter} = 20$ successive iterations of the full [**Step#1+Step#2**] block, concluded by a finalization **Step#1** computation. This results in a typical cost range of $\sim 40\text{-}60 \cdot 10^3$ CPU time hours for a complete SRoll2 run, depending on the frequency used, i.e. depending on the total number of bolometers used in the run.

III.2.2 Data model

Now that the SRoll2 version of the mapmaking algorithm has been introduced, described in broad functions, objectives, and looping structure, as well as characterized by its inputs and outputs, it is time to present the data model it rests upon, i.e. detail the content of the term $\mathcal{N}$ in Eq.III.1 in the case of *Planck*.

To this end, I will remind the whole SRoll2 data model used in the **Step#1** of the algorithm, as presented in Delouis et al. (2019). However, emphasis in details will only be put on the elements of the model which prove essential and/or strongly impacting the building and analysis of the CO maps. This is why the data model described in this subsection, as well as the χ^2 minimization computation detailed in the next subsection, are limited to the **Step#1** of the algorithm.

The SRoll2 data model can be written in the following form*:

$$g_b M_{b,r,p}^{tot} = A_{b,r,p} \cdot \left(S_p^{Emi} + \sum_{f \in \mathfrak{F}} \delta L_b^f C_p^f \right) + \sum_{h=1}^{N_{harm}} \gamma_b^h V_{b,r,p}^h + O_{b,r} + D_{r,p}^{tot} \\ + \delta \rho_b \left. \frac{\partial(gM)}{\partial \rho} \right|_{b,r,p} + \delta \phi_{b,r,p} \left. \frac{\partial(gM)}{\partial \phi} \right|_{\phi_{b,r,p}} + F_{b,r,p}^{tot} + g_b n_{b,r,p} + g_b \mathfrak{N}_{b,r,p}, \quad (\text{III.8})$$

where I use the notations below:

- g_b is the *inverse* of the bolometer "gain", i.e. the *inverse* of the calibrated response of the bolometer chain,
- $M_{b,r,p}^{tot}$ is the total measured bolometer signal data,
- $A_{b,r,p} = [1, \rho_b \cos(2\phi_{b,r,p}), \rho_b \sin(2\phi_{b,r,p})]$ is the pointing vector giving the observed pixel p by the bolometer b at the considered ring r , and it allows to "unproject" a map back into its corresponding HPR data (it corresponds to the projector P introduced in the generic description of subsection III.1.1),
- ρ_b is the polarization efficiency of bolometer b ,
- $\phi_{b,r,p}$ is the ground-measured detector polarization angle for bolometer b with respect to the north-south axis of the Galactic coordinate system,
- S_p^{Emi} is the $[I_p^{Emi}, Q_p^{Emi}, U_p^{Emi}]$ sky emission map components value at pixel p , corresponding to the sky signal deprived of both the *solar* and *orbital* dipoles (detailed in Eq.III.12). $[I_p^{Emi}, Q_p^{Emi}, U_p^{Emi}]$ represent the intensity and polarization component maps of the common sky emissions seen by all bolometers,

*Injected in Eq.III.17.

- δL_b^f is the bolometer response difference coefficient, for the foreground emission f , with respect to its corresponding reference value L_{ref}^f . This reference value is taken as the average bolometer foreground response over all bolometers. δL_b^f is also called the bolometer bandpass foreground color-correction coefficient relative to the bolometers average bandpass,
- C_p^f is the $[I_p^f, Q_p^f, U_p^f]$ spatial template sky components maps of foreground f seen by all bolometers. In the case of non-polarized templates, we therefore have $Q_p^f = U_p^f = 0$,
- γ_b^h is the fitted empirical time-transfer function amplitude,
- $V_{b,r,p}^h$ is the *timeline* template of the empirical time-transfer function computed from timeline convolution with long time constant (e.g. $> 1s$),
- $O_{b,r}$ is the offset per pointing period ring r used to model the $1/f$ noise,
- $D_{r,p}^{tot}$ is the *total* dipole signal, sum of the *solar* and *orbital* dipoles,
- $\delta\rho_b$ is the polarization efficiency correction term,
- $\left. \frac{\partial(gM)}{\partial\rho} \right|_{b,r,p}$ is the template of the derivative of the signal with respect to the polarization efficiency, taken at the input value ρ_b (cf. Eq.III.7),
- $\delta\phi_{b,r,p}$ is the detector polarization angle correction term,
- $\left. \frac{\partial(gM)}{\partial\phi} \right|_{\phi_{b,r,p}}$ is the template of the derivative of the signal with respect to $\phi_{b,r,p}$, taken at the input value of the angle $\phi_{b,r,p}$ (cf. Eq.III.7),
- $F_{b,r,p}^{tot}$ is the sum of the *total* dipole and the galactic signal integrated over the FSL, resulting in a FSL only sky signal,
- $n_{b,r,p}$ is the pixel white noise (electronic + photon noise) in bolometer b , with variance $\sigma_{b,r,p}^2$ (note that the number of samples falling in pixel p from the ring r of bolometer b , called *hitcount*, is taken into account in the value of the $\sigma_{b,r,p}^2$),
- $\aleph_{b,r}$ is the signal distortion induced by the ADCNL residual effect for the bolometer b , at ring r .

This equation of the data model corresponds to Eq.III.1 in the generic description of SRoll.

In the above Eq.III.8, the values of the δL_b^f correspond, for the foreground f , to the bandpass **mismatch** colour-correction coefficient of the bolometer b with respect to a reference value L_{ref}^f , which can be written:

$$\forall f \in \mathfrak{F}, \forall b \in \mathfrak{B}, \delta L_b^f = L_b^f - L_{ref}^f, \quad (\text{III.9})$$

with L_b^f being the response coefficient of bolometer b to foreground f , as physically defined by Eq.III.5.

Closure conditions As was discussed for Eq.III.3, and illustrated in the case of SRoll2 by later Eq.III.24, the δL_b^f defined in Eq.III.9 require a *closure condition* for the minimization to be fully defined, i.e. SRoll2 requires a condition for the value of L_{ref}^f . Given that we have no additional information about the average foreground response coefficient, the *closure condition* is defined for the δL_b^f such that L_{ref}^f corresponds to the aforementioned average foreground response coefficient over all bolometers of the SRoll2 run, which writes:

$$L_{ref}^f = \overline{L^f}^{N_{bolo}} = \frac{1}{N_{bolo}} \sum_{b=1}^{N_{bolo}} L_b^f.$$

where $\overline{L^f}^{N_{bolo}}$ is the average value of the foreground response coefficients for the foreground emission f . To achieve this selection of the reference value, the closure condition is taken as:

$$\forall f \in \mathfrak{F}, \quad \sum_{b=1}^{N_{bolo}} \delta L_b^f = 0. \quad (\text{III.10})$$

This therefore ensure that we have:

$$\begin{aligned} \sum_{b=1}^{N_{bolo}} \delta L_b^f = 0 &\Leftrightarrow \sum_{b=1}^{N_{bolo}} (L_b^f - L_{ref}^f) = 0 \\ &\Leftrightarrow L_{ref}^f = \overline{L^f}^{N_{bolo}}. \end{aligned}$$

Similarly, the determination of the ring offsets, i.e. the $O_{b,r}$, used to model the $1/f$ noise, also needs a *closure condition* since SRoll2 does not provide information about the maps *monopoles* (average value in the whole timeline, therefore corresponding to that of the frequency maps). Consequently, this closure condition is defined by:

$$\sum_{b=1}^{N_{bolo}} \sum_{r=1}^{N_{ring}} O_{b,r} = 0.$$

Following the same logic, the *closure condition* for the polarization efficiency correction is defined by:

$$\sum_{b=1}^{N_{bolo}} \delta \rho_b = 0.$$

Note that in the case of the polarization efficiency, though SRoll determines small corrections, the average value of this parameter is already known from the input values, which we can use when applying the small corresponding corrections to the output signal.

To ensure that there is no introduction of any bias in the bolometers polarization angle, the following *closure condition* was also added for the detector polarization angle correction:

$$\forall p, \quad \sum_{b=1}^{N_{bolo}} \sum_{r=1}^{N_{ring}} \delta \phi_{b,r,p} = 0.$$

Note on the absence of closure condition for the $\delta g_{b,i}$ and $\gamma_{b,i}^h$ At this point, one might be wondering why a *closure condition* was defined for the parameters fitted by SRoll2, except for the parameters $\delta g_{b,i}$ and the $\gamma_{b,i}^h$. The reason resides in a fundamental difference between the signal contribution from which they are fitted and that of the other parameters: $\delta g_{b,i}$ and $\gamma_{b,i}^h$ are fitted on a signal which is *time-dependent* and **does not only**

project on the sky. Indeed, the $\delta g_{b,i}$ are fitted on the *orbital* dipole component, and the $\gamma_{b,i}^h$ on the template of the time-transfer function defined as a convolution of the timeline. In each case, the signal on which those parameters are fitted are time-dependent *within* the pixel, while the parameters depend only on b . This fitting of a parameter only depending on b being fitted on a signal depending on r *within* each pixel, provides us with many more independent equations, allowing an absolute fitting in the case of those parameters, i.e. a determination of the average value using the accurate knowledge of the signal on which they are fitted.

From the above definition of their *closure condition*, the bolometer RDCs, values fitted by SRoll2, can be written:

$$\forall b \in \mathfrak{B}, \forall f \in \mathfrak{F}, \delta L_b^f = L_b^f - \overline{L^f}^{N_{bolo}}. \quad (\text{III.11})$$

The value fitted by SRoll2 for each foreground is then a *differential* correction term with respect to the average foreground f signal over all bolometers of the considered run. This average foreground signal is implicitly present in the data model defined by Eq.III.8 as it is part of the sky emission signal S^{Emi} . If we want to detail the elements composing this sky emission, it could be expressed as follows:

$$S_p^{Emi} = S_p^{CMB} + \sum_{f \in \mathfrak{F}} \overline{L^f}^{N_{bolo}} S_p^f, \quad (\text{III.12})$$

where $\overline{L^f}^{N_{bolo}} S_p^f = \overline{L^f}^{N_{bolo}} [I_p^f, Q_p^f, U_p^f]$ is the average signal maps of foreground f over all bolometers used in the considered run, and S_p^{CMB} is the $[I_p^{CMB}, Q_p^{CMB}, U_p^{CMB}]$ CMB anisotropies signal map components. However, these elements composing the sky signal S^{Emi} , detailed in the above equation, are never directly accessed by SRoll2 in a run, and only the terms appearing in Eq.III.8 are computationally accessible and modelled in SRoll2.

Since SRoll2 only fits the values of the response *difference* coefficients, i.e. the δL_b^f (because it only works by fitting its parameters on differences, cf Eq.III.3 and Eq.III.21), and since it does not have access to the average values $\overline{L^f}^{N_{bolo}}$, it cannot either determine the response coefficients L_b^f . To determine the L_b^f , an additional information would be necessary, in the form of a *global* calibrator for the values of the $\overline{L^f}^{N_{bolo}}$. Thankfully, this will not limit our main goal of producing full-sky ^{12}CO and ^{13}CO maps, as to this end the *difference* coefficients δL_b^f are sufficient, cf. subsection IV.2.1.

As previously discussed, one of the main changes in the data model from the SRoll1 to the SRoll2 version of the mapmaking algorithm consists in fitting **temporally stable** bolometer responses, while redefining completely the ADCNL determination, using a model of the effect of the ADCNL dependent **both** on ADU and on *time*. In SRoll2, the fitted bolometer response can be decomposed as follows:

$$g_{b,i+1} = g_{b,i} - \delta g_{b,i}. \quad (\text{III.13})$$

At each iteration i , the minimization provides a determination of the $\delta g_{b,i}$, which allows an update of the bolometer response values. The $\delta g_{b,i}$ must then converge towards zero as SRoll2 iteratively converges towards a fit of its data model, i.e. $\forall b, \lim_{i \rightarrow \infty} \delta g_{b,i} = 0$. It will

prove useful later to rewrite the above Eq.III.13 as follows:

$$\begin{aligned}
 g_{b,i} &= g_{b,i+1} + \delta g_{b,i} \\
 &= g_{b,i+1} \left(1 + \frac{\delta g_{b,i}}{g_{b,i+1}} \right) \\
 &= g_{b,i+1} (1 + \tilde{\delta} g_{b,i}),
 \end{aligned} \tag{III.14}$$

where I defined $\tilde{\delta} g_{b,i} = \frac{\delta g_{b,i}}{g_{b,i+1}}$. Indeed, what SRoll2 *actually* fits in its data model is the value of $\tilde{\delta} g_{b,i}$, which allows the computation of the new response values $g_{b,i+1}$. Similarly to the initial definition, $\tilde{\delta} g_{b,i}$ must also converge towards zero as SRoll2's minimization iteratively converges, i.e. $\forall b, \lim_{i \rightarrow \infty} \tilde{\delta} g_{b,i} = 0$.

The parameters fitted in the SRoll2 data model all correspond to low-intensity effects in the signal. In order to work on the signal at the scale of these effects, we subtract from the signal the well-known and substantial total dipole contribution, as well as the low-amplitude effect of the FSL. Since we need to fit **separately** the chain responses and the ADCNL contribution (because the 2 effects are degenerate), we also subtract the latest determination of the ADCNL term $\aleph_{b,r,i}$, which starts at the first iteration $i = 0$ as $\aleph_{b,r,i=0} = 0$.

Therefore, the SRoll2 minimization is computed on the remaining part of the signal, called the *reduced* bolometer signal, and which can then be written:

$$g_{b,i} \dot{M}_{b,r,p}^{tot} = g_{b,i} M_{b,r,p}^{tot} - D_{r,p}^{tot} - F_{b,r,p}^{tot} - g_{b,i} \aleph_{b,r,i}. \tag{III.15}$$

At the beginning of the iteration i :

- ★ We *know* the values of $g_{b,i}$ and $\aleph_{b,r,i}$, which were determined at the previous iteration;
- ★ We primarily *aim to determine*:

- ① at **Step#1** the values of $\tilde{\delta} g_{b,i}$ (to deduce those of the $g_{b,i+1}$), $\gamma_{b,i}^h$, $\delta L_{b,i}^f$, $O_{b,r,i}$, $\delta \rho_{b,i}$, and $\delta \phi_{b,r,p,i}$,
- ② at **Step#2** the values of $\aleph_{b,r,i+1}$.

Keep in mind that the data model described here, as well as the minimization detailed in the next subsection, correspond to the computation performed at **Step#1 only**. This is why, in the equations written here, the term for the ADCNL $\aleph_{b,r,i}$ only appears with the index of the *current* iteration i . This term is fixed during the computation of **Step#1**, and its next estimation $\aleph_{b,r,i+1}$ only appears in **Step#2**, which I will not detail in this work. The χ^2 minimization of the **Step#1** is then detailed in the next subsection.

I can now detail the model of the *reduced* bolometer signal in SRoll2. By injecting the expression of Eq.III.14 in the above Eq.III.15, it becomes:

$$g_{b,i} \dot{M}_{b,r,p}^{tot} = (1 + \tilde{\delta} g_{b,i}) g_{b,i+1} M_{b,r,p}^{tot} - D_{r,p}^{tot} - F_{b,r,p}^{tot} - g_{b,i} \aleph_{b,r,i}. \tag{III.16}$$

Then, using Eq.III.8, and developing the factor $(1 + \tilde{\partial}g_{b,i})$, the above Eq.III.16 becomes:

$$\begin{aligned}
g_{b,i}\dot{M}_{b,r,p}^{tot} = & A_{b,r,p} \cdot \left(S_p^{Emi} + \sum_{f \in \mathfrak{F}} \delta L_{b,i}^f C_p^f \right) + \sum_{h=1}^{N_{harm}} \gamma_{b,i}^h V_{b,r,p}^h + O_{b,r,i} \\
& + \delta \rho_{b,i} \left. \frac{\partial(gM)}{\partial \rho} \right|_{b,r,p} + \delta \phi_{b,r,p,i} \left. \frac{\partial(gM)}{\partial \phi} \right|_{\phi_{b,r,p}} + g_{b,i+1} n_{b,r,p} \\
& + g_{b,i+1} \aleph_{b,r,i} - g_{b,i} \aleph_{b,r,i} \\
& + \tilde{\partial}g_{b,i} \left(A_{b,r,p} \cdot S_p^{Emi} + D_{r,p}^{tot} + \sum_{h=1}^{N_{harm}} \gamma_{b,i}^h V_{b,r,p}^h + O_{b,r,i} + F_{b,r,p}^{tot} \right. \\
& \quad + A_{b,r,p} \cdot \sum_{f \in \mathfrak{F}} \delta L_{b,i}^f C_p^f + \delta \rho_{b,i} \left. \frac{\partial(gM)}{\partial \rho} \right|_{b,r,p} + \delta \phi_{b,r,p,i} \left. \frac{\partial(gM)}{\partial \phi} \right|_{\phi_{b,r,p}} \\
& \quad \left. + g_{b,i+1} \aleph_{b,r,i} + g_{b,i+1} n_{b,r,p} \right). \tag{III.17}
\end{aligned}$$

For ease of notations, we additionally introduce the systematic effects term:

$$\begin{aligned}
\epsilon_{b,r,p,i} = & \sum_{h=1}^{N_{harm}} \gamma_{b,i}^h V_{b,r,p}^h + A_{b,r,p} \cdot \sum_{f \in \mathfrak{F}} \delta L_{b,i}^f C_p^f + O_{b,r,i} \\
& + \delta \rho_{b,i} \left. \frac{\partial(gM)}{\partial \rho} \right|_{b,r,p} + \delta \phi_{b,r,p,i} \left. \frac{\partial(gM)}{\partial \phi} \right|_{\phi_{b,r,p}} + F_{b,r,p}^{tot}. \tag{III.18}
\end{aligned}$$

From the intermediary development above, and using the term defined by Eq.III.18, by regrouping all terms with $\aleph_{b,r,i}$ together, and doing the same with the term containing $n_{b,r,p}$, we can finally rewrite the *reduced* bolometer signal as follows*:

$$\begin{aligned}
g_{b,i}\dot{M}_{b,r,p}^{tot} = & A_{b,r,p} \cdot \left(S_p^{Emi} + \sum_{f \in \mathfrak{F}} \delta L_{b,i}^f C_p^f \right) + \sum_{h=1}^{N_{harm}} \gamma_{b,i}^h V_{b,r,p}^h + O_{b,r,i} \\
& + \delta \rho_{b,i} \left. \frac{\partial(gM)}{\partial \rho} \right|_{b,r,p} + \delta \phi_{b,r,p,i} \left. \frac{\partial(gM)}{\partial \phi} \right|_{\phi_{b,r,p}} + (g_{b,i+1} + \tilde{\partial}g_{b,i}g_{b,i+1})n_{b,r,p} \\
& - \delta g_{b,i} \aleph_{b,r,i} + \tilde{\partial}g_{b,i}g_{b,i+1} \aleph_{b,r,i} \\
& + \tilde{\partial}g_{b,i} \left(A_{b,r,p} \cdot S_p^{Emi} + D_{r,p}^{tot} + \epsilon_{b,r,p,i} \right) \tag{III.19} \\
= & A_{b,r,p} \cdot \left(S_p^{Emi} + \sum_{f \in \mathfrak{F}} \delta L_{b,i}^f C_p^f \right) + \sum_{h=1}^{N_{harm}} \gamma_{b,i}^h V_{b,r,p}^h + O_{b,r,i} \\
& + \delta \rho_{b,i} \left. \frac{\partial(gM)}{\partial \rho} \right|_{b,r,p} + \delta \phi_{b,r,p,i} \left. \frac{\partial(gM)}{\partial \phi} \right|_{\phi_{b,r,p}} + g_{b,i} n_{b,r,p} \\
& + \tilde{\partial}g_{b,i} \left(A_{b,r,p} \cdot S_p^{Emi} + D_{r,p}^{tot} + \epsilon_{b,r,p,i} \right).
\end{aligned}$$

In Eq.III.19, since we aim to fit the values of the $\tilde{\partial}g_{b,i}$ from differences between various observations, the good convergence of the algorithm **requires** that we *know* the term $(A_{b,r,p} \cdot S_p^{Emi} + D_{r,p}^{tot} + \epsilon_{b,r,p,i})$. By masking the sky regions with strong signal gradients, which may

*Injected in Eq.III.22.

cause spurious leakage from bandpass mismatch and beam anisotropy, we ensure that for all pixels kept for the minimization, and at all iteration, we have (cf. [Planck Collaboration Int. XLVI \(2016\)](#), [Planck Collaboration III \(2020\)](#), [Delouis et al. \(2019\)](#)):

$$\forall i \forall p, \epsilon_{b,r,p,i} \ll A_{b,r,p} \cdot S_p^{Emi} + D_{r,p}^{tot}. \quad (\text{III.20})$$

This allows to neglect the $\epsilon_{b,r,p,i}$ term in Eq. III.19. The only part of the term left that we **need** to know is the term on which the $\delta g_{b,i}$ are fitted, i.e. $(A_{b,r,p} \cdot S_p^{Emi} + D_{r,p}^{tot})$, condition met through the input of a very accurate $D_{r,p}^{tot}$ (to a relative precision of $\sim 10^{-4}$) and an initial estimation of S_p^{Emi} from the HFI 2018 *Planck* legacy frequency maps.

III.2.3 Mapmaking parameters computation (Step#1)

As previously mentioned, the SROLL algorithm achieves the determination of its parameters by making use of the redundancy in the observational data which is twofold: first because each detector makes multiple observations of each pixel p at different points in time, but also because each pixel is seen at each time simultaneously by multiple bolometers. The last and key quantity used to fit the various parameters thanks to the data redundancy is called the *vector residual* for a detector b and pointing ring r , as it was introduced for Eq. III.3. It corresponds to the difference between the observation of a pixel p made by a bolometer b in a ring r and the average observation of the same pixel p by all bolometers at all rings reprojected onto the aforementioned observation. For reasons mentioned in the previous subsection, the *vector residual* is defined on the *reduced* bolometer signal.

To be able to express the *vector residual*, we need beforehand to define the collections of the various parameters and quantities as was generically defined for Eq. III.2. These collections are vectors or matrices built from the collection of values taken by a quantity at each observation of a given pixel p , i.e. the collection of observations of pixel p by all bolometers and at all rings passing by the considered pixel. All collections are each appropriately weighed by the corresponding inverse of white noise variance $1/\sigma_{b,r,p}^2$. Given that said pixel p has been observed a total of N_{hit} times during the mission, then these collections have at least one of their dimensions equal to N_{hit} . Below is a list of all the collections required, where the index of each observation of pixel p are written as an additional index, up to N_{hit} , on the indices b and r . Therefore, a specific observation is characterized by the doublet (b_k, r_k) with $k \in \llbracket 1, N_{hit} \rrbracket$. When a quantity depends on only one of the 2 indices, the specific observation is then characterized by either b_k or r_k , following the aforementioned dependence.

$\mathbb{A}_p$ This matrix is the collection of all the values of $A_{b,r,p}$ for the observations of pixel p at all bolometers b and all rings r . It can be written:

$$\begin{aligned} \mathbb{A}_p &= \left(\begin{array}{c} \vdots \\ A_{b_k, r_k, p} / \sigma_{b_k, r_k, p}^2 \\ \vdots \end{array} \right)_{k \in \llbracket 1, N_{hit} \rrbracket} \\ &= \left(\begin{array}{ccc} \vdots & \vdots & \vdots \\ \frac{1}{\sigma_{b_k, r_k, p}^2} & \frac{\rho_{b_k}}{\sigma_{b_k, r_k, p}^2} \cos(2\phi_{b_k, r_k, p}) & \frac{\rho_{b_k}}{\sigma_{b_k, r_k, p}^2} \sin(2\phi_{b_k, r_k, p}) \\ \vdots & \vdots & \vdots \end{array} \right)_{k \in \llbracket 1, N_{hit} \rrbracket}. \end{aligned}$$

$\dot{\mathbb{M}}_{p,i}$ This vector is the collection of all the values of $g_{b,i}\dot{M}_{b,r,p}^{tot}$ for the observations of pixel p at all bolometers b and all rings r . It can be written:

$$\dot{\mathbb{M}}_{p,i} = \left(\begin{array}{c} \vdots \\ g_{b_k,i}\dot{M}_{b_k,r_k,p}^{tot}/\sigma_{b_k,r_k,p}^2 \\ \vdots \end{array} \right)_{k \in \llbracket 1, N_{hit} \rrbracket}.$$

$\partial\mathbb{S}_{p,i}$ This vector is the collection of all the values of $\partial g_{b,i}(A_{b,r,p} \cdot S_p^{Emi} + D_{r,p}^{tot})$ for the observations of pixel p at all bolometers b and all rings r . It can be written:

$$\partial\mathbb{S}_{p,i} = \left(\begin{array}{c} \vdots \\ \partial g_{b_k,i}(A_{b_k,r_k,p} \cdot S_p^{Emi} + D_{r_k,p}^{tot})/\sigma_{b_k,r_k,p}^2 \\ \vdots \end{array} \right)_{k \in \llbracket 1, N_{hit} \rrbracket}.$$

$\mathbb{C}_{p,i}^f$ This vector is the collection of all the values of $\delta L_{b,i}^f A_{b,r,p} \cdot C_p^f$ for the observations of pixel p at all bolometers b and all rings r . It can be written:

$$\mathbb{C}_{p,i}^f = \left(\begin{array}{c} \vdots \\ \delta L_{b_k,i}^f A_{b_k,r_k,p}/\sigma_{b_k,r_k,p}^2 \\ \vdots \end{array} \right)_{k \in \llbracket 1, N_{hit} \rrbracket} \cdot C_p^f.$$

$\mathbb{V}_{p,i}^h$ This vector is the collection of all the values of $\gamma_{b,i}^h V_{b,r,p}^h$ for the observations of pixel p at all bolometers b and all rings r . It can be written:

$$\mathbb{V}_{p,i}^h = \left(\begin{array}{c} \vdots \\ \gamma_{b_k,i}^h V_{b_k,r_k,p}^h/\sigma_{b_k,r_k,p}^2 \\ \vdots \end{array} \right)_{k \in \llbracket 1, N_{hit} \rrbracket}.$$

$\mathbb{O}_{p,i}$ This vector is the collection of all the values of $O_{b,r,i}$ for the observations of pixel p at all bolometers b and all rings r . It can be written:

$$\mathbb{O}_{p,i} = \left(\begin{array}{c} \vdots \\ O_{b_k,r_k,i}/\sigma_{b_k,r_k,p}^2 \\ \vdots \end{array} \right)_{k \in \llbracket 1, N_{hit} \rrbracket}.$$

Note: One can remark that the resulting vector $\mathbb{O}_{p,i}$ only depends on p , although its elements do not explicitly depend on p . Though the index p does not appear in the elements of vector $\mathbb{O}_{p,i}$, the dependence on p is implicitly present in the set of rings on which this collection is built. Indeed, the vector $\mathbb{O}_{p,i}$ is only built on rings which *sample* the pixel p , i.e. on rings which observe the pixel p .

Table III.4: Dimensions of all non-scalar quantities appearing in the equations detailing the data model and minimization computed in SROLL2.

Quantities	Dimension
$\hat{\mathbb{M}}_{p,i}, \hat{\mathbb{S}}_{p,i}, \mathbb{C}_{p,i}^f, \mathbb{V}_{p,i}^h, \mathbb{O}_{p,i}, \mathbf{q}_{p,i}, \mathbf{\Phi}_{p,i}, \mathbf{n}_p, \boldsymbol{\sigma}_{p,i}, \mathbb{S}_{p,i}^{eval}, \hat{\mathbb{M}}_{p,i}^{out}$	$N_{hit} \times 1$
$A_{b,r,p}$	1×3
$T_{b,r,p}$	$1 \times N_{hit}$
$\mathbb{A}_p$	$N_{hit} \times 3$
$\mathbb{A}_p^\top, (\mathbb{A}_p^\top \mathbb{A}_p)^{-1} \mathbb{A}_p^\top$	$3 \times N_{hit}$
$(\mathbb{A}_p^\top \mathbb{A}_p)$	3×3

$\mathbf{q}_{p,i}$ This vector is the collection of all the values of $\delta\rho_{b,i} \frac{\partial(gM)}{\partial\rho} \Big|_{b,r,p}$ for the observations of pixel p at all bolometers b and all rings r . It can be written:

$$\mathbf{q}_{p,i} = \begin{pmatrix} \vdots \\ \delta\rho_{b_k,i} \frac{\partial(gM)}{\partial\rho} \Big|_{b_k,r_k,p} / \sigma_{b_k,r_k,p}^2 \\ \vdots \end{pmatrix}_{k \in \llbracket 1, N_{hit} \rrbracket}.$$

$\mathbf{\Phi}_{p,i}$ This vector is the collection of all the values of $\delta\phi_{b,r,p,i} \frac{\partial(gM)}{\partial\phi} \Big|_{\phi_{b,r,p}}$ for the observations of pixel p at all bolometers b and all rings r . It can be written:

$$\mathbf{\Phi}_{p,i} = \begin{pmatrix} \vdots \\ \delta\phi_{b_k,r_k,p,i} \frac{\partial(gM)}{\partial\phi} \Big|_{\phi_{b_k,r_k,p}} / \sigma_{b_k,r_k,p}^2 \\ \vdots \end{pmatrix}_{k \in \llbracket 1, N_{hit} \rrbracket}.$$

$\mathbf{n}_p$ This vector is the collection of all the values of $n_{b,r,p}$ for the observations of pixel p at all bolometers b and all rings r . It can be written:

$$\mathbf{n}_p = \begin{pmatrix} \vdots \\ n_{b_k,r_k,p} / \sigma_{b_k,r_k,p}^2 \\ \vdots \end{pmatrix}_{k \in \llbracket 1, N_{hit} \rrbracket}.$$

The respective dimensions of all the collections defined above are listed in Table III.4. I also list the dimensions of some other collections which are defined later in this subsection, all non-scalar quantities appearing in this section, as well as some products of aforementioned quantities and collections, which often appear in this section's equations.

The **data** vector residual can then be written as:

$$R_{b,r,p,i} = g_{b,i} \hat{M}_{b,r,p}^{tot} - T_{b,r,p} \hat{\mathbb{M}}_{p,i}, \quad (\text{III.21})$$

where $T_{b,r,p}$ is the projection matrix used to project into a single average HPR corresponding to the observation of pixel p by the bolometer b at ring r , the matrix built from all the

observations made of said pixel p , i.e. from all bolometers and all rings inside it. $T_{b,r,p}$ corresponds to the value $P(\mathbb{P}^\top \mathbb{P})^{-1} \mathbb{P}^\top$ introduced in Eq.III.2. It can be written:

$$T_{b,r,p} = A_{b,r,p} \left(\mathbb{A}_p^\top \mathbb{A}_p \right)^{-1} \mathbb{A}_p^\top.$$

Also, the **data** vector residual expressed above corresponds to the left side of Eq.III.3, i.e. to the term noted R in Eq.III.3 of the generic description of SRoll.

Injecting Eq.III.19 (while applying the property of Eq.III.20) into Eq.III.21, and using the definition of $\tilde{M}_{p,i}$, the **data** vector residual is described by the SRoll2 data model as:

$$\begin{aligned} R_{b,r,p,i} = & A_{b,r,p} \cdot \left(S_p^{Emi} + \sum_{f \in \mathfrak{F}} \delta L_{b,i}^f C_p^f \right) + \sum_{h=1}^{N_{harm}} \gamma_{b,i}^h V_{b,r,p}^h + O_{b,r,i} + \delta \rho_{b,i} \left. \frac{\partial(gM)}{\partial \rho} \right|_{b,r,p} \\ & + \delta \phi_{b,r,p,i} \left. \frac{\partial(gM)}{\partial \phi} \right|_{\phi_{b,r,p}} + \tilde{\partial} g_{b,i} \left(A_{b,r,p} \cdot S_p^{Emi} + D_{r,p}^{tot} \right) + g_{b,i} n_{b,r,p} \\ & - T_{b,r,p} \left(\mathbb{A}_p \cdot S_p^{Emi} + \sum_{f \in \mathfrak{F}} \mathbb{C}_{p,i}^f + \sum_{h=1}^{N_{harm}} \mathbb{V}_{p,i}^h + \mathbb{O}_{p,i} + \mathbf{e}_{p,i} + \mathbf{\Phi}_{p,i} + \mathbf{n}_p + \tilde{\partial} \mathbb{S}_{p,i} \right). \end{aligned} \quad (\text{III.22})$$

Then, making use of the fact that $T_{b,r,p} \mathbb{A}_p \cdot S_p^{Emi} = A_{b,r,p} \cdot S_p^{Emi}$, and gathering together the terms referencing each of the modelled systematic effects, the above Eq.III.22 becomes:

$$\begin{aligned} R_{b,r,p,i} = & \tilde{\partial} g_{b,i} (A_{b,r,p} \cdot S_p^{Emi} + D_{r,p}^{tot}) - T_{b,r,p} \tilde{\partial} \mathbb{S}_{p,i} + O_{b,r,i} - T_{b,r,p} \mathbb{O}_{p,i} \\ & + \sum_{f \in \mathfrak{F}} \left(\delta L_{b,i}^f A_{b,r,p} \cdot C_p^f - T_{b,r,p} \mathbb{C}_{p,i}^f \right) + \sum_{h=1}^{N_{harm}} \left(\gamma_{b,i}^h V_{b,r,p}^h - T_{b,r,p} \mathbb{V}_{p,i}^h \right) \\ & + \delta \rho_{b,i} \left. \frac{\partial(gM)}{\partial \rho} \right|_{b,r,p} - T_{b,r,p} \mathbf{e}_{p,i} + \delta \phi_{b,r,p,i} \left. \frac{\partial(gM)}{\partial \phi} \right|_{\phi_{b,r,p}} - T_{b,r,p} \mathbf{\Phi}_{p,i} \\ & + g_{b,i} n_{b,r,p} - T_{b,r,p} \mathbf{n}_p. \end{aligned} \quad (\text{III.23})$$

Following the model given by Eq.III.23, we can now define the **model** vector residual, which is the quantity built from the templates – known and **fixed** input values – and parameters – parameters fitted to minimize the χ^2 later defined by Eq.III.25 –, as follows:

$$\begin{aligned} \mathcal{R}_{b,r,p,i} = & \tilde{\partial} g_{b,i} M_{b,r,p}^{eval} - T_{b,r,p} \tilde{\partial} \mathbb{S}_{p,i}^{eval} + \tilde{O}_{b,r,i} - T_{b,r,p} \tilde{\mathbb{O}}_{p,i} \\ & + \sum_{f \in \mathfrak{F}} \left(\tilde{\delta} L_{b,i}^f A_{b,r,p} \cdot C_p^f - T_{b,r,p} \tilde{\mathbb{C}}_{p,i}^f \right) + \sum_{h=1}^{N_{harm}} \left(\tilde{\gamma}_{b,i}^h V_{b,r,p}^h - T_{b,r,p} \tilde{\mathbb{V}}_{p,i}^h \right) \\ & + \tilde{\delta} \rho_{b,i} \left. \frac{\partial(gM)}{\partial \rho} \right|_{b,r,p} - T_{b,r,p} \tilde{\mathbf{e}}_{b,r,p,i} + \tilde{\delta} \phi_{b,r,p,i} \left. \frac{\partial(gM)}{\partial \phi} \right|_{\phi_{b,r,p}} - T_{b,r,p} \tilde{\mathbf{\Phi}}_{b,r,p,i}, \end{aligned} \quad (\text{III.24})$$

where a quantity X is noted $\tilde{X}$ when it refers to a quantity determined by SRoll2 and affected by noise and error, and where $M_{b,r,p}^{eval}$ and $\tilde{\partial} \mathbb{S}_{p,i}^{eval}$ are respectively the template value taken for $(A_{b,r,p} \cdot S_p^{Emi} + D_{r,p}^{tot})$ and the template value taken for $\tilde{\partial} \mathbb{S}_{p,i}$. $M_{b,r,p}^{eval}$ is built from the template maps entered as input for the sky signal as introduced in subsection III.2.1.a), and with the added template of $D_{r,p}^{tot}$. $\tilde{\partial} \mathbb{S}_{p,i}^{eval}$ is built the same way as $\tilde{\partial} \mathbb{S}_{p,i}$ is defined, using the template $M_{b,r,p}^{eval}$, i.e. it is the vector of the collection of all values of $\tilde{\partial} g_{b,i} M_{b,r,p}^{eval}$ for the

observations of pixel p at all bolometers b and all rings r . It can be written:

$$\widetilde{\mathbf{S}}_{p,i}^{eval} = \begin{pmatrix} \vdots \\ \widetilde{\mathbf{g}}_{b_k,i} M_{b_k,r_k,p}^{eval} / \sigma_{b_k,r_k,p}^2 \\ \vdots \end{pmatrix}_{k \in \llbracket 1, N_{hit} \rrbracket}.$$

Note that the **model** vector residual corresponds to the term noted $\mathcal{R}$ in the generic description of SRoll, and expressed in Eq.III.3.

Using these **data** and **model** vector residuals, for each iteration, the minimization of the following χ^2 is computed at the end of **Step#1**:

$$\chi_i^2 = \sum_{b=1}^{N_{bolo}} \sum_{r=1}^{N_{ring}} \sum_{p=1}^{N_{pix}} \frac{(R_{b,r,p,i} - \mathcal{R}_{b,r,p,i})^2}{\sigma_R^2}, \quad (\text{III.25})$$

with σ_R^2 being the variance of the noise impacting the *data* vector residual $R_{b,r,p,i}$. Using the definition of the *data* vector residual given by Eq.III.21, and given that the known variance of noise affecting the measure of $M_{b,r,p}^{tot}$ is $\sigma_{b,r,p}^2$, the variance σ_R^2 can be written:

$$\sigma_R^2 = (g_{b,i} \sigma_{b,r,p})^2 + T_{b,r,p} \sigma_{p,i}^2$$

where $\sigma_{p,i}^2$ is the collection of all the values of $(g_{b,i} \sigma_{b,r,p})^2$ for the observations of pixel p at all bolometers b and all rings r , and can be written:

$$\sigma_{p,i}^2 = \begin{pmatrix} \vdots \\ (g_{b_k,i} \sigma_{b_k,r_k,p})^2 \\ \vdots \end{pmatrix}_{k \in \llbracket 1, N_{hit} \rrbracket}.$$

It is to note that $\sigma_{p,i}^2$ only depends on p , although its elements do not explicitly depend on p . As in the case of the vector $\mathbb{O}_{p,i}$, the dependence on p is implicitly present in the set of rings on which this collection is built.

From this, the χ^2 minimization computed at the end of each **Step#1**, defined by Eq.III.25, can be expressed as follows:

$$\chi_i^2 = \sum_{b=1}^{N_{bolo}} \sum_{r=1}^{N_{ring}} \sum_{p=1}^{N_{pix}} \frac{(R_{b,r,p,i} - \mathcal{R}_{b,r,p,i})^2}{(g_{b,i} \sigma_{b,r,p})^2 + T_{b,r,p} \sigma_{p,i}^2}. \quad (\text{III.26})$$

In the χ^2 minimization of Eq.III.26, the denominator $[(g_{b,i} \sigma_{b,r,p})^2 + T_{b,r,p} \sigma_{p,i}^2]$, corresponding to the variance σ_R^2 , contains the factor $g_{b,i}$. It is **essential** to not forget this factor as to not let the determination of the bolometer chain responses be biased by the noise level. Without it, the higher the level of noise, the smaller the fitted value of the $g_{b,i}$, since the best numerical solution to reduce the noise is to decrease the value of the gains. This denominator term thereby ensures a responses determination free of such bias.

Finally, from the estimates of $\widetilde{\mathbf{g}}_{b,i}$, $\widetilde{\mathbf{O}}_{b,r,i}$, $\widetilde{\mathbf{L}}_{b,i}^f$, and $\widetilde{\gamma}_{b,i}^h$ provided by the χ^2 minimization defined by Eq.III.26, each **Step#1** of each iteration concludes with the computation of the *cleaned* and *calibrated* bolometer signal, followed by its projection into *cleaned* frequency maps. Each *cleaned* and *calibrated* bolometer signal is then obtained by removing from

the signal the fitted instrumental systematic effects and applying the newly determined calibration, which is computed as follows:

$$\begin{aligned} \widetilde{M}_{b,r,p,i}^{out} = \frac{1}{g_{b,i+1}} & \left(g_{b,i} \dot{M}_{b,r,p}^{tot} - \widetilde{O}_{b,r,i} - \sum_{f \in \mathfrak{F}} \widetilde{\delta L}_{b,i}^f A_{b,r,p} \cdot C_p^f - \sum_{h=1}^{N_{harm}} \widetilde{\gamma}_{b,i}^h V_{b,r,p}^h \right) \\ & - \widetilde{\delta g}_{b,i} T_{b,r,p} S_{p,i}^{eval} + \widetilde{\delta \rho}_{b,i} \left. \frac{\partial(gM)}{\partial \rho} \right|_{b,r,p} + \widetilde{\delta \phi}_{b,r,p,i} \left. \frac{\partial(gM)}{\partial \phi} \right|_{\phi_{b,r,p}}, \end{aligned} \quad (\text{III.27})$$

where $g_{b,i+1}$ can be determined from Eq.III.14 as $g_{b,i+1} = \frac{g_{b,i}}{1 + \widetilde{\delta g}_{b,i}}$, and by defining $S_{p,i}^{eval}$ as the collection of all the values of $M_{b,r,p}^{eval}$ for the observations of pixel p at all bolometers b and all rings r , which can be written:

$$S_{p,i}^{eval} = \begin{pmatrix} \vdots \\ M_{b_k,r_k,p}^{eval} / \sigma_{b_k,r_k,p}^2 \\ \vdots \end{pmatrix}_{k \in \llbracket 1, N_{hit} \rrbracket}.$$

The cleaned signal defined above corresponds to the cleaned signal appearing in Eq.III.4 for the generic description of SRoll, in which the collection of all the hereby cleaned signals was noted $\mathbb{M} - \mathbb{N}$.

The final computation defined by the above Eq.III.27 is different in the case of the **100GHz** frequency band, in that the polarization efficiency correction and the detector polarization angle correction are not used to clean the signal. The determination of these corrections was observed to be too noisy and erroneous along this frequency band, therefore the terms $\widetilde{\delta \rho}_{b,i} \left. \frac{\partial(gM)}{\partial \rho} \right|_{b,r,p}$ and $\widetilde{\delta \phi}_{b,r,p,i} \left. \frac{\partial(gM)}{\partial \phi} \right|_{\phi_{b,r,p}}$ are not included in Eq.III.27 when applied to a 100GHz SRoll2 run.

As mentioned before in subsection III.1.1.c), Eq.III.27 above performs the signal correction I refer to as the bolometers **realignment**. This corresponds in said equation to the term $\sum_{f \in \mathfrak{F}} \widetilde{\delta L}_{b,i}^f A_{b,r,p} \cdot C_p^f$ being subtracted from the signal of each bolometer b . Since the

bolometer RDCs $\widetilde{\delta L}_b^f$ have been defined as the bolometer response *difference* with respect to the average over all bolometers used in the run (cf. Eq.III.11), and since this response difference is subtracted from the signal of each bolometer for each of the foregrounds f , all bolometers foregrounds signal are therefore *realigned* to the same **average** response over all bolometers. This is an essential step to correct for the intensity-to-polarization leakage due to bandpass mismatch, but will also have implications in the way I compute the new ^{12}CO and ^{13}CO maps in the next chapter.

Eventually, this allows for the *cleaned* and *calibrated* bolometer signal provided by Eq.III.27 to be projected into the output *cleaned* frequency sky maps, which can be written:

$$\widetilde{S}_{p,i}^{out} = \left(\mathbb{A}_p^\top \mathbb{A}_p \right)^{-1} \mathbb{A}_p^\top \widetilde{\mathbb{M}}_{p,i}^{out}, \quad (\text{III.28})$$

where $\widetilde{\mathbb{M}}_{p,i}^{out}$ is the collection of all the values of $\widetilde{M}_{b,r,p,i}^{out}$ for the observations of pixel p at all bolometers b and all rings r , and can be written:

$$\widetilde{\mathbb{M}}_{p,i}^{out} = \begin{pmatrix} \vdots \\ \widetilde{M}_{b_k,r_k,p,i}^{out} / \sigma_{b_k,r_k,p}^2 \\ \vdots \end{pmatrix}_{k \in \llbracket 1, N_{hit} \rrbracket}.$$

This equation corresponds exactly to Eq.III.4 in the generic description of SRoll.

Two additional outputs are provided by SRoll2 and will be the base of the new full-sky ^{12}CO and ^{13}CO maps computation in the next chapter. These other outputs are the *cleaned* signal sky maps per bolometer, which can be written:

$$\tilde{S}_{b,p,i}^{\text{out}} = A_{b,r,p}^\top \tilde{M}_{b,r,p,i}^{\text{out}}, \quad (\text{III.29})$$

and the maps of the precision matrix components for each bolometer used in the considered SRoll2 run, which are defined and computed as follows:

$$\begin{aligned} \Sigma_{b,p}^{-1} &= \begin{pmatrix} II_{b,p}^{\text{out}} & IQ_{b,p}^{\text{out}} & IU_{b,p}^{\text{out}} \\ IQ_{b,p}^{\text{out}} & QQ_{b,p}^{\text{out}} & QU_{b,p}^{\text{out}} \\ IU_{b,p}^{\text{out}} & QU_{b,p}^{\text{out}} & UU_{b,p}^{\text{out}} \end{pmatrix} \\ &= \begin{pmatrix} w_{b,p} & w_{b,p}\rho_b \cos(2\phi_{b,p}) & w_{b,p}\rho_b \sin(2\phi_{b,p}) \\ w_{b,p}\rho_b \cos(2\phi_{b,p}) & w_{b,p}\rho_b^2 \cos^2(2\phi_{b,p}) & w_{b,p}\rho_b^2 \cos(2\phi_{b,p}) \sin(2\phi_{b,p}) \\ w_{b,p}\rho_b \sin(2\phi_{b,p}) & w_{b,p}\rho_b^2 \cos(2\phi_{b,p}) \sin(2\phi_{b,p}) & w_{b,p}\rho_b^2 \sin^2(2\phi_{b,p}) \end{pmatrix}, \end{aligned} \quad (\text{III.30})$$

where $\Sigma_{b,p}^{-1}$ is the precision matrix of bolometer b for pixel p , $\phi_{b,p}$ is the average detector polarization angle over all rings in the pixel p , and $w_{b,p}$ is the inverse of the variance of noise in pixel p of bolometer b defined by $w_{b,p} = \frac{\mathfrak{h}\mathfrak{c}_{b,p}}{\sigma_{b,p}^2}$, noting as $\mathfrak{h}\mathfrak{c}_{b,p}$ the hitcount in pixel p of bolometer b summed over the rings falling in the said pixel p . These maps will be essential later for the computation of the ^{12}CO and ^{13}CO maps produced in this work. The precision matrix $\Sigma_{b,p}^{-1}$ does not depend on i as it is only computed at the end of the algorithm, in the final independent computation of **Step#1**, when $i = N_{\text{iter}}$.

Step#2 After determining an estimate for almost all its parameters and computing cleaned maps in **Step#1**, the last remaining parameter to be fitted at the iteration i is $\aleph_{b,r,i+1}$.

In the model of SRoll2, all residual effects left at the end of **Step#1** are interpreted as caused by the ADCNL. Therefore, **Step#2** consists in fitting a model for the ADCNL built on spline functions in *time* and in *ADU* on the remaining systematic residuals. This ADCNL model and its fitting computation is detailed in Delouis et al. (2019) and will not be detailed here.

This final fit provides an estimate of the value of $\aleph_{b,r,i+1}$, which can then be used in the next iteration. Though I will not use it in this work, it can be noted that the full spline functions fitting of the SRoll2 ADCNL model is also a part of SRoll2's output.

III.2.4 SRoll2 final residuals

The final residuals of the systematic effects corrected by SRoll2 are gathered and summed up in Fig.III.14, taken from the Fig.24 of Delouis et al. (2019), and plotting the corresponding power spectra of said residuals. The $1/f$ noise is the dominating residual at large scales (low multipole ℓ), for all frequencies. While the 353GHz band is dominated by the errors on the polarization efficiency and polarization orientation angle, introducing distortions of the vastly dominant thermal dust emission, the main residue at all other frequencies (100-143-217GHz) remains that of the ADCNL, despite a clear improvement compared to the results from the *Planck* legacy release, plotted in Fig.52 from Planck Collaboration III (2020).

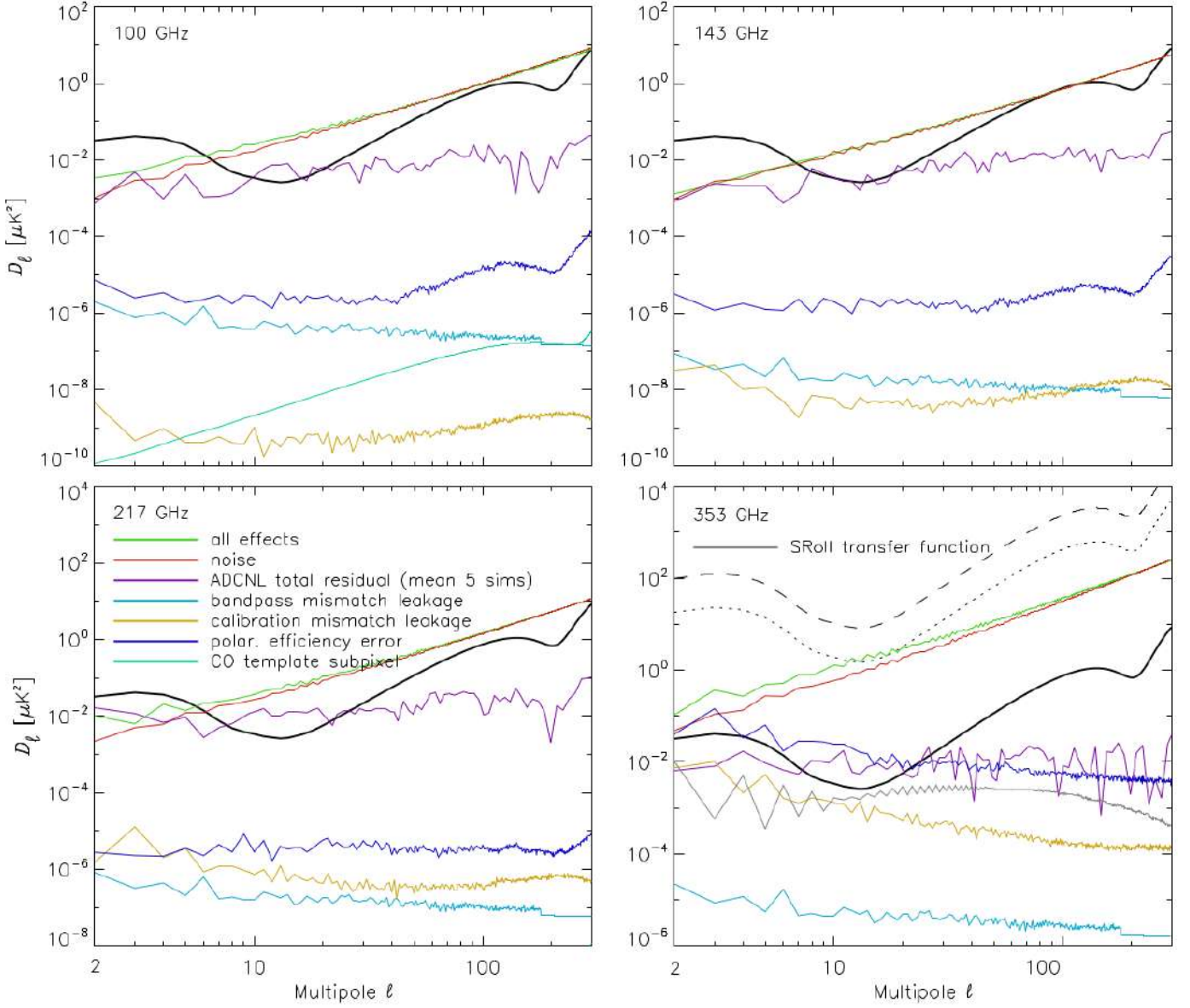


Fig. III.14 Polarization power spectra in D_ℓ^{EE} , showing the noise (red) and the main systematic residuals: ADCNL remaining after cleaning (purple), polarization efficiency (dark blue), bandpass-mismatch leakage (light blue), leakage from calibration mismatch (orange), and the sum of all these (green). SRoll2 residual empirical transfer function (gray), and CO template subpixel effect (turquoise) have not been included in the sum. The fiducial CMB power spectrum is shown as a black lines. At 353 GHz, the systematic residuals are dominated by the uncertainty on the polarization efficiency, that has been fitted at the noise level for the lowest multipoles. At other frequencies, ADCNL is still the instrumental systematic effect that has the highest residuals.

Taken from [Delouis et al. \(2019\) Fig.24](#).

Input foreground template accuracy Another characterization they computed in [Delouis et al. \(2019\)](#) is the evaluation of the impact of the input foreground template accuracy on the bandpass mismatch determination and correction. They carried out this test using a simple model of sky with 2 foregrounds: the 100GHz map assumed to be dominated by the CO bandpass mismatch, and the 353GHz dominated by the dust emission bandpass mismatch. On this toy model, they computed SRoll2 runs iteratively, starting with degraded foregrounds where a colored noise is added with the same power spectrum as the foregrounds. From the SRoll2 frequency maps output by this first run from degraded foreground templates, they build new CO and dust maps, used as new input templates for a subsequent run. They observe, following this iterative approach, a convergence of the template towards stable maps, as shown in Fig.14 of [Delouis et al. \(2019\)](#), converging close to the initial 2 components building this simple model. This shows the capacity of the SRoll2 solution to determine a good fit of the corresponding coefficients, even if the input template is not exactly identical to the corresponding sky emission.

This test actually inspired the try for a general iterative approach on SRoll2, which I explored as part of my PhD work, but which showed to require more time and computational hours than I could reasonably invest into in order to achieve a convergence. The general idea of this iterative approach, as well as some of its intermediary results, are presented in Section IV.4.

III.3 SRoll2 simulations

Simulations have been employed as a powerful tool for characterization purposes in both *Planck*'s LFI and HFI throughout the *Planck* mission data analysis history. The availability of accurate and consistent simulations is essential for various types of results and methods qualification:

- ⇒ Checking the quality and accuracy of tools measuring the instrumental characteristics from the data,
- ⇒ Quantifying the systematic residuals resulting from the algorithms used to treat and clean the signal through the full data analysis pipeline,
- ⇒ Ascertain the accuracy and quality of the methods used to extract scientific results from the data,
- ⇒ Evaluating the biases and uncertainty on the various results produced from the data.

To this end, prior to the mission launch, an extensive parametric sky model, called the *Planck* Sky Model (PSM), was designed to produce all-sky maps of the main galactic and extra-galactic emissions composing the *Planck* instruments data, as detailed in [Delabrouille et al. \(2013\)](#). This model was already built to produce sky maps accounting for parts of the various instruments specificities, such as the beam, frequency band specific response properties, polarization properties, scanning scheme, and noise parameters. The models of the sky emission sources were updated after the mission launch following the evolution of the available observational knowledge of the aforementioned sky model components.

Since the characterisation steps described above are very specific to each of the LFI and HFI instruments and analysis pipelines, the various simulations used for said characterization were partly developed independently within the respective research groups. However, some of these results assessments require consistent and concerted qualification from both LFI and HFI. Also, the production of many Monte Carlo realizations, necessary for the

simulations construction, shows to be a computationally very costly task. Therefore, an effort was made to conjointly develop a suite of code to provide the *Planck* collaboration with a complete set of self-consistent simulations for all detectors at all *Planck* frequencies, called the full focal plane (FFP) simulations (cf. [Planck Collaboration XII \(2016\)](#)). This set of simulations was later slightly updated and improved for the qualification of the 2018 *Planck* legacy release (cf. Appendix A of [Planck Collaboration III \(2020\)](#)), resulting in a version called FFP10.

In my work, I also used these simulations for any quality checks and assessment of my results, such as the estimation of the uncertainty of the RDCs' determination, the level of noise in my new ^{12}CO and ^{13}CO maps, and the evaluation of some sources of error and biases in my method, as will be seen in Chapter IV. I also use these simulations to estimate a level of confidence in the detection of CO signal at high latitude and low column density from my new maps in later Section V.3.

In this section, I will briefly introduce the FFP simulation production pipeline, and the elements composing the PSM it is based upon. I will then introduce the slight modifications and adaptations to the FFP10 simulations that were performed to use them for the qualification of SRoll2 and its results.

III.3.1 Simulation pipeline

The full FFP simulations production pipeline is represented in the schematic of Fig.III.15, as taken from the Fig.A.1 of [Planck Collaboration III \(2020\)](#).

The inputs are comprised of the main elements and parameters necessary to simulate the data from the complete *Planck* mission for all bolometers at all frequencies, which requires to provide the mission and instrumental characteristics, as well as the sky emission composition. Note that the latter category of inputs will be discussed succinctly in the next subsection.

The characteristics of the instruments and the mission operation comprise (cf. [Planck Collaboration XII \(2016\)](#), [Planck Collaboration III \(2020\)](#)):

- ❑ The pointing data for each bolometer,
- ❑ The satellite trajectory data,
- ❑ The flagging of discarded regions of the timeline,
- ❑ The full bolometers' scanning beams, accounting also for the FSL effect,
- ❑ The bolometers' bandpasses,
- ❑ Various realisations of accurately modelled noise affecting each bolometer, produced to correspond to each of the measured detectors' noise power spectral density,
- ❑ The known instrumental systematic effects, such as:
 - Bolometer signal time-response convolution and residuals from time response deconvolution,
 - Noise auto-correlation due to deglitching,
 - The ADCNL effect affecting HFI,
 - $1/f$ noise,
 - 4-K line residuals,

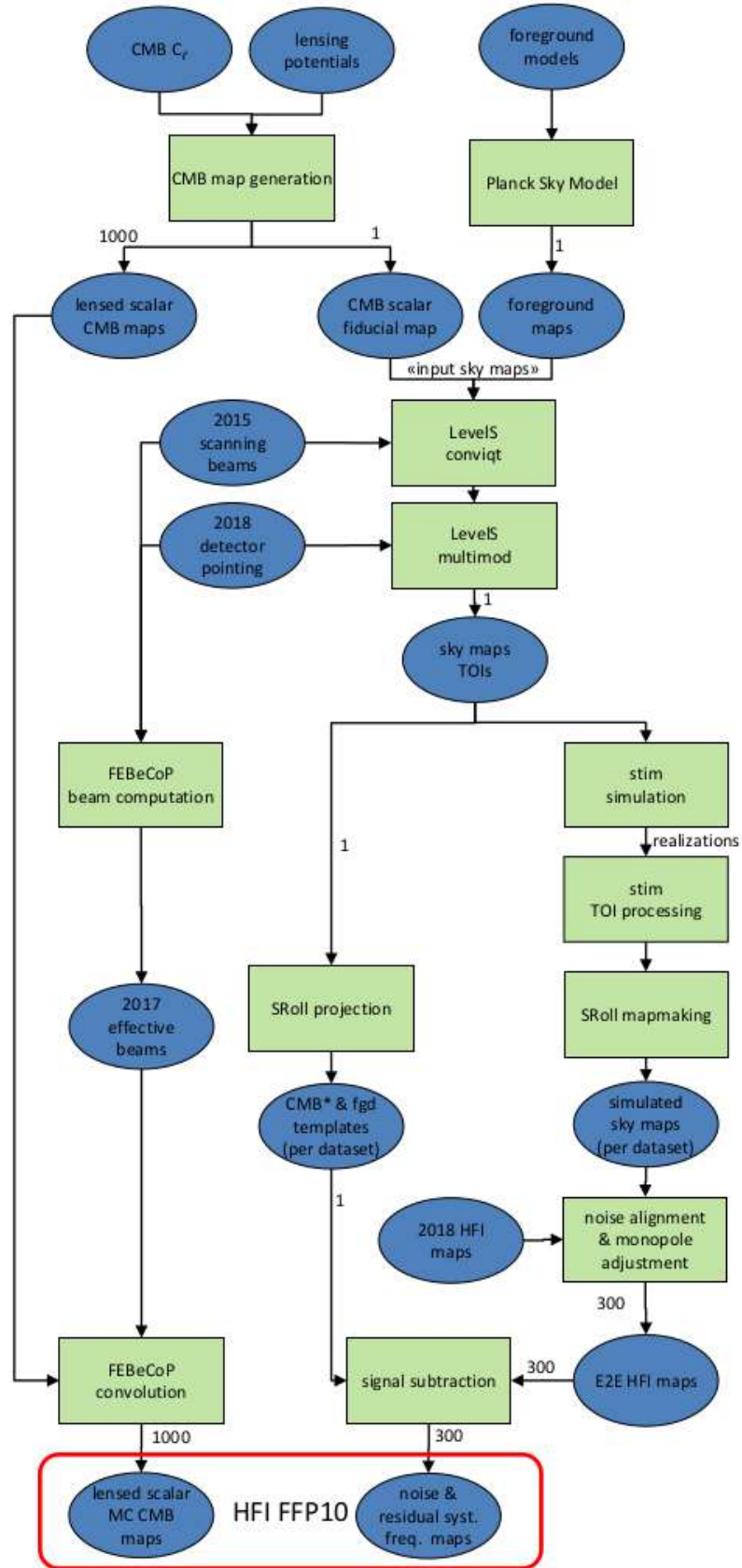


Fig. III.15 Schematic of the HFI simulation pipeline. The numbers are the number of realizations. The red frames show the products available in the Planck Legacy Archive. Taken from Fig.A.1 of *Planck Collaboration III* (2020).

- Compression/decompression of the transmitted data.

Using the input PSM and the mission operation parameters and data, sky signal TOIs are produced, as would be detected by ideal detectors aboard the *Planck* satellite. Then, using all the instruments' characterisation parameters, all systematic instrumental effects, noise, glitches, scanning beams convolution, etc, are applied on said TOIs, to produce both end-to-end HFI and LFI simulation maps and TOIs.

Since SRoll2 operates on TOI data, we use the TOI simulations produced to characterize the algorithm's operation and results quality.

III.3.2 Sky model

The sky model used for the FFP10 simulations is based on the PSM (cf. [Delabrouille et al. \(2013\)](#)), composed of multiple realisations produced from the same power spectrum of the CMB anisotropies, both in intensity and polarization, and a sum of modelled foreground emission sources, both galactic and extragalactic.

The CMB anisotropies realisations are built from 3 contributions constructed separately: the scalar CMB sky affected by gravitational lensing, CMB sky maps of primordial tensor modes, and primordial non-gaussianity of the CMB intensity and polarization signal (cf. [Planck Collaboration XII \(2016\)](#)).

The sky model of foreground emission is made of modelled signal from:

- ◆ Thermal dust, with I (intensity) component built by applying the Generalized Needlet Internal Linear Combination (GNILC) method (see [Remazeilles et al. \(2011\)](#)) on the 353GHz map of the *Planck* 2015 legacy release, and with Q and U built from a blend of the *Planck* 2015 353GHz map polarization (near the galactic disk, and for the 10 first spherical harmonics) and a statistical model developed in the work of [Vansyngel et al. \(2017\)](#),
- ◆ Spinning dust (cf. [Planck Collaboration XX \(2011\)](#), [Planck Collaboration X \(2016\)](#), [Planck Collaboration XII \(2016\)](#)),
- ◆ Synchrotron (cf. [Planck Collaboration XII \(2016\)](#)),
- ◆ Free-free (cf. [Dickinson et al. \(2003\)](#), [Planck Collaboration XII \(2016\)](#)),
- ◆ CIB, based on the distribution of individual galaxies in template maps based on the distribution of dark matter at a range of relevant redshifts (cf. [Planck Collaboration XII \(2016\)](#)),
- ◆ Unresolved faint infrared and radio point sources (cf. [Planck Collaboration III \(2020\)](#)),
- ◆ Sunyaev-Zeldovich effect from galaxy clusters, both thermal and kinetic (cf. [Planck Collaboration III \(2020\)](#)),
- ◆ CO $J = 1 \rightarrow 0$, $J = 2 \rightarrow 1$, and $J = 3 \rightarrow 2$ lines, with the $J = 1 \rightarrow 0$ line map modelled from the composite CO survey performed by [Dame et al. \(2001\)](#), and with the $J = 2 \rightarrow 1$ and $J = 3 \rightarrow 2$ lines built by scaling the $J = 1 \rightarrow 0$ previously constructed using the line emission ratios: $\frac{J=2 \rightarrow 1}{J=1 \rightarrow 0} = 0.65$, and $\frac{J=3 \rightarrow 2}{J=2 \rightarrow 1} = 0.50$ (cf. [Delabrouille et al. \(2013\)](#)).

III.3.3 SRoll2 updates & peculiarity

In the scope of the qualification of SRoll2, the FFP10 simulations were slightly updated, and their use in the running of SRoll2 entails a small adaptation at the beginning of the run in order to take into account the added separation of the ^{12}CO and ^{13}CO components, as I will detail here.

Apart from this small change and updates, using SRoll2 on the FFP10 simulation TOIs consists only in using as input templates the sky components of the PSM, and feeding to SRoll2 the updated simulated TOIs.

The SRoll2 update on the FFP10 simulations consists in a twofold modification:

1. The solar dipole is updated in amplitude and direction to that from the latest *Planck* determination: $d = 3362.71\mu\text{K}_{\text{CMB}}$, $l = 264.21^\circ$, and $b = 48.253^\circ$,
2. A kinetic Doppler quadrupole is added at each frequency, with amplitudes of: $0.84\mu\text{K}_{\text{CMB}}$ at 100GHz, $1.00\mu\text{K}_{\text{CMB}}$ at 143GHz, $1.35\mu\text{K}_{\text{CMB}}$ at 217GHz, and $2.10\mu\text{K}_{\text{CMB}}$ at 353GHz.

The aforementioned kinetic Doppler quadrupole corresponds to the quadrupole component ($\ell = 2$) of the Doppler effect due to the *Planck* satellite's motion with respect to the CMB frame, for which the expression was given in Eq.II.2. Its effect, and the need to take it into account, as well as an estimation of its value at each *Planck* frequency, was already presented in the work of [Notari & Quartin \(2015\)](#). While the Solar dipole, i.e. dipole component stemming from the same Doppler effect, is of the order of $\beta = v/c$ (cf. Eq.II.1 for notation details), the quadrupole is of the order of β^2 (cf. [Notari & Quartin \(2015\)](#)).

Aside from these modifications, it can be noted that the FFP10 simulation does not contain a model of the ^{13}CO emission, which was consistent with the absence of separation of the 2 ^{12}CO and ^{13}CO isotopologues in the *Planck* legacy releases, but which no longer matches the data model of SRoll2. In order to account for the ^{13}CO emission, an additional step is added at the very beginning of the SRoll2 run (in the case of a run of the FFP10 simulation data). Since SRoll2 expects a ^{13}CO emission template map, one is built for runs on simulations and is constructed from the SRoll2 frequency maps using the ground-based determination of the ^{13}CO response coefficients. The added starting step consists then in adding to the simulations HPRs a ^{13}CO signal for SRoll2 to fit its corresponding RDCs. To this end, the ^{13}CO emission template map is projected back into HPRs and added to the input simulation HPRs, multiplied by the corresponding bolometer specific ^{13}CO conversion coefficient as computed in the work of [Planck Collaboration XIII \(2014\)](#). After this signal is added to the input HPRs, SRoll2 can correctly fit the corresponding ^{13}CO RDCs, and check in output the correspondence between the fitted value and that used as input.

We will see, in subsection IV.1.3.d), that I use in my work these same SRoll2 simulations for characterisation of my results. However, I modify it by changing the ^{13}CO emission template map used, as well as the input coefficients employed for the addition of the ^{13}CO template emission into the input HPRs.

Chapter IV

Building new CO maps

Starting from the full description of the SRoll2 mapmaking algorithm provided in the previous chapter, I will now introduce the original methods and products provided by the endeavours of my PhD work. First, I will define the modifications I devised and applied to the typical operating conditions of SRoll2, as well as some of its inputs, in order to optimize its use to fulfil the central goal of my work: the production of new ^{12}CO and ^{13}CO fullsky maps. I will then present the main results from this new altered version of SRoll2, which I call SRoll2.1, and characterize their quality and consistency.

Secondly, from the main outputs of the SRoll2.1 algorithm, I developed a separation method to extract the ^{12}CO and ^{13}CO emissions from the rest of the signal, and build their respective fullsky maps. I thoroughly present the method, as well as the resulting maps.

Thirdly, I explore probable sources of error and bias which rise from the specific nature of SRoll2.1, and from the way I exploit its outputs, i.e. the main structure of my ^{12}CO and ^{13}CO maps' construction scheme. This either allows to ensure the quality of the overall method, or is an occasion to suggest potential solutions, and pathways towards various improvements.

Finally, I present an iterative scheme on SRoll2.1, which was originally devised as an integral part of the new SRoll2.1 version, but proved to require an investment in time and computational resources that I could not afford in the scope of my PhD. I present the intermediary results from this iterative scheme, as well as potential ways to achieve its convergence.

IV.1 SRoll2.1, SRoll2 made for the CO maps

In this work, all maps produced were computed using the various outputs from the SRoll2 mapmaking algorithm presented in Chapter III. Our all-sky CO emission maps are then computed using a χ^2 minimisation algorithm on the said outputs, as is presented in depth in Section IV.2.1 below.

The SRoll mapmaking algorithm version used in this work corresponds to its second iteration, called SRoll2, as it was introduced and presented in Section III.2. While the algorithm structure remains unchanged, we still adapted the typical operating conditions of the algorithm compared to what was done in the previous work of Delouis et al. (2019). This change consists in adding the bolometers from a second frequency band to the SRoll2 run. Indeed, until the work performed in Delouis, J.-M. et al. (2021), this algorithm had only been applied on sets of bolometers belonging to the same frequency band among the 4 *polarized* ones spanning the HFI data (100-143-217-353 GHz). Conveniently, the algorithm was built in a way that does not limit the number of bolometers used, nor the various

frequency bands they belong to. This mixing of bolometers from different frequency bands in SRoll2, while unnecessary to achieve the algorithm's first objectives, is critical to this work. Such an expansion of the data set fed into SRoll2 can be considered a double-edged sword: while it broadens the amount of information SRoll2 can use to fit its parameters (and especially those key to the later computation of the new ^{12}CO and ^{13}CO maps from SRoll2's outputs), it also complicates the sky model that we need to use to accurately describe the full extent of the input of such an extended SRoll2 run and ensure an optimal fitting of the said parameters.

These changes, with respect to the addition of a second frequency band in the SRoll2 run, are presented and explained in subsection IV.1.1. Then a necessary additional modification of the sky mask used by SRoll2.1 is described in subsection IV.1.2. These modifications are subsequently included into the SRoll2 data model in subsection IV.1.3, presenting how the inputs, outputs, and equations from Section III.2, as well as the simulations previously introduced in Section III.3, are slightly modified by our original use of SRoll2. This is followed in subsection IV.1.4 by the summary of the results I obtained for the ^{12}CO and ^{13}CO coefficients, as well as other foreground emission coefficients, and their comparison to the previous results from Planck.

This slightly altered version of the SRoll2 algorithm is referred to in this work as **SRoll2.1**, as it is not a whole new revision of the mapmaking algorithm but an extended version of its second generation.

Figure IV.1, adapted from Fig. II.10, reminds the steps of the HFI data analysis pipeline presented in Section II.3, but where the last 2 steps are adapted to reflect how the work detailed in this manuscript connects to this pipeline. These 2 steps consist of the use of the SRoll2.1 algorithm, followed by the computation of our fullsky ^{12}CO and ^{13}CO maps using the outputs of the SRoll2.1 run (see Section IV.2).

In this work, all templates used as inputs of the SRoll2.1 algorithm are the same as those used in SRoll2 (cf. Delouis et al. (2019)), except when explicitly mentioned. A summary of all those changes is given in Section IV.1.3.a).

IV.1.1 Addition of the 143GHz

As previously mentioned, though I use the original unmodified SRoll2 algorithm, there is one way in which my use differs from the one first introduced. In order to better meet our main goal, i.e. obtain better fullsky ^{12}CO and ^{13}CO emission maps, for which we must optimize the CO RDCs determination, we added the bolometers from the 143GHz band to those containing the sought CO line signal. This helps in 2 major ways and a minor one, as described below, but also requires 2 additional changes :

1. The addition of a new foreground template component, presented in subsection IV.1.1.b),
2. The modification of the closure condition given on the CO coefficients averaged over all bolometers, later detailed in subsection IV.1.3.c).

In this work, I produced full-sky ^{12}CO and ^{13}CO emission maps for the 2 transitions $J = 1 \rightarrow 0$ and $J = 2 \rightarrow 1$. As later developed in Section IV.2, the **2 maps** of ^{12}CO and ^{13}CO emissions for 1 rotational line are computed from the output of **one** SRoll2.1 run. This entails that I carried out the execution of SRoll2.1 in 2 typical settings:

1. To produce ^{12}CO and ^{13}CO maps for $J = 1 \rightarrow 0$: the SRoll2.1 run is computed using all bolometers from the 100GHz and the 143GHz bands, and is referred to in this work as a SRoll2.1 100-143GHz run,

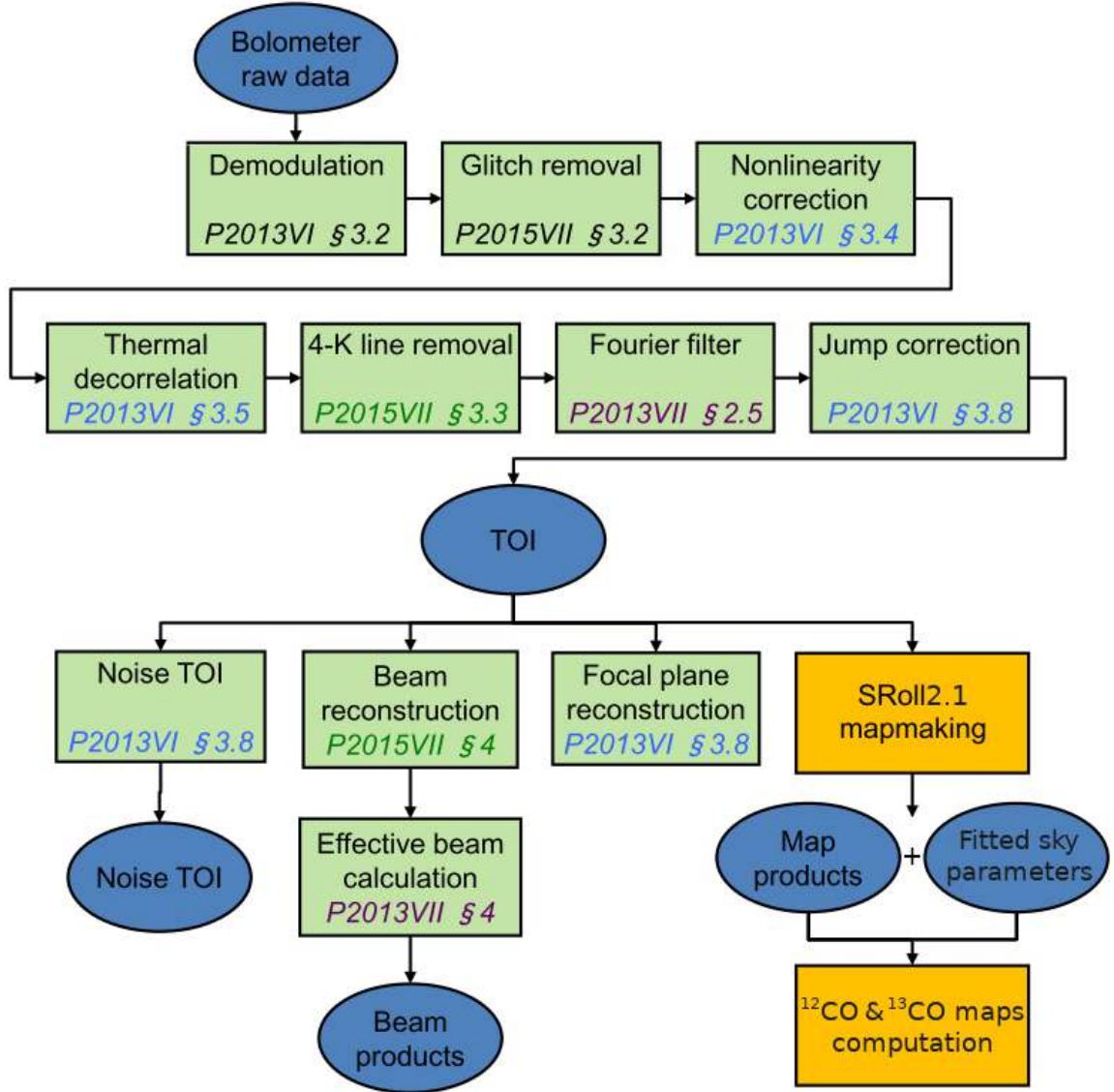


Fig. IV.1 Schematic of the HFI data analysis pipeline, with references to sections from the relevant *Planck* legacy publications at each step. The last steps are adapted to the work presented in this thesis, where our new modified use of SROLL2, called "Sroll2.1" is presented in Section IV.1, and the computation of new ^{12}CO and ^{13}CO emissions fullsky maps is presented in Section IV.2. Adapted from *Planck Collaboration III* (2020), Fig.1.

2. To produce ^{12}CO and ^{13}CO maps for $J = 2 \rightarrow 1$: the SRoll2.1 run is computed using all bolometers from the 217GHz and the 143GHz bands, and is referred to in this work as a SRoll2.1 217-143GHz run.

Each of these 2 configurations requires some specific inputs, as will be detailed in subsections IV.1.2 and IV.1.3.a).

a) What is gained from this added frequency band

One characteristic specific to the 143GHz band is the absence of CO line emission in the bandpass at this frequency. This can be exploited in our work to fulfill 3 main objectives : **firstly** to add information on all non-CO contributions at the CO maps computation step, allowing us to build our CO maps at a higher resolution, **secondly** to improve the separation of the thermal Dust and the CO emission foregrounds in SRoll, **thirdly** to allow a numerical determination, i.e. from the data, of the CO coefficients' average value, by making use of the 143GHz band as an *absolute* reference of zero-emission for ^{12}CO and ^{13}CO .

Improving the CO maps computation, and the resulting maps' resolution

The most impacting effect of adding the 143GHz bolometers to the SRoll2.1 run comes into play at the step of the CO map computation, described in Section IV.2.1. This last computation step is performed using the outputs of the SRoll2.1 run to build 5 resulting maps : $\mathcal{I}^{other}$, $\mathcal{Q}^{other}$, $\mathcal{U}^{other}$, corresponding to the averaged sky emission along the bolometers used to compute the maps **excluding** the ^{12}CO and ^{13}CO emission, and $\mathcal{M}^{12CO}$ and $\mathcal{M}^{13CO}$, respectively corresponding to the full-sky maps of ^{12}CO and ^{13}CO signal in a given rotational line. These 5 maps are built by computing a χ^2 minimisation on the cleaned frequency maps *per bolometer* output by SRoll2.1 at the pixel level, i.e. one χ^2 minimisation is computed for each pixel in our maps. In the end, each of these χ^2 minimisations boils down to the inversion of a matrix built from the data of our various frequency maps *per bolometer* (more precision in Section IV.2.1). Taking the case of the $J = 1 \rightarrow 0$ CO line as an example, if we only use the data from the bolometers at the 100GHz band, then each χ^2 minimization in each pixel fits 5 values (one for each map we compute) from 8 measurements (1 measurement per bolometer used). This is statistically risky, as the chances to end up with pixels in which we try to invert an ill-conditioned matrix get really high. By using the bolometers at the 143GHz band, we cannot use their data to determine the 2 CO maps ($\mathcal{M}^{12CO}$ and $\mathcal{M}^{13CO}$) since the bolometers at this frequency are not sensitive to any CO line signal, but we can use it to determine the 3 other maps we build ($\mathcal{I}^{other}$, $\mathcal{Q}^{other}$, and $\mathcal{U}^{other}$). If we go back to our example of the $J = 1 \rightarrow 0$ CO line, this leads us to using 19 measures to determine 3 values (8 PSB bolometers at 100GHz, and 11 bolometers at 143GHz – 8 PSBs and 3 SWBs – to compute the 3 maps $\mathcal{I}^{other}$, $\mathcal{Q}^{other}$, and $\mathcal{U}^{other}$), and 8 out of these 19 measures to determine 2 additional values (the 8 PSBs at 100GHz to determine the 2 maps $\mathcal{M}^{12CO}$ and $\mathcal{M}^{13CO}$). This is mathematically more robust and allows us to compute our CO maps at higher resolution while still ensuring the matrix invertibility at all pixels. Since it adds information to our χ^2 minimisation computation, it is also expected to reduce the noise in our resulting CO maps, as will be seen in Section IV.2.3.a). Additionally, we expect that distinct bolometers display different systematic instrumental errors, especially bolometers from different frequency bands, helping to decrease the degeneracy between the instrumental errors and the signal.

Thermal dust and CO separation Given the dust signal intensity at the considered frequencies (100GHz, 143GHz, and 217GHz) paired with the fact that the CO and dust

emissions are significantly spatially correlated, the correct separation of the ^{12}CO and ^{13}CO emission from that of the dust is an issue that bears keeping in mind. Indeed, the signal from these bolometers is dominated by 2 components : the CMB signal and the thermal Dust emission, with comparable contributions to the total signal intensity (cf. Fig.I.10, and Fig.II.15). However, as was seen in Section III.2.1.a), there is enough surface in the foreground templates where the CO and dust are not correlated to allow their separation by the SRoll2 algorithm. Indeed, as seen in Fig.III.8, there are areas in the Taurus region where the CO signal vanishes rapidly (at the edges of the molecular cloud) while the dust signal slowly decreases without vanishing. Also, from what was discussed in subsection I.4.3, given the intensity range expected of the CO signal from high-latitude regions, one can expect that all regions outside of the dense molecular clouds have a negligible contribution to the CO RDC determination. These CO RDCs are then entirely determined by SRoll2.1 from the data present in the Taurus region as observed by Goldsmith et al. (2008), i.e. the non-null region in our CO template. Therefore, the region of the sky where the dust coefficients are most efficiently determined, thanks to the CO and dust emissions being sufficiently uncorrelated, corresponds to all the non-masked regions *outside* of the Taurus region, since all other molecular clouds are masked in SRoll2.1 as will be detailed in subsection IV.1.2. This aforementioned part of the sky corresponds to 56.5% of the total sky surface in the case of a 100-143GHz Sroll2.1 run, and 57.3% of the total sky surface in the case of a 143-217GHz Sroll2.1 run, as computed from the masks presented in subsection IV.1.2.

Using the 143GHz band adds signal where the thermal dust is present in the absence of any CO signal. This presence of the dust emission signal in absence of CO emission adds statistical information to the SRoll2.1 algorithm to separate these two highly spatially correlated signals, allowing for the determination of better dust and CO bolometer coefficients for all bolometers used in any considered SRoll2.1 run. However this addition of the 143GHz band, effectively increasing the complexity of our sky model, requires to account for this increase in complexity, namely by taking into account the spatial variation of the dust SED, as discussed in the next subsection.

CO coefficients average value determination Secondly, as introduced in Chapter III, SRoll2 outputs are entirely stemming from the response *difference* between bolometers, which does not provide the information necessary for the determination of the RDC average values for the ^{12}CO and ^{13}CO . This can be solved by making use of the 143GHz band bolometers as a reference for this global calibration. This is numerically performed by changing the closure condition defining the reference values $L_{ref}^{12\text{CO}}$ and $L_{ref}^{13\text{CO}}$, as will be extensively explained in the following subsection IV.1.3.c). However, this use of the 143GHz bands' bolometers as a global calibrator of the CO coefficients average is possible only if we start from the hypothesis that **no** CO line is present in the data at this frequency. While this is true, we still expect some molecular lines' emissions to be present in this band and to be spatially correlated to the CO emission. Thankfully, these are expected to be less intense than a CO line and also not completely correlated spatially, which decreases the magnitude of the impact these emissions may have on the resulting ^{12}CO and ^{13}CO maps. This issue is discussed further in the subsection IV.3.5. It is also useful to compare our results to those obtained previously from the ground-based measurements used in the official *Planck* CO maps determination. While this use of the 143GHz band as an *absolute* calibrator of CO emission could theoretically be impacted by the presence of molecular emission in the 143GHz band, we will see that the effect is negligible in the case of the $J = 1 \rightarrow 0$ CO maps, and that its effect is complicated to estimate in the case of the $J = 2 \rightarrow 1$ CO maps because of additional maps calibration issues. This impact is further detailed in subsection IV.3.5,

where I will show that it does **not** impact the calibration of the ^{12}CO and ^{13}CO maps.

b) The need for a new template : DustVar

Given the slight differences observed in the thermal Dust emission at the various HFI frequency bands, as has been shown in the work of [Planck Collaboration III \(2020\)](#), it is necessary to add information into the algorithm to ensure that this spatial SED variation is taken into account, however small as it may be. This spatial SED variation is shown in Fig. IV.2, which is taken from [Planck Collaboration III \(2020\)](#) (where it is Fig.20). A very similar pattern can be seen at all the frequencies, that varies mostly depending on Galactic latitude.

In the previous works, such as [Planck Collaboration III \(2020\)](#), this dust spatial SED variation had been taken into account for the determination of the Solar dipole but not in the mapmaking procedure. This is because up to and including SRoll2 ([Delouis et al. \(2019\)](#)), the mapmaking algorithm was only used on *one* frequency band at a time. Since we decided, for the reasons listed above, to make SRoll2.1 run on 2 frequency bands at the same time, this effect has to be taken into account in the data model.

As previously introduced in subsection III.1.2, the recent work of [Delouis, J.-M. et al. \(2021\)](#) evaluated this spatial variation of SED for the thermal dust emission, using a model based on a dust emission described as a MBB. From this model, 2 maps are fitted to account for the SED spatial variation of the dust: using the 545GHz map as a reference for the dust, they propagate said reference map the other frequencies' intensity, and fit the 2 maps respectively weighed by the first and second derivative of the dust MBB with respect to the spectral index. These 2 fitted maps are then combined (using the aforementioned weighing) to build a final *additive correction* map per frequency. The resulting fitted additive term they determined for each frequency (except for the reference frequency of 545GHz) are displayed in Fig.IV.3. This plot gives us an idea of the type of correction we need to add into the SRoll2.1 sky model to account for the dust SED spatial variation.

To this end, the last structural modification specific to SRoll2.1 consists in the use of an additional foreground component in the data model, which I call : *DustVar*. This foreground component then corresponds to the spatial SED variation of the thermal dust emission between the 2 frequencies used in any considered SRoll2.1 run. It is treated in the model as any other foreground component, for which a RDC is determined for each bolometer, using a template map **fixed** for the whole run, and finally *realigned* by SRoll2.1 before projecting the HPRs into frequency maps (see next section for more precision on DustVar's integration into the SRoll2.1 data model).

DustVar template construction Since the foreground emissions are modelled as additive terms in the SRoll2 and SRoll2.1 models (cf. Eq.III.8), the template for this component is built from the difference between the SRoll2 frequency maps 545 – 353GHz in units of K_{CMB} for the intensity, and from the difference between the SRoll2 frequency maps 353 – 217GHz in units of K_{CMB} for the polarization, after removing the CMB anisotropies. This subtraction is computed by renormalizing the 545GHz intensity and the 217GHz polarization maps, following the dust emission law given in [Planck Collaboration X \(2016\)](#) (MBB emission as described by Eq.II.3, with $T_{dust} = 21$ K and $\beta_{dust} = 1.51$) to bring it to an equivalent spectral radiance at 353GHz, i.e. the construction of the DustVar template

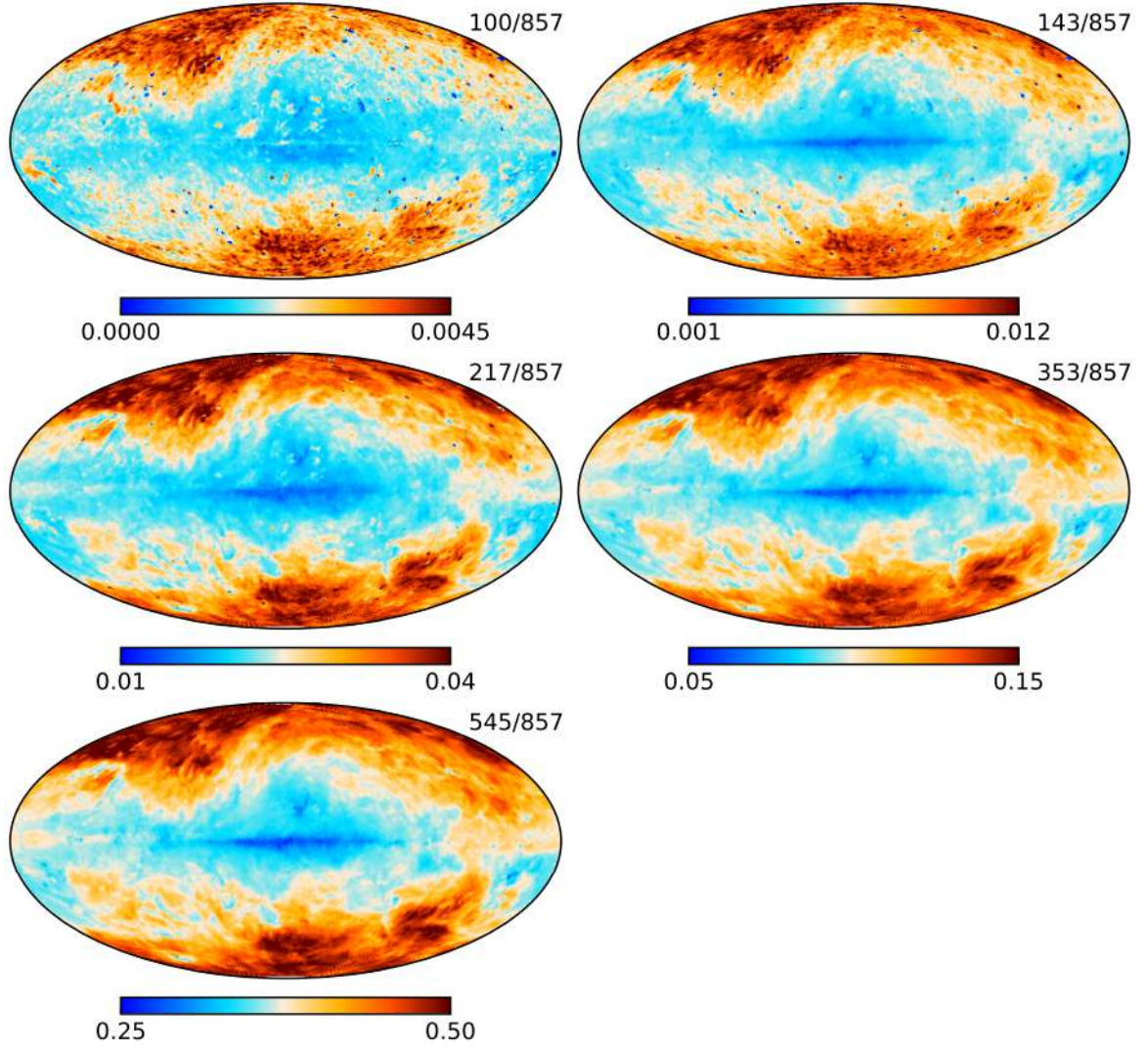


Fig. IV.2 Ratio between maps at other frequencies and the 857GHz map. At 100 and 143 GHz, low-frequency foregrounds (synchrotron, free-free) had been removed before taking the ratio. The CMB anisotropies had been removed for the 100 to 353GHz maps. These ratios are thus indicative of the Galactic dust SED variations. Taken from *Planck Collaboration III* (2020), Fig.20.

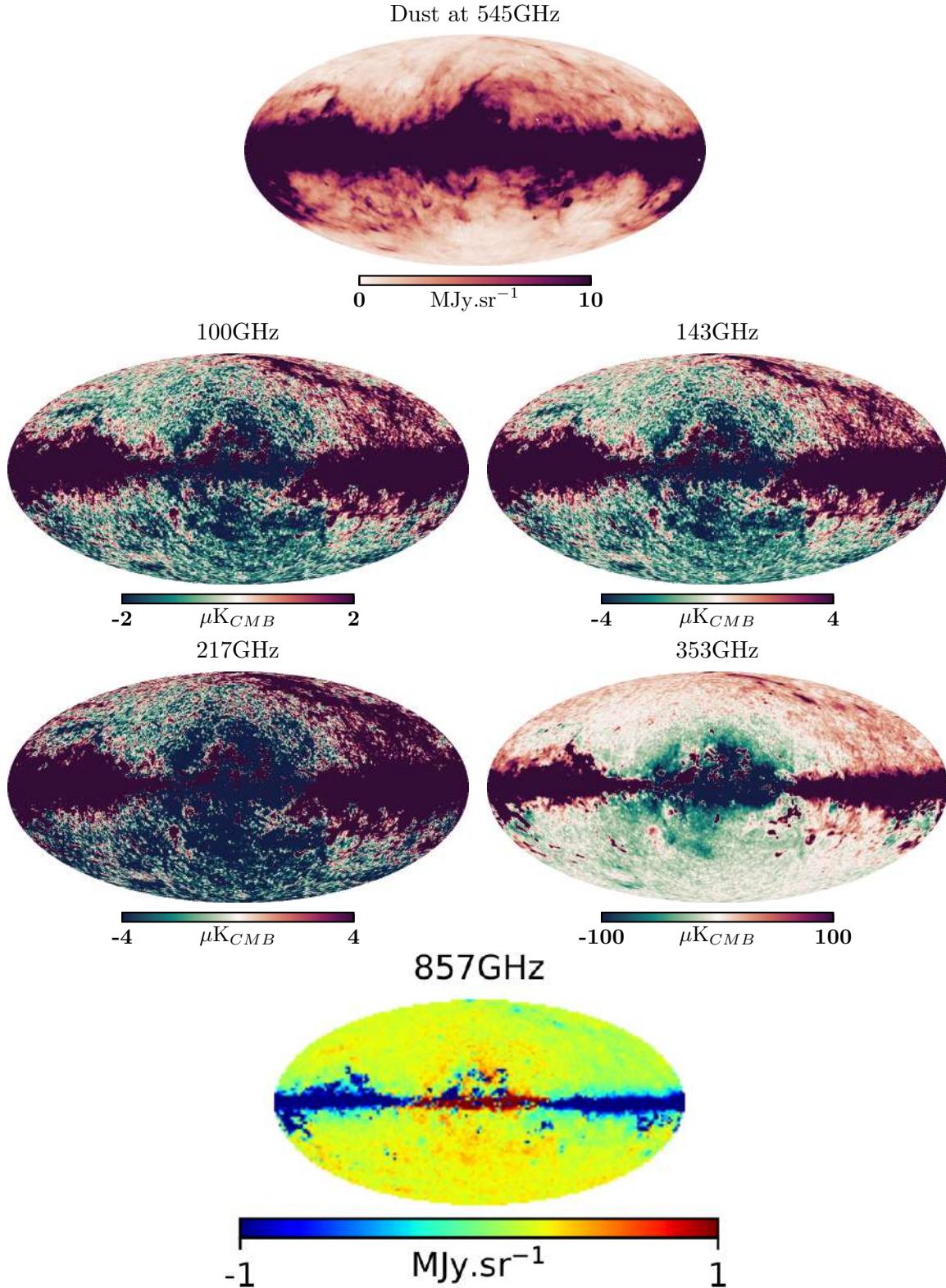


Fig. IV.3 Dust model maps: the top map shows the Dust template extracted from the 545GHz final map to be propagated to lower frequencies with the average SED. The other maps show the additive correction accounting for the spatial variation of the dust SED, for each of the other HFI frequencies. Maps are at a resolution of $N_{side} = 128$.

Data taken from [Delouis, J.-M. et al. \(2021\)](#), inspired from Fig.12, and 857GHz correction map taken from said Fig.12.

can be written as:

$$\begin{aligned} M_I^{DustVar} &= \frac{I_{353}}{I_{545}} M_I^{545, SRoll2} - M_I^{353, SRoll2}, \\ M_Q^{DustVar} &= M_Q^{353, SRoll2} - \frac{I_{353}}{I_{217}} M_Q^{217, SRoll2}, \\ M_U^{DustVar} &= M_U^{353, SRoll2} - \frac{I_{353}}{I_{217}} M_U^{217, SRoll2}, \end{aligned}$$

where $M_I^{DustVar}$, $M_Q^{DustVar}$, and $M_U^{DustVar}$ are respectively the maps of the I , Q , and U Stokes parameters of the DustVar template, I_ν is taken as the dust MBB emission defined by Eq.II.3, and $M_S^{\nu, SRoll2}$ is the SRoll2 Stokes parameter $\mathcal{S}$ of the frequency map for the frequency band ν , *after subtraction* of the CMB anisotropies. One can see the resulting I, Q, U Stokes parameters of the DustVar component construction in Fig. IV.4. We can see that the Q and U polarized components of the template are very noisy and display a non-gaussian spatial distribution of noise caused by the *Planck* mission scanning strategy. I therefore discarded these polarized components of the template to avoid the introduction of noise correlated with the input signal. From this construction, only the non-polarized intensity map is then kept, i.e. the I Stokes component, and this template is used as a non-polarized contribution.

One can note that since I use the 353GHz frequency map to build this new template, some residual CO signal from the $J = 3 \rightarrow 2$ line can be present in the DustVar template map. However, its effect is expected to be negligible for 2 main reasons:

1. At this frequency, the signal contribution of the summed ^{12}CO and ^{13}CO lines is at most of the order of 1% (cf. [Planck Collaboration XIII \(2014\)](#)), in the denser regions of the galaxy (galactic plane and inside well-known dense molecular clouds),
2. As will be explained in the next subsection IV.1.2, the masks I use in SRoll2.1 mask all known regions of CO emission from the galactic plane and known molecular clouds, with the exception of the Taurus region, as mapped by [Goldsmith et al. \(2008\)](#). Therefore, the largest area of determination of the DustVar template response coefficients by SRoll2.1, i.e. the higher latitudes, corresponds to areas of negligible CO emission at all lines (cf. [Planck Collaboration XIII \(2014\)](#), [Dame & Thaddeus \(2022\)](#), [Fixsen et al. \(1999\)](#)).

The use of such an implementation for the dust SED spatial variation is motivated by 2 main reasons. The first is the ease of integration into the SRoll2 algorithm's framework : the addition of a new foreground component to the data model fits well into the existing architecture, making it both more convenient to implement as well as to check and characterize later on. The second is that the main features of this spatial SED variation differ mildly from one frequency band to the other, as can be seen in Fig. IV.2. Whereby the hypothesis made for the SRoll2.1 sky model that this SED variation can be considered to be a single additive component map, weighed by the bolometer specific bandpass.

The contribution of this additional template is also very small, leading to small corrections when performing the realignment. Indeed, the DustVar RDC determined by SRoll2.1 used for its correction within a frequency band are of the order of $\sim 10^{-2}$ (cf. Appendix.D), for a template showing intensity variations of up to $\sim 10\mu\text{K}_{CMB}$, i.e. leading to corrections smaller than $\lesssim 0.1\mu\text{K}_{CMB}$. And even the biggest correction from one frequency band to the other (see later Eq.IV.1 and subsection IV.1.3.c), happening when realigning the bolometers from the 143GHz band onto the 217GHz band in the case of a SRoll2.1 run using these 2 frequency bands, amounts to an average correction using the DustVar template multiplied by a factor of 0.384 (cf. Eq.IV.1 using RDCs presented in Appendix.D).

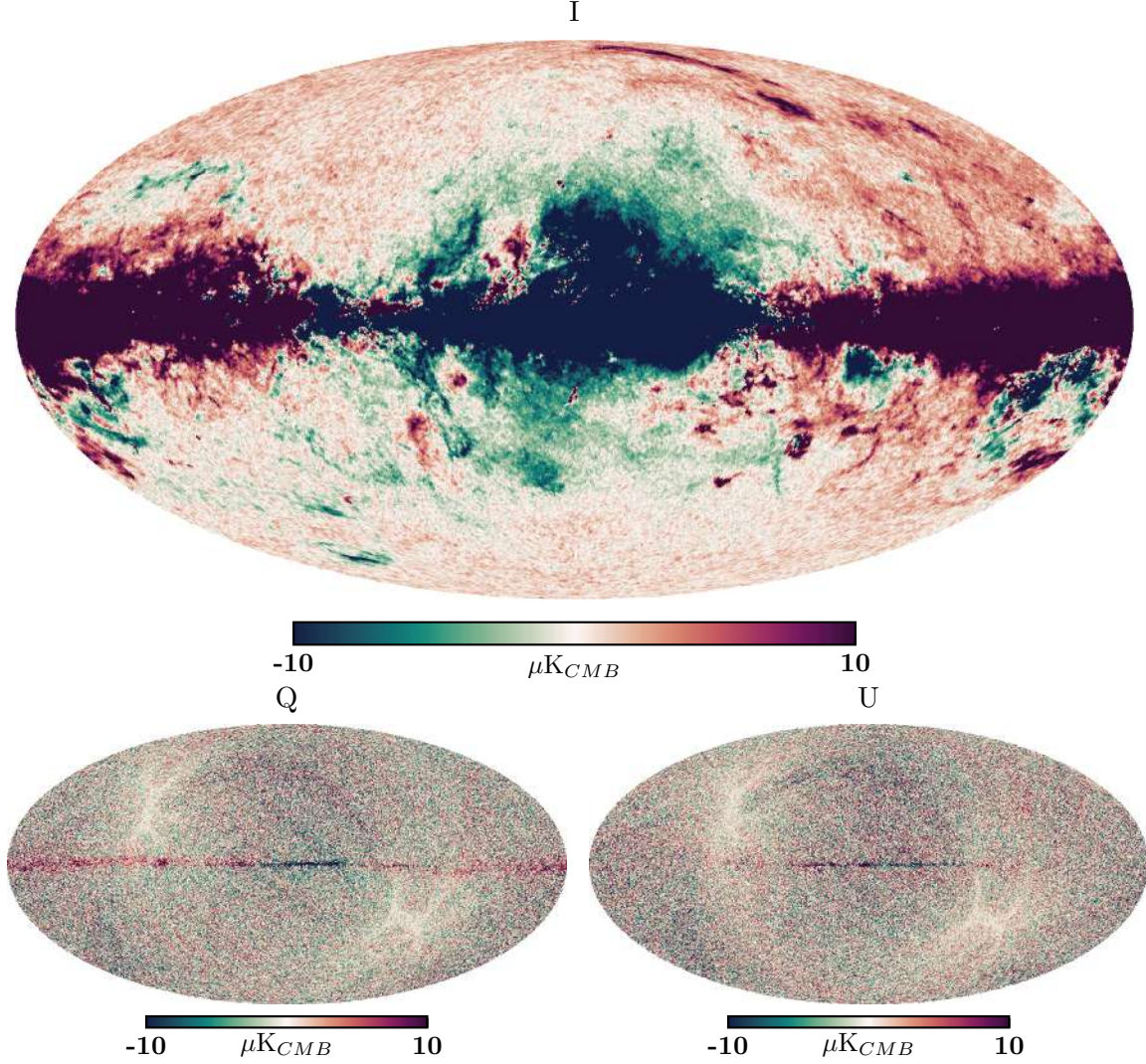


Fig. IV.4 SRoll2.1 DustVar component template. Obtained from the difference between the 545GHz and 353 GHz frequency maps. **Top:** I Stokes parameter, **Bottom left:** Q Stokes parameter (*discarded* and unused in SRoll2.1), **Bottom right:** U Stokes parameter (*discarded* and unused in SRoll2.1).

Additionally, despite a very different construction method, my new DustVar template is very similar to the additive correction term for the dust SED spatial variation determined in the work of Delouis, J.-M. et al. (2021). Indeed, since my DustVar template is built from the difference between the 545GHz and 353GHz SRoll2 frequency maps, and since the frequency band used as a reference in Delouis, J.-M. et al. (2021) is also 545GHz, my DustVar template should be very similar to their fitted dust SED spatial variation additive correction for the 353GHz band. Therefore, I plotted the comparison between these 2 maps in Fig. IV.5, where each map is displayed, as well as the plot of their correlation. More precisely, the scatter plot of their correlation is computed between the 353GHz additive correction fitted in Delouis, J.-M. et al. (2021) and my DustVar template *multiplied by a factor $\times 10$* . I am not sure where this difference of a factor of 10 comes from, but it is not an issue as my template is only used to determine the corresponding DustVar RDCs within Sroll2.1, which are *multiplicative* with the template (cf. Eq. III.8), i.e. this factor will simply be *absorbed* by the RDC fit. For reference, the identity function is plotted in *blue* in the correlation scatter plot of Fig. IV.5. It shows a good spatial consistency between the 2 maps.

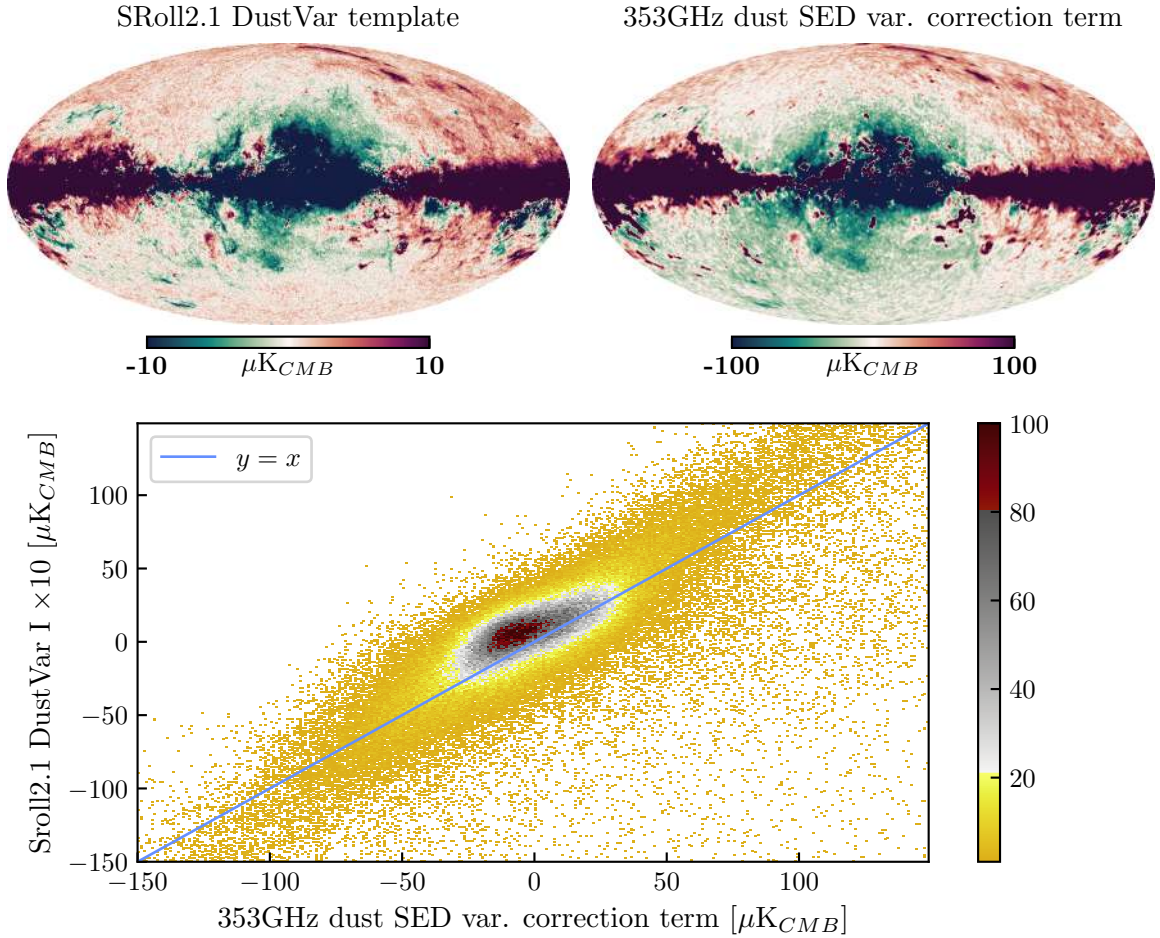


Fig. IV.5 Comparison between the SRoll2.1 DustVar template map and the 353GHz fitted dust SED spatial variation additive correction from [Delouis, J.-M. et al. \(2021\)](#). **Top left:** SRoll2.1 DustVar template intensity map; **Top right:** 353GHz additive correction fitted in [Delouis, J.-M. et al. \(2021\)](#). **Bottom:** Correlation scatter plot between the 353GHz additive correction fitted in [Delouis, J.-M. et al. \(2021\)](#) (x-axis), and the SRoll2.1 DustVar template map multiplied by a factor $\times 10$ (y-axis). The **blue** line is the identity function.

It is to note that this DustVar template could be improved by using the additive maps determined by [Delouis, J.-M. et al. \(2021\)](#), or a combination of said maps. However, when these maps were produced, the costly computations needed to qualify my method were already fairly advanced, and I did not have the computational resources at my disposal to change my definition of this template at this point.

IV.1.2 Modification of the SRoll2 sky mask

The mask used in a run SRoll is an essential part to ensure a good convergence of the algorithm, and it is another element which was adapted for the specific goals of SRoll2.1. As introduced in Section [III.2.1.a\)](#), this mask is used to select which pixels are considered and which are masked during the SRoll run.

a) The problem of *decorrelation*

Since our goal is the production of ^{12}CO and ^{13}CO maps, it is imperative to ensure the determination of the best possible CO coefficients. To this end, we need to minimize the risks of a phenomenon which I call: *decorrelation*. This *decorrelation* refers to the occurrence of having a foreground template that differs from the real signal present in the data. More generally, it can then be defined as the detrimental effect on the quality of the parameter fitting caused by using an incorrect template and/or instrumental systematics model in the SRoll run. When such a decorrelation occurs, the parameters determination quality is deteriorated. In the case of our main goal, if our CO emission template decorrelates from the CO signal present in the data, our CO coefficients determined by SRoll will be incorrect because underestimated, leading to incorrect ^{12}CO and ^{13}CO maps.

Since our ^{12}CO and ^{13}CO templates are composed of the G08 survey Goldsmith et al. (2008), limited to the Taurus region, the rest of the sky is of null value in the said templates, as can be seen in Fig.III.10. This creates the risk of decorrelation in every pixel that is null in the ^{12}CO and ^{13}CO templates while having a non-null ^{12}CO and ^{13}CO contribution in the *real* data, i.e. all the sky signal originating from dense molecular clouds other than the Taurus region observed by Goldsmith et al. (2008). All these dense cloud pixels will then inevitably induce decorrelation for the CO if not masked. Therefore, our mask needs to be modified to also mask all the aforementioned pixels.

b) SRoll2.1 masks construction

These new masks are constructed by following 3 main steps:

1. The selection of a starting mask, excluding all known high gradient or time-dependant point sources,
2. The masking of all regions of the sky containing CO emission signal, to prevent the apparition of the previously discussed *decorrelation*,
3. The addition in the mask, as a **non**-masked region, of the Taurus region as surveyed by Goldsmith et al. (2008), i.e. the region displayed in Fig.III.10.

Starting mask In the case of a SRoll2.1 100-143GHz run, aiming to build the $J = 1 \rightarrow 0$ ^{12}CO and ^{13}CO maps, I start from the mask used on the 100GHz band in both SRoll2 (Delouis et al. (2019)) and by the *Planck* collaboration in the SRoll1 (Planck Collaboration III (2020)) generation. This corresponds to the mask displayed in the upper left part of Fig.III.7.

In the case of a SRoll2.1 217-143GHz run, aiming to build the $J = 2 \rightarrow 1$ ^{12}CO and ^{13}CO maps, I did not have access to the mask used on the 217GHz band by the *Planck* collaboration, nor did I find a sure way to recompute an exact copy. I circumvented this limitation by building an alternative starting mask for the SRoll2.1 217-143GHz run. I used the *Planck* legacy HFI 217GHz point source mask, which cut signal points exceeding a threshold of $\text{SNR} > 10$, from the 2013 PR1 release. I computed the new starting mask by convolving this point source mask with a top-hat beam profile with a radius of 1.3° , in order to obtain a starting mask with similar point source masking size as the one observed in the 217GHz band *Planck* collaboration mask from Fig.III.7.

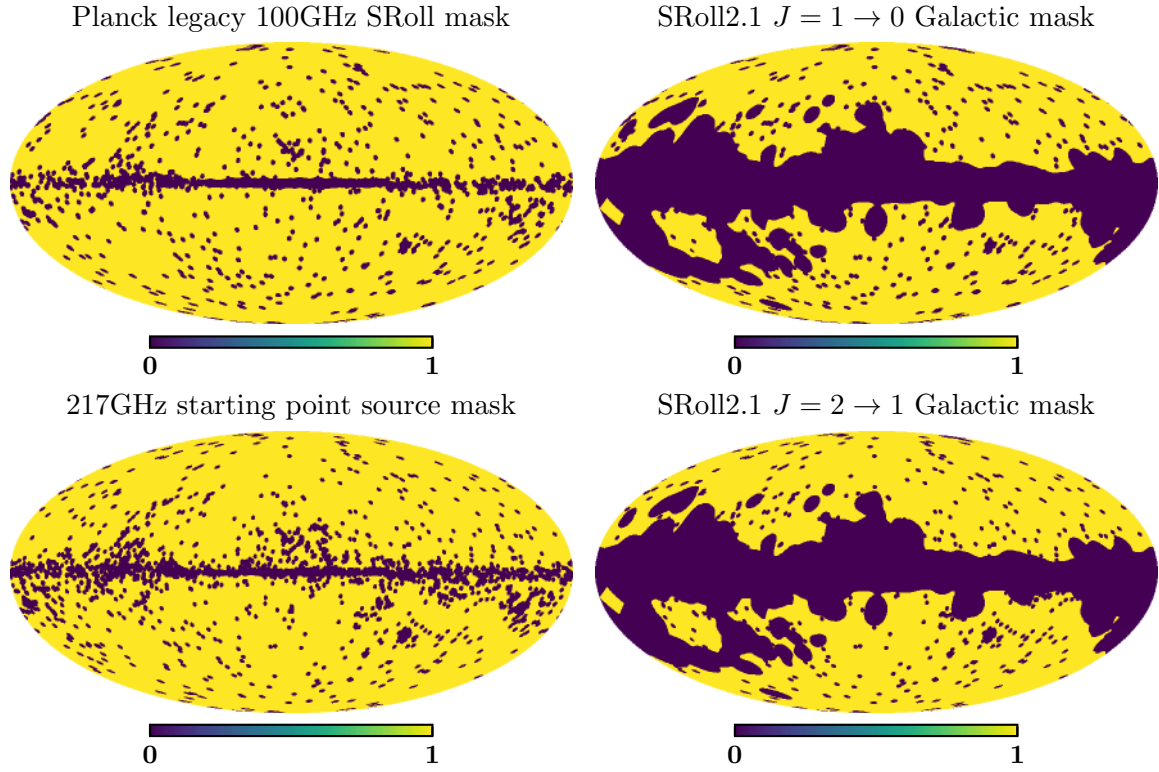


Fig. IV.6 *SRoll2.1* masks compared to the corresponding starting mask of their construction. **Left:** *SRoll2.1* starting masks for building the new masks, **top** is the Planck collaboration *SRoll1* mask for the 100GHz band, **bottom** results from the convolution of the HFI 217GHz Planck legacy point source mask from PR1 SNR > 10 threshold with a top-hat beam profile of 1.3° radius. **Right:** *SRoll2.1* mask, **top** corresponds to a $J = 1 \rightarrow 0$ 100-143GHz run, **bottom** corresponds to a $J = 2 \rightarrow 1$ 217-143GHz run.

CO signal masking In order to determine the regions of the sky which may have a non zero ^{12}CO and ^{13}CO contribution, I make use of the recent survey performed by Dame & Thaddeus (2022). It is a composite survey built up from multiple sub-surveys of the ^{12}CO $J = 1 \rightarrow 0$ line emission covering roughly three quarters of the entire sky in total. In this manuscript, this survey will often be referred to as the D22 survey. The sky map from this survey is then smoothed by a wide Gaussian beam with FWHM = 5° , and only the pixels of intensity inferior to 0.01K.km.s^{-1} in the resulting smoothed map are kept **unmasked** for our *SRoll2.1* mask.

If this step is carried out incorrectly and some CO signal was to remain in the non-masked region, this would directly increase the error on the response coefficient determination, but also be visible in a resulting incorrect calibration of the ^{12}CO and ^{13}CO maps built from the outputs of *SRoll2.1*, as presented in Section IV.2. This will be proven not to be the case in Section V.1, where we will confront the newly built ^{12}CO and ^{13}CO maps to external observation data.

Taurus region inclusion The Taurus region is included in the masks by keeping **unmasked** any pixel of non-zero value in the HEALPix projected survey from Goldsmith et al. (2008), i.e. from the map plotted in Fig.III.10.

The resulting masks, compared to their respective starting point source mask, are shown in Fig. IV.6.

These new masks ensure a minimal risk of decorrelation for the ^{12}CO and ^{13}CO components, while leaving a sufficient area of the sky for the determination of all the other parameters fitted by SRoll2.1.

IV.1.3 Modifications of SRoll2 model and equations

Now that all the central modifications of the original SRoll2 algorithm's mode of operation characterizing the SRoll2.1 version have been introduced, I will present in this subsection how these changes affect the structure of the algorithm presented in Section III.2. I will first summarize the inputs and outputs of SRoll2.1, by reminding those presented for SRoll2 while emphasizing the changes and addition implied by the new SRoll2.1 version. I will then present how the equations of the data model and χ^2 minimization computations change from the SRoll2 to the SRoll2.1 version. Finally, I will detail the last modification of this new version of the mapmaking algorithm, namely the closure condition of the difference response coefficients.

Note that no changes were brought to the iteration scheme of SRoll2, which means that every aspect of the algorithm's looping structure presented in subsection III.2.1.c) are still fully relevant in the case of the SRoll2.1 version.

a) Input and output changes and addition

I summarize here the inputs and outputs modifications implied by the SRoll2.1 version with respect to SRoll2.

Inputs:

I remind in Table IV.1 all the inputs necessary for a SRoll2.1 run, where any quantity which is either added or modified in any way with respect to the inputs of SRoll2 presented in subsection III.2.1.a) are emphasized by a green color.

Dust template changes We modified the polarized part of the dust template with respect to the SRoll2 version. Since the SRoll2 algorithm cleaned the main sources of intensity-to-polarization leakage in the data signal, it was able to produce cleaner polarization components for the resulting frequency maps compared to the previous SRoll1 version used in the *Planck* legacy release. Given that the thermal dust is the dominating contribution to the foregrounds signal in the 353GHz band (by a factor of at least ~ 100 , cf. Fig. I.10 and Fig. 28 of *Planck Collaboration I (2014)*), we defined the Q and U Stokes parameters of the dust template as the Q and U Stokes parameters of the SRoll2 353GHz frequency maps after subtracting from it the signal of the CMB anisotropies.

In this case, we are not concerned with potential CO residuals at 353GHz since only the *polarised* components of the dust are taken using the SRoll2 353GHz frequency map. Not only is the total CO signal contribution at this frequency **at most** of about $\sim 1\%$ (cf. *Planck Collaboration XIII (2014)*), but the CO emission is expected to display a polarization fraction of **at most** $\sim 1\%$ (cf. *Greaves et al. (1999)*).

Additionally, the median value of each Stokes parameter map of the dust template was subtracted. This was originally done in order to avoid modifying the monopole (or zero-level) of the bolometers signal when performing the realignment, since SRoll cannot determine the *absolute* value of the monopoles. Indeed, when computing the realignment, the monopole

Table IV.1: Nature and source of all inputs for the SRoll2.1 mapmaking algorithm. If the input is explicitly mentioned in the data model or convergence computation of SRoll2.1 (or SRoll2), then the corresponding notation is also given.

Quantity	Notation	Source
TOI signal from bolometers, compressed into HPR	$M_{b,r,p}^{tot}$	<i>Planck</i> full TOI processing pipeline, exempted from its ADCNL correction
Parameters: Initial calibration Monopoles NEP Initial polarization efficiency	$1/g_{b,i=0}$ ρ_b	<i>Planck</i> 2015 release determination <i>Planck</i> 2015 release determination <i>Planck</i> 2013 release determination Ground-based measurements
Mask		Built for SRoll2.1 depending on the run setting (100-143GHz or 217-143GHz), cf. subsection IV.1.2
Templates: Foreground emissions: Dust DustVar $^{12}\text{CO} / ^{13}\text{CO}$ Free-free Synchrotron FSL Sky signal map template Polarization corrections templates Empirical time-transfer function template	C_p^f C_p^{Dust} $C_p^{DustVar}$ $C_p^{^{12}\text{CO}} / C_p^{^{13}\text{CO}}$ C_p^{FF} C_p^{Sync} $F_{b,r,p}^{tot}$ $M_{b,r,p}^{eval}$ $\left. \frac{\partial M}{\partial \rho} \right _{b,r,p}, \left. \frac{\partial M}{\partial \phi} \right _{\phi_{b,r,p}}$ $V_{b,r,p}^h$	Intensity from the <i>Commander</i> separation of the <i>Planck</i> 2015 release, polarization from the CMB subtracted 353GHz SRoll2 maps Intensity component of the difference between the 545GHz and 353GHz SRoll2 maps after CMB subtraction, cf. subsection IV.1.1.b) for more details G08 Taurus survey Goldsmith et al. (2008) <i>Commander</i> separation from <i>Planck</i> 2015 release <i>Commander</i> separation from <i>Planck</i> 2015 release <i>Planck</i> 2015 release determination 1° smoothed <i>Planck</i> 2018 release HFI frequency maps, containing the <i>total</i> dipole <i>Planck</i> 2018 release HFI frequency maps, down-scaled to a HealPix resolution of $N_{side} = 128$ Template HPRs fitted from the data TOIs, see subsection III.1.1.e)
Pointing		<i>Planck</i> 2013 release determination
Total dipole HPR	$D_{r,p}^{tot}$	<i>orbital</i> dipole computed from pointing data assuming a time-invariant amplitude <i>solar</i> dipole from <i>Planck</i> 2018 release with constant direction ($l = 264.021^\circ$, $b = 48.253^\circ$) and amplitude ($3362.08\mu\text{K}_{\text{CMB}}$), cf. Planck Collaboration III (2020)
Flags of discarded rings		<i>Planck</i> 2015 data flagging Planck Collaboration VII (2016)
Hitcount	$\mathfrak{h}\mathfrak{c}_{b,r,p}$	Compression procedure of TOI data into HPRs
ADU distribution model		Built from sparse raw data gap filled using a simplified sky model

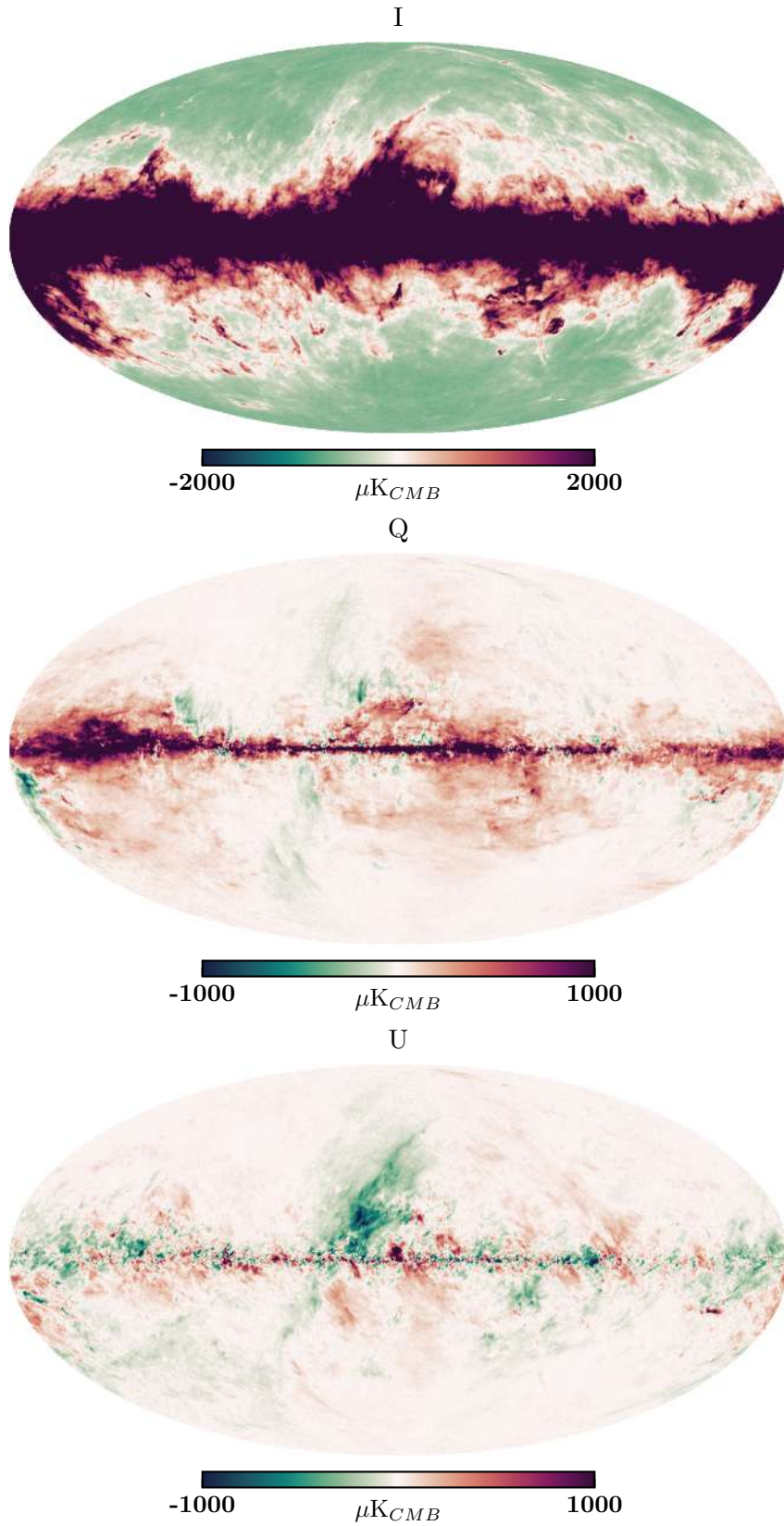


Fig. IV.7 *S*Roll2.1 template of the dust foreground emission. The median value was subtracted in each map. The resolution is that of the *S*Roll2.1 templates: $N_{side} = 128$. Top: *I* Stokes intensity component; Middle: *Q* Stokes polarization component; Bottom: *U* Stokes polarization component.

of the templates (multiplied by the corresponding RDC) are removed from the signal before projecting the resulting frequency – or *per bolometer* – maps (cf. Eq.III.27 for SRoll2, and later Eq.IV.1 for SRoll2.1). The impact is negligible in the case of the free-free and synchrotron foregrounds (with template maps median values of the order of $1\mu\text{K}_{CMB}$, multiplied by RDCs ranging between 10^{-3} - 10^{-1}), as well as for the DustVar template (with a median value less than $1\mu\text{K}_{CMB}$, multiplied by RDCs of the order of 10^{-2}), but not for the dust (with a median value greater than $700\mu\text{K}_{CMB}$, multiplied by RDCs ranging between 3×10^{-4} - 2×10^{-3}). In any case, we later showed that the monopoles of the bolometers' signal impact the ^{12}CO and ^{13}CO maps computation only negligibly, as detailed in subsection IV.3.3, thereby making the monopole value of our SRoll2.1 templates insignificant for the building of my ^{12}CO and ^{13}CO maps. This precaution was then of negligible impact to my results, but I never changed it, which explains the shape of the maps in Fig.IV.7. The resulting component maps of the SRoll2.1 dust template are displayed in Fig.IV.7.

Outputs:

I did not display a Table of outputs for the SRoll2.1 algorithm because it is unchanged from that of the SRoll2 algorithm, i.e. the outputs of the SRoll2.1 algorithm correspond to those listed in Table.III.3. The only implicit difference is for the output *foregrounds colour-correction coefficient*, noted $\widetilde{\delta L}_{b,i=N_{iter}}^f$, which is determined for one additional foreground component: the added template *DustVar*. Therefore, the implicit difference is that the ensemble spanned by the f index also comprises the DustVar template.

b) Equations and model changes

As previously introduced in the above subsection IV.1.1, the SRoll2.1 algorithm was used in 2 typical settings, depending on the CO rotational line present in the data. The equations describing the SRoll2 data model and its computed χ^2 minimization at each **Step#1** are only modified in 2 ways:

- The set spanned by the bolometer index b , depending on the set of frequency bands used in the run (100-143GHz or 217-143GHz),
- The set spanned by the foreground component index f , which comprises an additional element compared to SRoll2: the *DustVar* template.

The new sets of b Given my use of 2 frequency bands in any given SRoll2.1 run performed, I can define 3 sets of bolometers that the index b may span:

1. I call $\mathfrak{B}$ the set of all bolometers used in the computed SRoll2.1 run, with $\text{Card}(\mathfrak{B}) = N_{bolo}$. In the case of my 2 typical run settings, this corresponds to $N_{bolo} = 19$ for a 100-143GHz run, and to $N_{bolo} = 23$ for a 217-143GHz run,
2. $\mathcal{B}_{CO}$ is the set of bolometers containing the expected CO signal, with $\text{Card}(\mathcal{B}_{CO}) = N_{CO}$, i.e. either $N_{CO} = 8$ bolometers from the 100GHz band if we aim to compute the map of the $J = 1 \rightarrow 0$ rotational line, or $N_{CO} = 12$ bolometers from the 217GHz band for $J = 2 \rightarrow 1$,
3. $\mathcal{B}_{143}$ is the set of bolometers from the 143GHz frequency band that contains **no** CO line, with $\text{Card}(\mathcal{B}_{143}) = N_{143} = 11$.

The new set of f For the reasons explained above, we introduced an additional foreground template for the SRoll2.1 version of the mapmaking algorithm. This modification implies a change in the set of foreground components spanned by the f index: f now spans the ensemble of total size $N_{comp} = 6$, defined as:

$$\mathfrak{F} = \left\{ Dust, \mathbf{DustVar}, Free-free, Synchrotron, {}^{12}CO, {}^{13}CO \right\}.$$

From the way I have introduced and noted the modifications specific to the SRoll2.1 version, almost none of the equations from subsections III.2.2 and III.2.3 need rewriting, as the changes have been made *implicit*. Consequently, the definitions of the following terms, introduced in the previous subsections III.2.2 and III.2.3, also remain valid to describe the SRoll2.1 version of the mapmaking algorithm:

- ✓ The data model written in Eq.III.8,
- ✓ The *inverse* bolometer chain response decomposition, in equations III.13 and III.14,
- ✓ The systematic effects term defined in Eq.III.18,
- ✓ The *reduced* bolometer signal expressed in equations III.15 and III.19,
- ✓ The *data* and *model* vector residuals, respectively defined by equations III.21 and III.24 (also the *data* vector residual model description given by Eq.III.23),
- ✓ The *cleaned* frequency maps, both averaged and per bolometer, respectively given by equations III.28 and III.29,
- ✓ The output precision matrix defined in Eq.III.30,
- ✓ All the matrix and vectors of the various collections defined in subsection III.2.3 and used in the aforementioned equations.

From the definitions listed above, the changes characteristic to the SRoll2.1 version only appear through the modification of the definition of the set $\mathfrak{F}$ and the introduction of the new sets $\mathfrak{B}$, $\mathcal{B}_{CO}$, and $\mathcal{B}_{143}$. It also means that the bolometers chain responses are determined and calibrated exactly the same as was done in SRoll2, since they are treated the same and still calibrated on the same orbital dipole signal.

The χ^2 minimization, computed at each **Step#1** of the algorithm, can then be defined similarly to its expression of Eq.III.26:

$$\chi_i^2 = \sum_{b \in \mathfrak{B}} \sum_{r=1}^{N_{ring}} \sum_{p=1}^{N_{pix}} \frac{(R_{b,r,p,i} - \mathcal{R}_{b,r,p,i})^2}{(g_{b,i} \sigma_{b,r,p})^2 + T_{b,r,p} \sigma_{p,i}}.$$

Realignment modification Finally, the last computation step adapted for the goal of ${}^{12}CO$ and ${}^{13}CO$ maps production is that of the *cleaned* and *calibrated* bolometer signal, given by Eq.III.27 in the case of SRoll2. The modification consists in **not** performing the realignment of the ${}^{12}CO$ and ${}^{13}CO$ signals in the data. The benefit of such **non**-realignment is that in the resulting output *per bolometer* cleaned frequency maps, all foreground contributions of the signal are brought to the same average response **except** for the ${}^{12}CO$ and ${}^{13}CO$ signals. Keeping these bolometers response differences to the ${}^{12}CO$ and ${}^{13}CO$ emissions is paramount to build their respective full-sky maps since we rely on these differences to build

the said maps. This means that, using the $\widetilde{\delta L}_{b,i=N_{iter}}^f$ values determined by SRoll2.1, we will be able to compute a new χ^2 minimization in order to extract the signal of the ^{12}CO and ^{13}CO rotational lines from the rest of the data, as is detailed below in subsection IV.2.1. The final cleaned, calibrated, and realigned bolometer signal can then be written, in the case of SRoll2.1:

$$\begin{aligned} \widetilde{M}_{b,r,p,i}^{out} = \frac{1}{g_{b,i+1}} & \left(g_{b,i} \dot{M}_{b,r,p}^{tot} - \widetilde{O}_{b,r,i} - \sum_{f \in \mathfrak{F} \setminus \{^{12}\text{CO}, ^{13}\text{CO}\}} \widetilde{\delta L}_{b,i}^f A_{b,r,p} \cdot C_p^f - \sum_{h=1}^{N_{harm}} \widetilde{\gamma}_{b,i}^h V_{b,r,p}^h \right) \\ & - \widetilde{\delta g}_{b,i} T_{b,r,p} \mathbb{S}_{p,i}^{eval} + \widetilde{\delta \rho}_{b,i} \left. \frac{\partial M}{\partial \rho} \right|_{b,r,p} + \widetilde{\delta \phi}_{b,r,p,i} \left. \frac{\partial M}{\partial \phi} \right|_{\phi_{b,r,p}}. \end{aligned} \quad (\text{IV.1})$$

As a precaution, similarly to what was done in SRoll2, the corrections of the polarization efficiency and detector polarization angle are **not** applied for the signal of bolometers from the **100GHz frequency band**, i.e. the terms $\widetilde{\delta \rho}_{b,i} \left. \frac{\partial(gM)}{\partial \rho} \right|_{b,r,p}$ and $\widetilde{\delta \phi}_{b,r,p,i} \left. \frac{\partial(gM)}{\partial \phi} \right|_{\phi_{b,r,p}}$ disappear from Eq.IV.1 above for all bolometers from the 100GHz frequency band.

c) Closure condition modification

While the *closure condition* for the foreground RDCs is changed for various reasons explained below, those of the other parameters remain unchanged with respect to their definition introduced for the SRoll2 version:

$$\sum_{b \in \mathfrak{B}} \sum_{r=1}^{N_{ring}} O_{b,r} = 0, \quad \sum_{b \in \mathfrak{B}} \delta \rho_b = 0, \quad \forall p, \quad \sum_{b \in \mathfrak{B}} \sum_{r=1}^{N_{ring}} \delta \phi_{b,r,p} = 0.$$

The only visible difference with respect to the definition from SRoll2 stems from the new set spanned by the bolometers index b , but still implies the same physical meaning: SRoll does not give access to the monopole value in the maps, hence we shall not modify it inside the algorithm.

I think relevant to **emphasize** that while the definition of the foreground RDCs *technically* remains unchanged, keeping the *closure condition* defined for SRoll2 would imply, given the modification in the set spanned by b , an alteration in the nature of the *average* value of foreground response coefficients, and thereupon, also that of the RDCs. Indeed, the average value of foreground response coefficients, for the foreground emission f , is now defined over the set $\mathfrak{B}$, which can be written:

$$\forall f \in \mathfrak{F}, \quad \overline{L}^{\mathfrak{B}} = \frac{1}{N_{bolo}} \sum_{b \in \mathfrak{B}} L_b^f,$$

and from this definition, **if** the *closure condition* defined for SRoll2 is kept, the bolometer RDCs would be defined as follows:

$$\forall b \in \mathfrak{B}, \forall f \in \mathfrak{F}, \quad \delta L_b^f = L_b^f - \overline{L}^{\mathfrak{B}}.$$

This means that the foreground RDCs would be defined with respect to the average value over the **2 frequencies** used in the SRoll2.1 run. This is the main reason for the modification of the closure condition on the aforementioned coefficients, as well as part of the need for the added DustVar foreground template.

Consequently, the closure condition is modified in SRoll2.1 for the bolometers' foregrounds RDCs. The new closure condition can take one of 2 forms depending on the foreground component:

- If the foreground is **not** from a CO rotational line emission, i.e. $f \in \{\text{Dust}, \text{DustVar}, \text{Free-free}, \text{Synchrotron}\}$, then its closure condition can be written:

$$\forall f \in \{\text{Dust}, \text{DustVar}, \text{Free-free}, \text{Synchrotron}\}, \sum_{b \in \mathcal{B}_{CO}} \delta L_b^f = 0, \quad (\text{IV.2})$$

- If the foreground is from a CO rotational line emission, i.e. $f \in \{^{12}\text{CO}, ^{13}\text{CO}\}$, then its closure condition can be written:

$$\forall f \in \{^{12}\text{CO}, ^{13}\text{CO}\}, \sum_{b \in \mathcal{B}_{143}} \delta L_b^f = 0. \quad (\text{IV.3})$$

Note that in each of the 2 aforementioned cases, the difference between the *new* closure condition and the one given by Eq.III.10, is the set over which the sum of coefficients is forced to zero: in SRoll2, this set is the entirety of the bolometers used in the run, whereas in SRoll2.1, it is only a subset of the used bolometers, i.e. a subset of $\mathfrak{B}$. I explain below why this subset differs between CO and non-CO foregrounds, and how the subset is chosen in each case.

CO foreground case The *new* definition of the closure condition for the ^{12}CO and ^{13}CO foreground emissions rests upon the same hypothesis that forms the central motivation for using the 143GHz band as the second frequency band in the SRoll2.1 run: it is a frequency band in which **no** CO line is present. The most straightforward way of writing this physical condition would be the following:

$$\forall f \in \{^{12}\text{CO}, ^{13}\text{CO}\}, \overline{L^f}^{\mathcal{B}_{143}} = \frac{1}{N_{143}} \sum_{b \in \mathcal{B}_{143}} L_b^f = 0. \quad (\text{IV.4})$$

Having a *physical* knowledge of the value of $\overline{L^f}^{\mathcal{B}_{143}}$ provides us with a global calibrator for the L_b^f . Now let us show that the *new* closure condition expressed in Eq.IV.3 is tantamount to making the fitted δL_b^f coincide with the values of the L_b^f . By introducing the definition of the foreground RDCs δL_b^f , given by Eq.III.9, into the expression of the *new* closure condition given by Eq.IV.3, we get:

$$\begin{aligned} \forall f \in \{^{12}\text{CO}, ^{13}\text{CO}\}, \sum_{b \in \mathcal{B}_{143}} \delta L_b^f = 0 &\Leftrightarrow \sum_{b \in \mathcal{B}_{143}} (L_b^f - L_{ref}^f) = 0 \\ &\Leftrightarrow N_{143} \overline{L^f}^{\mathcal{B}_{143}} - N_{143} L_{ref}^f = 0 \\ &\Leftrightarrow L_{ref}^f = \overline{L^f}^{\mathcal{B}_{143}}. \end{aligned}$$

We can now inject the above result back into the definition given by Eq.III.9, and use the physical condition expressed in Eq.IV.4, to write:

$$\begin{aligned} \forall f \in \{^{12}\text{CO}, ^{13}\text{CO}\}, \forall b \in \mathfrak{B}, \delta L_b^f &= L_b^f - L_{ref}^f \\ &= L_b^f - \overline{L^f}^{\mathcal{B}_{143}} \\ &= L_b^f. \end{aligned} \quad (\text{IV.5})$$

Consequently, in the SRoll2.1 version, the fitted values of the $\delta L_b^{^{12}\text{CO}}$ and the $\delta L_b^{^{13}\text{CO}}$ respectively coincide with those of the $L_b^{^{12}\text{CO}}$ and the $L_b^{^{13}\text{CO}}$. The ^{12}CO and ^{13}CO RDCs of

the bolometers from the 143GHz frequency band should then be distributed around zero within their respective uncertainty ranges. This will be checked in subsection IV.1.4.b). It should be noted that, as previously mentioned, signal from other molecules is expected to be present in the bandpass of the bolometers at 143GHz, and should also be spatially correlated to the ^{12}CO and ^{13}CO emissions since they should all be located in molecular cloud regions of sufficient density to prevent the molecules from being broken by incoming high energy photons and cosmic rays. The presence of such emissions breaks the physical condition set by Eq. IV.4 and we should have $\overline{L}^{\mathcal{B}_{143}} > 0$. This molecular signal can also impact the $\delta L_b^{12\text{CO}}$ and the $\delta L_b^{13\text{CO}}$ distributions for the bolometers from the 143GHz band. This impact is further detailed in subsection IV.3.5. Despite this potential perturbation, we will see that a value of $\overline{L}^{\mathcal{B}_{143}}$ significantly greater than zero does not impact the calibration of my final ^{12}CO and ^{13}CO maps.

Non-CO foreground case The *new* definition of the closure condition for the *non*-CO foregrounds, given by Eq. IV.2, is chosen with our main objective in mind: producing *new* ^{12}CO and ^{13}CO emission maps. As will be detailed below in Section IV.2, the aforementioned new maps are built using the signal of the bolometers from the frequency band containing CO rotational line emission, i.e. bolometers at 100GHz for the $J = 1 \rightarrow 0$ and 217GHz for the $J = 2 \rightarrow 1$. For this reason, I chose to realign all *non*-CO foregrounds to the average foreground response along bolometers from $\mathcal{B}_{\text{CO}}$, i.e. along the CO containing frequency band. Indeed, the realignment defined in Eq. IV.1 uses the foreground templates to compute this realignment. Since my foreground templates are imperfect, I prefer to minimize the amount of correction using these templates performed along the CO containing frequency band. By defining the closure condition as expressed by Eq. IV.2, I ensure that the realignment is indeed computed along the CO containing frequency band, and that the most substantial correction is applied on the bolometers at the 143GHz frequency band, which are not used to determine the ^{12}CO and ^{13}CO maps, reducing the risk of leakage from other foreground components into my CO maps due to templates' imperfection.

d) Simulations modifications and limitations

As was presented in Section III.3, SRoll2 was evaluated and characterized on a set of simulations, built from an update of the FFP10 simulation pipeline (cf. Planck Collaboration III (2020)). In this work, I used the same aforementioned set of simulations to characterize both the results of SRoll2.1, as well as to investigate the presence of CO signal at high latitudes, therefore originating from a CO emission mechanism not driven by opacity, in my resulting ^{12}CO and ^{13}CO maps.

The only slight modification on these simulations brought with the SRoll2.1 version is the ^{13}CO template map used for runs on simulation TOIs, as well as the parameters used to add the ^{13}CO template to each of the bolometers signal in input, as presented in subsection III.3.3. Also, since a SRoll2.1 run is performed on 2 frequency bands, one containing CO emission and the 143GHz band as a zero-CO reference, all the parameters of ^{13}CO template addition to the input signal are **null** for all bolometers along the 143GHz frequency band.

In the SRoll2 characterization from simulations, the ^{13}CO template employed was a very crude map built from a determination based on the ground-based ^{13}CO response coefficients. In an attempt to use a ^{13}CO template more representative of the true sky emission, both in term of overall emission distribution and intensity, but also in terms of orthogonality with the other foreground templates, and especially try to better approximate the orthogonality with the ^{12}CO template, I built a new ^{13}CO template map for the SRoll2.1 simulations.

This new ^{13}CO template map is built from an earlier result of ^{13}CO map constructed from the SRoll2.1 output, following the method I developed during my PhD, and which will be extensively presented in Section IV.2 below. To ensure that only real ^{13}CO signal is kept in this data map, and to render it fit as a SRoll2.1 template, I treated the map by:

- ✓ Putting to **zero** all pixels outside of the latitude range: $-35^\circ < b < 35^\circ$,
- ✓ Putting to **zero** all pixels of negative value in the map,
- ✓ Putting to **zero** all pixels with an intensity below 1K.km.s^{-1} in the high sky-coverage ^{12}CO survey of Dame & Thaddeus (2022),
- ✓ Downgrading the map to the SRoll2.1 templates' resolution of $N_{\text{side}} = 128$.

The new SRoll2.1 simulation ^{13}CO template map built from this procedure is displayed in Fig.IV.8, where I also plotted the ^{12}CO simulation template map as well as the *old* SRoll2 ^{13}CO simulation template map.

This procedure provided me with a ^{13}CO template map for the simulations more representative of the data, notably with respect to its level of orthogonality with the ^{12}CO template, which remains the PSM CO template (cf. Delabrouille et al. (2013)). This can be seen in Fig.IV.9, which represents the cross-spectrum between the ^{12}CO and ^{13}CO simulation template maps, both in the case of the old ^{13}CO template of SRoll2 (in yellow) and the newly constructed ^{13}CO template for SRoll2.1 (in blue). For comparison, I also plotted the cross-spectrum between the ^{12}CO and ^{13}CO maps from the data (in red), obtained from my method presented in Section IV.2 (it corresponds to the $J = 1 \rightarrow 0$ maps displayed in subsection IV.2.2). The cross-spectrum is computed using a mask hiding the galactic disk, keeping a fraction of sky $f_{\text{sky}} = 90\%$, and log-smoothed with a multipole fraction of $\ell_{\text{frac}} = 20$ (for more details on the cross-spectrum computation and the masks, see Appendix.C). We can see a clear improvement, as the cross-spectrum using the new ^{13}CO template better reproduces that of the data up $\ell \sim 140$. We can also see a strong cut of the data cross-correlation at the said value of $\ell \sim 140$, corresponding to the scale at which the correlation between the ^{12}CO and ^{13}CO emissions drops. A similar dip can be seen at the same scale, in both cases of simulation template cross-correlation, but drastically smaller: this is due to the cross-correlation being computed over almost the whole sky (galactic disk excluded), and the simulation template maps being null at high latitude. Therefore, the cross-correlations from the simulation templates are dominated, above said scales $\ell > 140$, by the correlations coming from the aforementioned zero-valued regions.

The parameters used to add the ^{13}CO template to the input simulation HPRs are then taken as the ^{13}CO RDCs determined by SRoll2.1 on the same data run from which the new ^{13}CO template was built. These coefficients are then normalized by their average value over the 100GHz band to keep the RDC distribution information while making the RDCs dimensionless, since the ^{12}CO and ^{13}CO *simulation* templates are in units of K_{CMB} , same as the HPR signal.

Limitations I was limited, in the scope of my PhD work, in the number of simulation runs I could expect to base my characterization upon. This is essentially limited by the great computational cost of each full SRoll2.1 run, landing in the range of $\sim 40\text{-}60 \times 10^3$ CPU hours for each run. Since I already had to consume many hours in my prospective approach of developing SRoll2.1, then trying to build the later abandoned iterative solution on SRoll (see Section IV.4), I ended up having to manage with the computational time that remained.

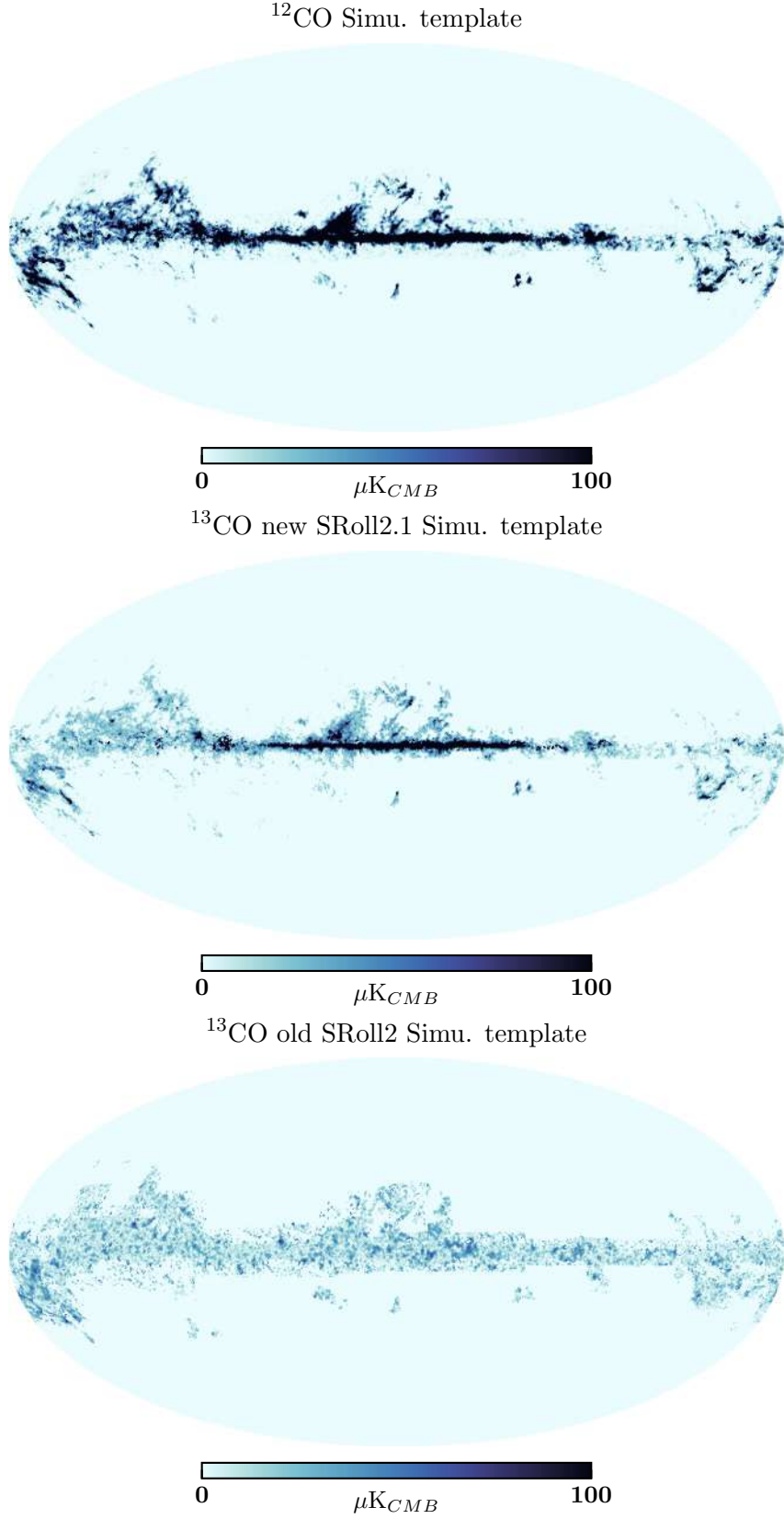


Fig. IV.8 *SRoll* simulation templates of the ^{12}CO and ^{13}CO foreground emissions. The resolution is that of the *SRoll2* and *SRoll2.1* templates: $N_{side} = 128$. Top: ^{12}CO simulation template; Middle: new *SRoll2.1* ^{13}CO simulation template; Bottom: old *SRoll2* ^{13}CO simulation template.

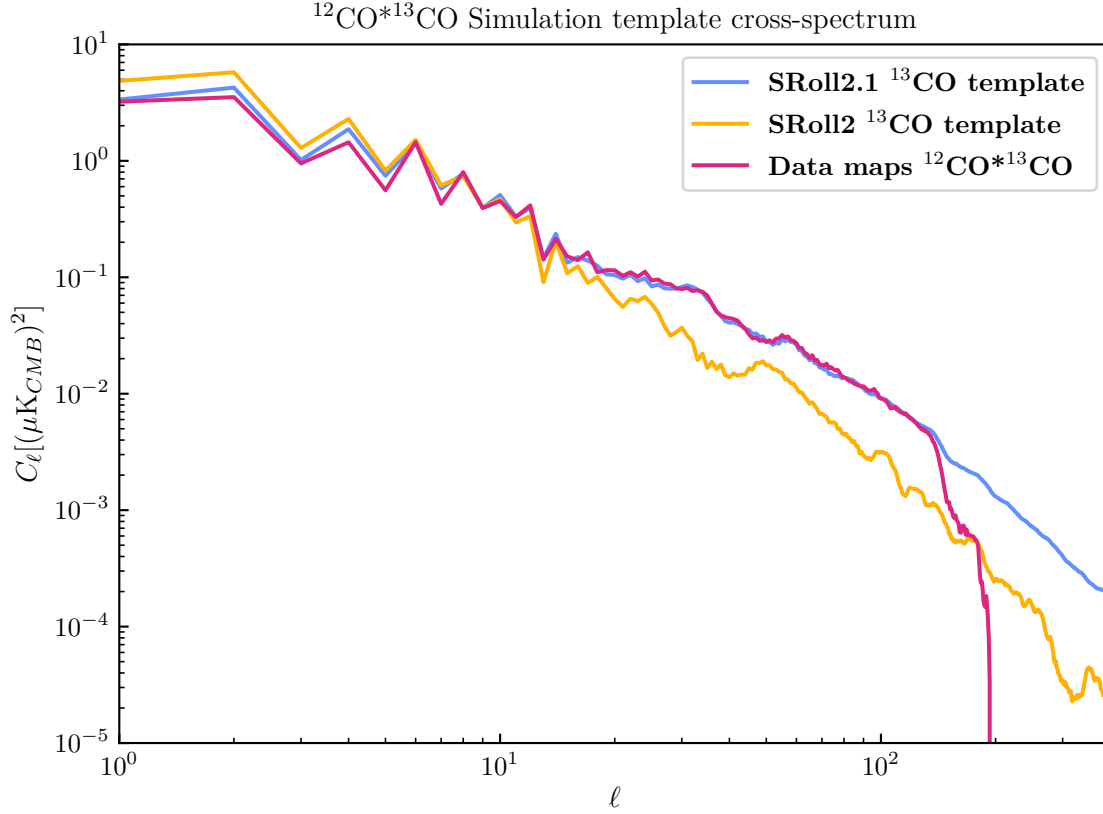


Fig. IV.9 Cross-spectrum of the ^{12}CO and ^{13}CO simulation template maps, computed using a mask keeping a sky fraction $f_{\text{sky}} = 90\%$, and hiding the galactic disk (cf. Appendix.C), as well as log-smoothed with a multipole fraction of $\ell_{\text{frac}} = 20$ (cf. Appendix.C). The ^{12}CO template is the one from the SROll2 simulations, corresponding to the PSM CO map. Blue: cross-spectrum using the new ^{13}CO template built for SROll2.1; Yellow: cross-spectrum using the old ^{13}CO template map from SROll2; Red: cross-spectrum between the ^{12}CO and ^{13}CO $J = 1 \rightarrow 0$ data maps built in the scope of this work (see Section IV.2).

Finally, only 10 full SROll2.1 100-143GHz simulation runs were run and kept for the qualification of my results, and will be used in the next sections of this chapter, as well as in the next chapter. I did not have enough computational hours remaining to compute 217-143GHz SROll2.1 simulation runs.

IV.1.4 Results on CO, dust, and DustVar coefficients from SROll2.1

I will now present all the results on the newly determined SROll2.1 foreground RDCs, in both cases of typical run performed in this work, i.e. both the 100-143GHz and 217-143GHz runs. Firstly, I need to introduce my method of error evaluation for the 217-143GHz run, on which I lack any simulation runs, as previously discussed in subsection IV.1.3.d). Secondly, I will present the SROll2.1 fitted RDCs for the ^{12}CO , ^{13}CO , dust, and DustVar foreground emissions. I then use the results for the 143GHz bolometers from both runs as a way to evaluate the consistency of the SROll2.1 sky model. Finally, I address the issue of the CO templates for the $J = 2 \rightarrow 1$ ^{12}CO and ^{13}CO coefficients determination, i.e. the fact that

we use a $J = 1 \rightarrow 0$ emission map as template of the $J = 2 \rightarrow 1$ line.

The full list of foreground RDCs fitted by SRoll2.1 can be found in the tables of Appendix.D.

a) Methods to compute the uncertainty

In the case of the 100-143GHz run, I have a set of 10 SRoll2.1 simulation runs to evaluate the uncertainty on the coefficients' determination. For each foreground RDC, the uncertainty can then be determined as the standard deviation of the distribution of the respective fitted coefficient along the set of the 10 simulations' results, i.e. the uncertainty on any δL_b^f can be defined as $\sigma_{\delta L_b^f}^{SIMU}$.

As for the 217-143GHz case, I do not have any SRoll2.1 simulation run on which I could compute the coefficients uncertainty. I therefore devised a method to evaluate the uncertainty on the fitted RDCs by only exploiting the data. The main benefit from this method is that the required SRoll2.1 runs can be downscaled in terms of RAM requirements, allowing me to compute them on the smaller ENS cluster located at MesoPSL. I checked and characterized the method on the 100-143GHz SRoll2.1 run first, for which I can compare this *alternative* method to the more conventional evaluation from the simulations.

Uncertainty evaluation from the data The main idea behind the method is to compute SRoll2.1 not on the full *Planck* mission data, but on various, mutually exclusive, subsets of the data. The hypothesis is then that, as long as each subset run is sufficiently representative of a complete one, these subset runs are only differentiated from one another by their noise, and are all affected by different realisations of noise since the subsets are mutually exclusive. If this hypothesis is verified, the standard deviation of the coefficients determination along these runs should correspond to the sought determination uncertainty multiplied by the inverse square-root of the fraction of total data covered by each subset, i.e. if each subset represents 1/10 of the total data, the RDCs' standard deviation along the subset runs amounts to the uncertainty on the RDCs multiplied by a factor $\sqrt{10}$. Notably, in order to still remain representative of the characteristic quality and systematics of a complete SRoll2.1 run, the selected data subsets must verify at least 2 conditions:

1. The selected subsets must be sufficiently spread throughout the mission, i.e. not fully concentrated on a single compact region of the timeline,
2. The subsets must be sufficiently representative of the full data, i.e. they must cover the full mapped sky with sufficient redundancy.

In order to satisfy these 2 requirements, the subset selection is performed by only considering **a regularly spaced subset of rings**. To this end, I make use of an additional parameter, called RSTEP, which can be passed to SRoll2.1 as an input, to perform such a selection from its input TOIs. When using the RSTEP parameter, i.e. setting it to $RSTEP > 1$, SRoll2.1 only considers *one ring out of every RSTEP rings*. This sparse but regular selection of the used parts of input TOIs ensures the first above condition, while the second is met as long as RSTEP is kept sufficiently small. We can then create similar subsets, yet still mutually exclusive, by shifting the starting ring. Such a subset selection allows to build a total of RSTEP mutually exclusive data subsets (by incrementing the starting ring number one by one up to RSTEP-1 times).

Hypothesis validation In order to validate this uncertainty evaluation method, I need to check the hypothesis defined above, i.e. that the coefficients determined on the subsets must vary only slightly from one subset to the other, and that the standard deviation of the coefficients determination must grow as $\sqrt{\text{RSTEP}}$, which can be written:

$$\forall f, \forall b, \sigma_{\delta L_b^f}^{\text{RSTEP}} \simeq C^{st} \times \sqrt{\text{RSTEP}}, \quad (\text{IV.6})$$

with $\sigma_{\delta L_b^f}^{\text{RSTEP}}$ the standard deviation of the determinations of the coefficient δL_b^f along the data subsets built using the parameter RSTEP. These conditions were tested by computing SRoll2.1 runs on data subsets built as described above for RSTEP values in the set $\text{RSTEP} \in \{10, 15, 20, 30\}$.

The condition of small variation with the starting ring (i.e. for a given value of RSTEP) can be verified by plotting the distribution of RDCs for various foregrounds along these data subsets runs. This is displayed in the case of the ^{12}CO foreground in Fig. IV.10. We can see that for $\text{RSTEP} \in \{10, 15, 20\}$ the distribution of the RDC averages stays stable, and that the distribution along the data subsets for each RDC is concentrated around the average value and spread evenly around it. For the case $\text{RSTEP} = 30$, the distributions around the average values start to get more irregular. Also, the values are compatible with the results computed on the complete SRoll2.1 run, displayed with the uncertainties computed on the simulations. The same plot, in the case of the ^{13}CO and dust foregrounds, can be seen in Appendix D.

The second condition, defined in Eq. IV.6, was checked using the same RSTEP condition, by comparing the standard deviation of the distributions of each RDC along the data subset runs, normalized by a factor $1/\sqrt{\text{RSTEP}}$. If the law of Eq. IV.6 is verified, the 4 standard deviations thereby computed for each RDC (one for each RSTEP value) should result in the same value. This comparison is plotted in Fig. IV.11 for the ^{12}CO , ^{13}CO , dust, and DustVar foreground components. The uncertainty on the standard deviations are computed from the uncertainty on the RDCs determined from the simulations. We can see that for all the foregrounds, the law is well verified for $\text{RSTEP} \in \{10, 15, 20\}$, but no longer is for $\text{RSTEP} = 30$. For this reason, the $\text{RSTEP} = 30$ case was ignored to compute the uncertainty from the data subsets.

With the founding hypotheses checked, we can now evaluate the uncertainty on the RDCs determination from the data. The final expression I retained for this computation is the average of the normalized standard deviation values for the 3 RSTEP cases $\text{RSTEP} \in \{10, 15, 20\}$, which can be written:

$$\Delta_{\delta L_b^f} = \frac{1}{3} \sum_{\text{RSTEP} \in \{10, 15, 20\}} \frac{\sigma_{\delta L_b^f}^{\text{RSTEP}}}{\sqrt{\text{RSTEP}}}. \quad (\text{IV.7})$$

The results from this method for the RDCs' uncertainty computation is compared to the results from the conventional computation from the set of 10 simulations in Table IV.2. In this table are listed the average uncertainties on the RDCs for each foreground, along a given frequency band (either 100GHz or 143GHz), computed from both methods. We can see that the uncertainty on the ^{12}CO and ^{13}CO foregrounds are well retrieved using the method of uncertainty determination from the data. For the other foregrounds, the uncertainty determination from the data tends to lead to an underestimation of the uncertainty compared to that computed from the simulations, with a factor varying from 1.8 (synchrotron at 143GHz) to 7.1 (free-free at 100GHz). I should be noted that the PSM used in the simulation is not perfect, and that this imperfect modeling of the various foreground

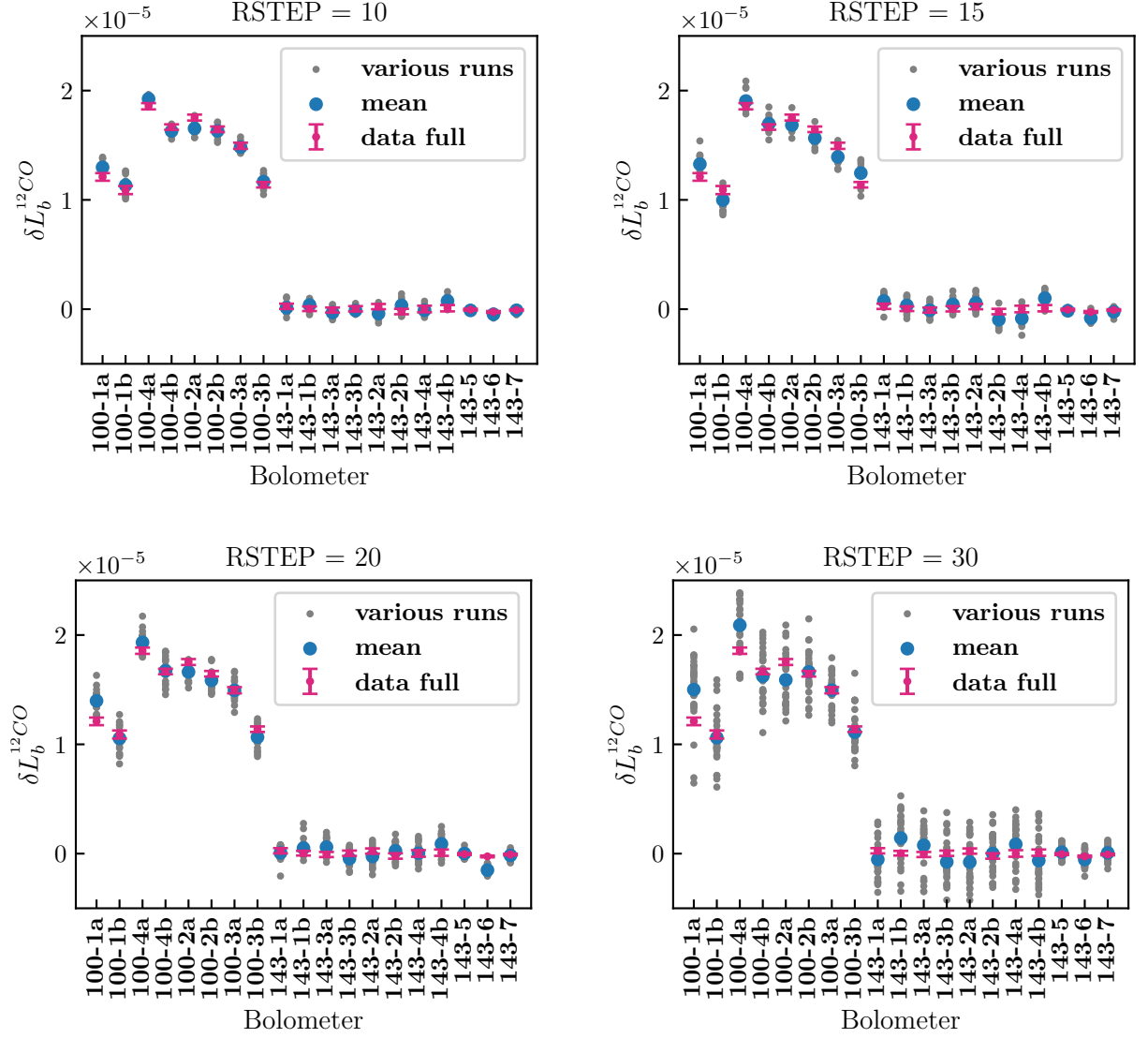


Fig. IV.10 ^{12}CO RDCs ($\delta L_b^{12\text{CO}}$) distributions (in **grey**) for data subsets runs with: RSTEP = 10 (**upper left**); RSTEP = 15 (**upper right**); RSTEP = 20 (**lower left**); RSTEP = 30 (**lower right**). In each case, the average along the corresponding bolometer's RDC distribution is plotted in **blue**, and the corresponding RDC computed from the complete SROLL2.1 run is plotted in **red** (with uncertainty bars determined from the simulations).

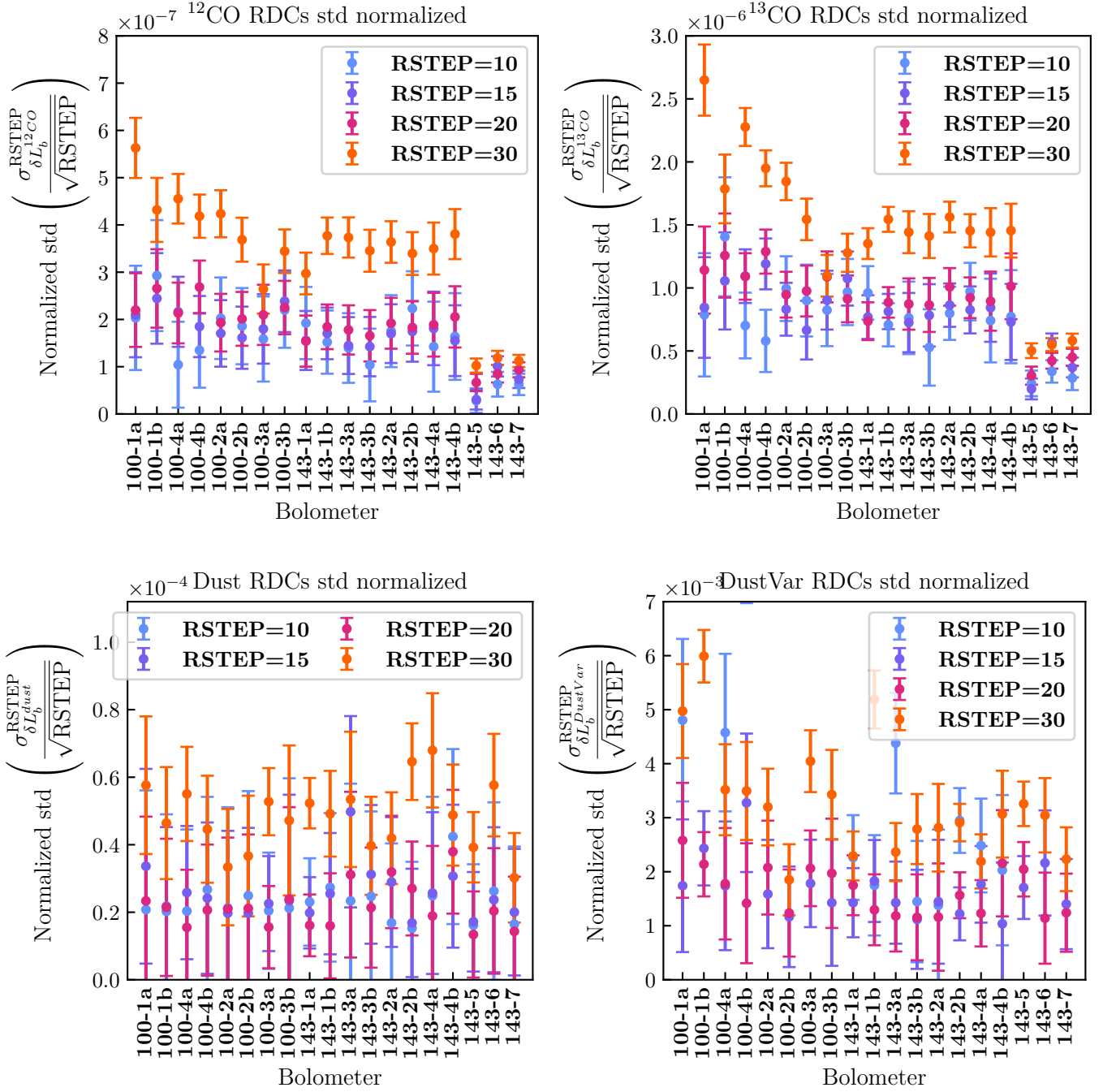


Fig. IV.11 Standard deviation of the distributions of each RDC along the data subset runs of same RSTEP, normalized by the corresponding $1/\sqrt{\text{RSTEP}}$ factor. For each RDC, a standard deviation is computed for each $\text{RSTEP} \in \{10, 15, 20, 30\}$. The uncertainty on the standard deviations is evaluated by propagating the RDC uncertainties computed from the simulations. **Upper left:** ^{12}CO RDCs; **Upper right:** ^{13}CO RDCs; **Lower left:** Dust RDCs; **Lower right:** DustVar RDCs.

Table IV.2: Comparison of the average uncertainty on the foregrounds RDCs determination computed either from the set of 10 SROLL2.1 simulation runs (rows **Simu.**), or from data subset runs as defined by Eq.IV.7 (rows **Data**).

Freq.	Method	^{12}CO	^{13}CO	Dust	DustVar	Free-free	Synchro.
100GHz	Data	2.04×10^{-7}	9.76×10^{-7}	2.21×10^{-5}	1.86×10^{-3}	9.83×10^{-4}	2.66×10^{-2}
	Simu.	2.90×10^{-7}	1.01×10^{-6}	9.18×10^{-5}	4.02×10^{-3}	6.95×10^{-3}	5.93×10^{-2}
143GHz	Data	1.41×10^{-7}	6.90×10^{-7}	2.39×10^{-5}	1.43×10^{-3}	9.53×10^{-4}	3.03×10^{-2}
	Simu.	2.04×10^{-7}	6.90×10^{-7}	7.49×10^{-5}	3.15×10^{-3}	5.06×10^{-3}	4.86×10^{-2}

emissions can also lead to an *overestimation* of the uncertainty from the 10 simulation runs. Despite this observed discrepancy between the methods, I have to make do with this method of uncertainty estimation for the 217-143GHz runs, and keep in mind of this underestimation bias.

In all the following plots displaying the fitted SROLL2.1 coefficients, I use the following conventions:

- ◆ For any result from a 100-143GHz run, the uncertainty displayed will be the one computed on the simulations, since I prefer to take the precaution of risking a potential *overestimation* of the uncertainty rather than an underestimation,
- ◆ For any result from a 217-143GHz run, the uncertainty displayed will be the one computed on the data subsets, as defined by Eq.IV.7, since it is the only one at my disposal.

The above conventions are the same used in the tables listing all RDC results in Appendix.D.

b) The response difference coefficients

I present here the RDCs of the ^{12}CO , ^{13}CO , dust, and DustVar foreground emissions resulting from the complete SROLL2.1 runs. Those of the free-free and synchrotron foreground are additionally displayed in Appendix.D.4.

^{12}CO and ^{13}CO I plotted in Fig.IV.12 the results from the 100-143GHz run, and in Fig.IV.13 those from the 217-143GHz run, for the ^{12}CO and ^{13}CO foreground components. The first notable result is the good spread of the RDC distributions compared to the noise. This can be measured from the ratio $\sigma_{\delta Lf}/\Delta_{\delta Lf}$ computed and displayed in the tables of Appendix.D. This ratio compares the standard deviation of the RDC distribution to the average uncertainty along the said distribution (stemming from noise and errors). The higher the ratio, the higher the distinct separation of the RDCs values along their distribution within the corresponding frequency band. This value is essential for the map computation, as the higher the ratio, the greater the potential quality of the maps resulting from a computation based on this distribution of response *differences*. The only exception here is the case of the ^{13}CO $J = 2 \rightarrow 1$ line with a ratio of only $\left. \frac{\sigma_{\delta L^{13}\text{CO}}}{\Delta_{\delta L^{13}\text{CO}}} \right|_{217} = 1.36$, compared to the ^{12}CO at the same frequency with a ratio of $\left. \frac{\sigma_{\delta L^{12}\text{CO}}}{\Delta_{\delta L^{12}\text{CO}}} \right|_{217} = 8.97$, as well as compared to

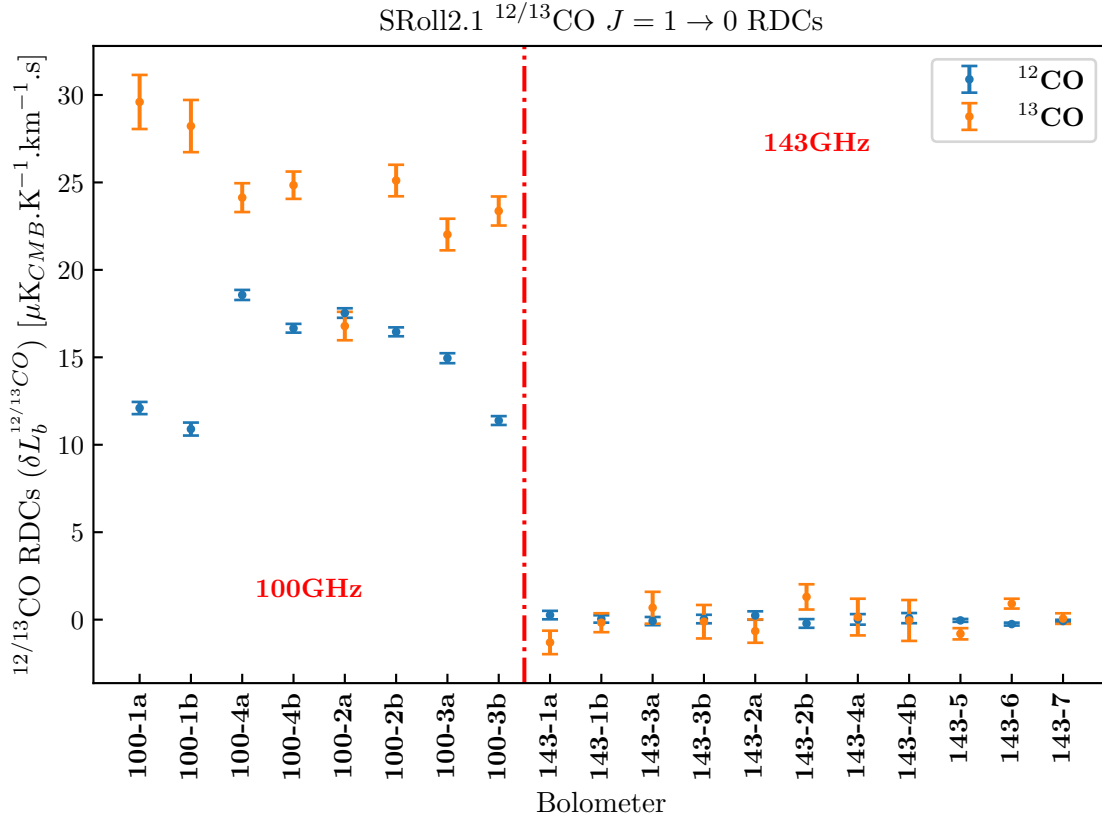


Fig. IV.12 ^{12}CO and ^{13}CO bolometer RDCs determined by SRoll2.1 from the 100-143GHz run, containing the $J = 1 \rightarrow 0$ line. The first 8 bolometers are from the 100GHz band, and the 11 last are from the 143GHz band.

the ^{12}CO and ^{13}CO $J = 1 \rightarrow 0$ lines, with ratios of respectively $\left. \frac{\sigma_{\delta L^{12\text{CO}}}}{\Delta_{\delta L^{12\text{CO}}}} \right|_{100} = 9.59$ and

$$\left. \frac{\sigma_{\delta L^{13\text{CO}}}}{\Delta_{\delta L^{13\text{CO}}}} \right|_{100} = 3.61.$$

The information provided by this ratio will also be useful to discuss the significance of the molecular emission from the 143GHz band, as will be presented in subsection IV.3.5.

These plots also allow a visual representation of the ^{12}CO and ^{13}CO SRoll2.1 closure condition defined by Eq. IV.3, and how it allows to use the 143GHz frequency band as an absolute calibrator for the ^{12}CO and ^{13}CO RDC averages, i.e. $\overline{\delta L^{12\text{CO}}}^{\text{BCO}}$ and $\overline{\delta L^{13\text{CO}}}^{\text{BCO}}$, **provided** the validity of the hypothesis that the molecular emission in the 143GHz band is negligible in intensity.

Dust I plotted in Fig. IV.14 the results from both the 100-143GHz and 217-143GHz runs, for the dust foreground component.

Though the uncertainty evaluation on the results of the 217-143GHz run are probably underestimated (by a factor of up to 4, as seen from Table. IV.2), we can see that the dust foreground RDCs, as those of the most intense foreground component, are very accurately determined (displaying a high ratio $\frac{\sigma_{\delta L^{\text{dust}}}}{\Delta_{\delta L^{\text{dust}}}}$, ranging from 3.3 to 75).

We can see a difference between the 8 first bolometers and the 3 last at 143GHz, and between the 8 first and the 4 last at 217GHz. This shows a RDC difference between the PSBs

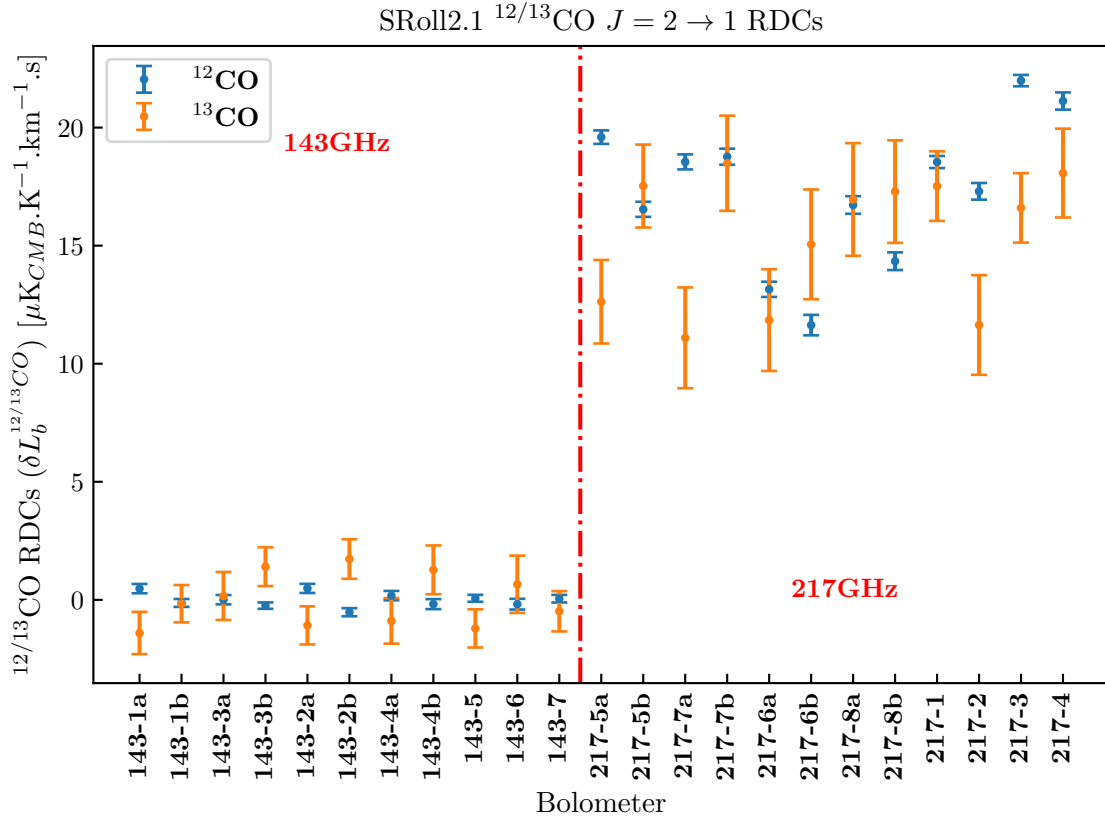


Fig. IV.13 ^{12}CO and ^{13}CO bolometer RDCs determined by SRoll2.1 from the 217-143GHz run, containing the $J = 2 \rightarrow 1$ line. The first 11 bolometers are from the 143GHz band, and the 12 last are from the 217GHz band.

and the SWBs, most probably stemming from the imperfection of the polarization part of the dust templates.

This figure also offers a visual representation of the dust SRoll2.1 closure condition defined by Eq. IV.3, and thereby the amount of correction subjected to the bolometers of the 143GHz band during the realignment of the dust foreground in order to be brought back to the average response along the CO containing frequency.

DustVar I plotted in Fig. IV.15 the results from both the 100-143GHz and 217-143GHz runs, for the DustVar foreground component.

We can see that the difference between the averages of the 2 bands is smaller for the 100-143GHz run (0.044) than for the 217-143GHz one (0.384). This could be expected since the DustVar template is built from the difference between the maps at 545GHz and 353GHz, and the farther we get spectrally from the origin of the template, the lesser we can expect its capacity to accurately represent the spatial variation of SED.

c) Comparison to CO results from *Planck* 2013

I compared my results on the ^{12}CO and ^{13}CO RDCs from SRoll2.1 to what was previously determined in the work of [Planck Collaboration XIII \(2014\)](#) for the determination of the *Planck* CO maps. To this end, I plotted the figure 2 from [Planck Collaboration XIII](#)

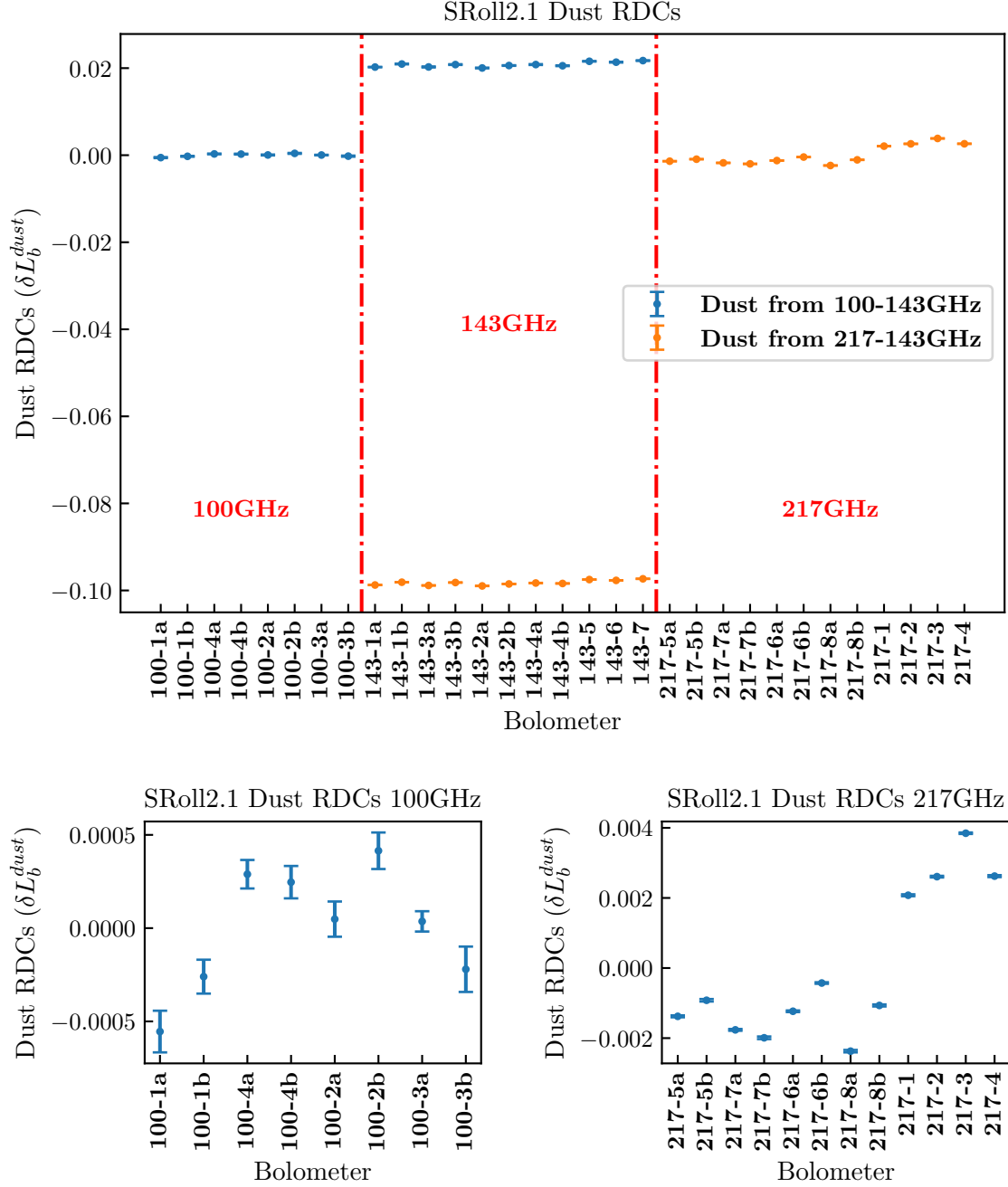


Fig. IV.14 **Top:** Dust bolometer RDCs determined by SRoll2.1 from both 100-143GHz and 143-217GHz runs. The first 8 bolometers are from the 100GHz band, the middle 11 bolometers are from the 143GHz band, and the 12 last are from the 217GHz band. **Bottom left:** Zoom from the top plot on the RDCs from the 100GHz frequency band; **Bottom right:** Zoom from the top plot on the RDCs from the 217GHz frequency band.

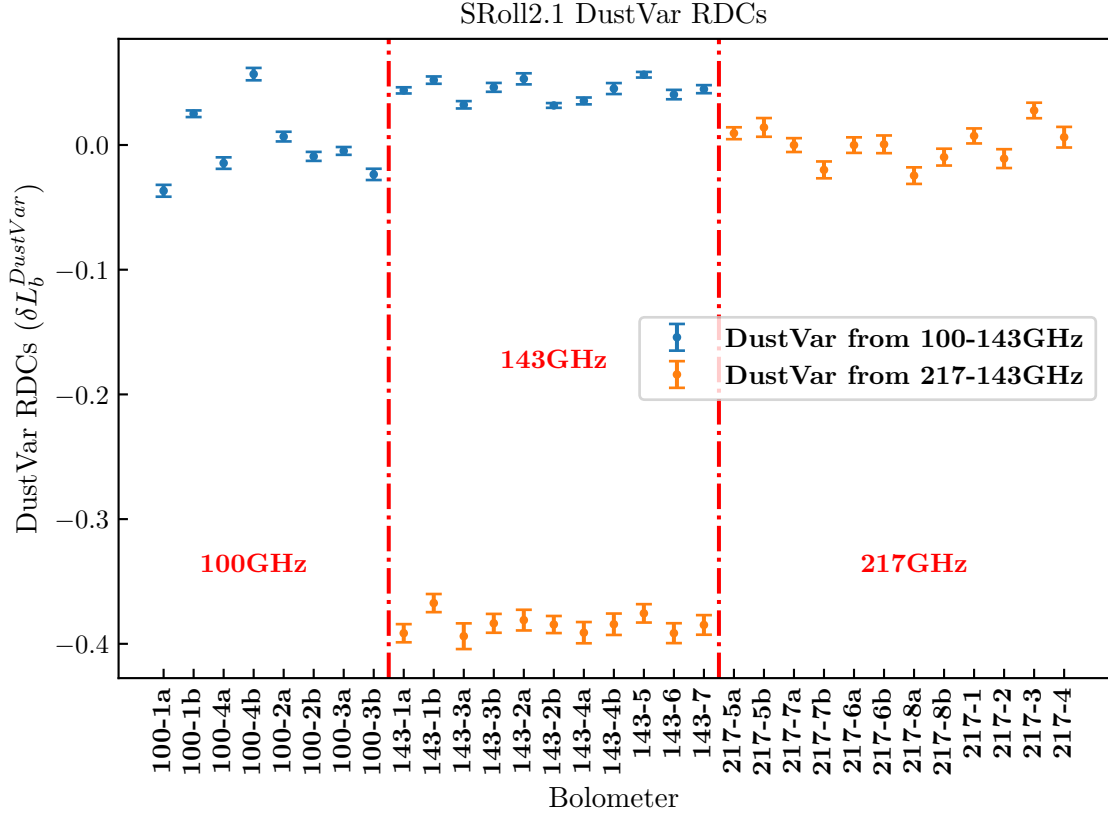


Fig. IV.15 *DustVar* bolometer RDCs determined by SROLL2.1 from both 100-143GHz and 143-217GHz runs. The first 8 bolometers are from the 100GHz band, the middle 11 bolometers are from the 143GHz band, and the 12 last are from the 217GHz band.

(2014), displaying the normalized conversion coefficients determined both from the ground-based measurement and from the computation carried out in their work for the CO maps construction, though only computed in the case of the ^{12}CO . Their determination of the ^{12}CO conversion coefficients was performed using template emissions for the CO (using the survey of Dame et al. (2001)) and the dust signals, and computed on the *pre-SRoll* frequency maps (see subsection II.4.1 for details). The results are displayed by plotting the average over each PSB pair, and individually for SWBs. I added to these plots my RDCs, which also *are* conversion coefficients since they were calibrated on the ^{12}CO and ^{13}CO templates, i.e. the survey of Goldsmith et al. (2008), and since the 143GHz was used to determine the *absolute* RDC averages, hereby their unit as discussed in subsection III.1.1.c). These figures are thus the same as the previously presented Figs. II.16 and II.17, but where I added my results from SROLL2.1 for comparison.

The results of this comparison are respectively shown in Figs. IV.16 and IV.17 for the 100GHz ($J = 1 \rightarrow 0$) and 217GHz ($J = 2 \rightarrow 1$) frequency bands. The values of the RDCs from the PSBs are computed as normalized bolometer pairs average, e.g. for the 100-1a and the 100-1b PSB pair from the 100GHz band, the value plotted for the ^{12}CO RDC would be $\frac{\delta L_{100-1a}^{12\text{CO}} + \delta L_{100-1b}^{12\text{CO}}}{2}$. The RDCs of the SWBs are just the normalized RDC values. Almost all my RDCs are compatible with the ground-based measurement in both ^{12}CO and ^{13}CO , for

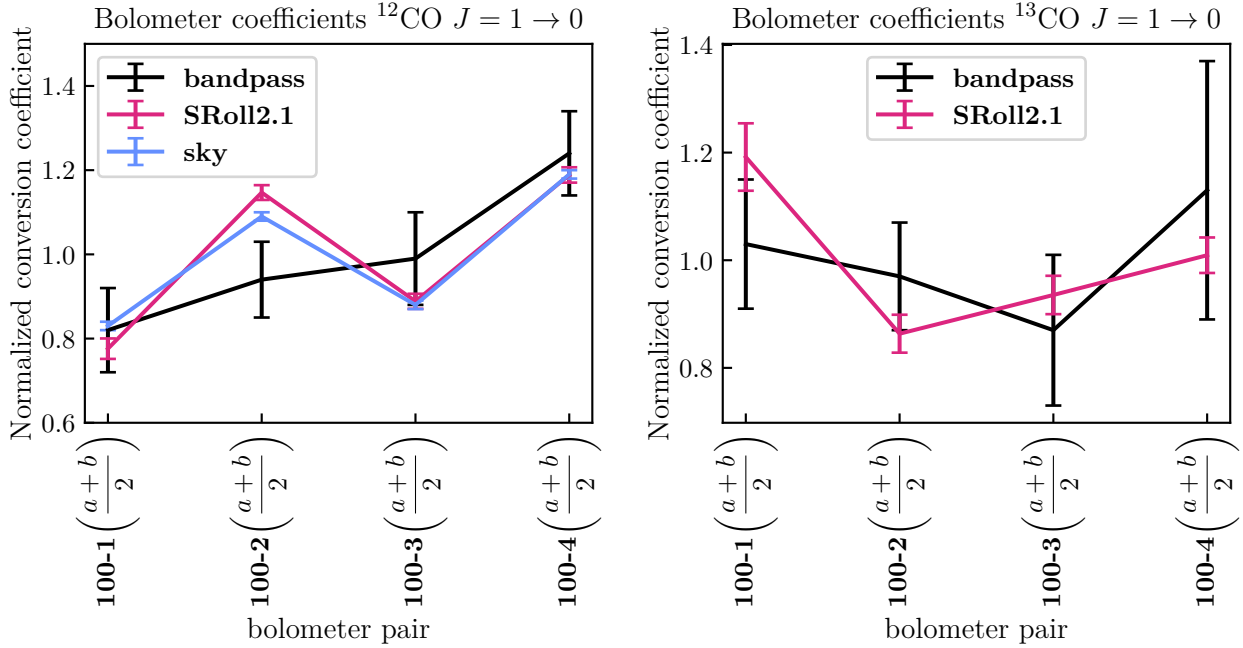


Fig. IV.16 CO RDCs for the 100GHz band, i.e. the $J = 1 \rightarrow 0$ CO line. Coefficients from PSBs are averaged over the pair, and all coefficients are normalized by the corresponding average $\overline{\delta L^{12/13CO} B_{CO}}$ over the full frequency band. Compared with the results from the *Planck* 2013 CO coefficients, cf. Fig. II.16. **Left:** ^{12}CO . **Right:** ^{13}CO .

both lines. Also, though the *Planck* ^{12}CO coefficients are not always compatible with my results, they display a very similar distribution. As for the results discrepancies, they can be due to the correction of additional systematics from SRoll2.1 and the completed *Planck* 2018 data analysis and cleaning pipeline compared to what was available as the time of the work of [Planck Collaboration XIII \(2014\)](#), impacting the CO coefficients determination. Also, the new separation of the ^{12}CO and ^{13}CO isotopologues performed in my work should play a role too in the results discrepancy observed. We can also see, from the RDC results listed in Appendix.D, that my results slightly deviate in average from the ground-based computation: $\overline{\delta L^{12CO} B_{100}} = 14.8 \pm 0.10 \mu\text{K}_{\text{CMB}}.\text{K}^{-1}.\text{km}^{-1}.\text{s}$ against a ground-based value of $14.2 \mu\text{K}_{\text{CMB}}.\text{K}^{-1}.\text{km}^{-1}.\text{s}$, and $\overline{\delta L^{13CO} B_{100}} = 24.3 \pm 0.37 \mu\text{K}_{\text{CMB}}.\text{K}^{-1}.\text{km}^{-1}.\text{s}$ against a ground-based value of $16.2 \mu\text{K}_{\text{CMB}}.\text{K}^{-1}.\text{km}^{-1}.\text{s}$ (cf. [Planck Collaboration XIII \(2014\)](#)). The effect of this average value is discussed and interpreted further in later subsection IV.3.5.

Additionally, we can see a great discrepancy between my average ^{12}CO and ^{13}CO RDC results and that from the ground-based computation for the 217GHz band:

$$\begin{aligned}\overline{\delta L^{12CO} B_{217}} &= 17.35 \pm 0.097 \mu\text{K}_{\text{CMB}}.\text{K}^{-1}.\text{km}^{-1}.\text{s}, \\ \overline{\delta L^{13CO} B_{217}} &= 15.4 \pm 0.57 \mu\text{K}_{\text{CMB}}.\text{K}^{-1}.\text{km}^{-1}.\text{s},\end{aligned}$$

against ground-based values of respectively $45.0 \mu\text{K}_{\text{CMB}}.\text{K}^{-1}.\text{km}^{-1}.\text{s}$ and $36.3 \mu\text{K}_{\text{CMB}}.\text{K}^{-1}.\text{km}^{-1}.\text{s}$ (cf. [Planck Collaboration XIII \(2014\)](#)). This is mainly due to the issue of the $J = 2 \rightarrow 1$ calibration. Indeed, as I have presented until now, the ^{12}CO and ^{13}CO foregrounds' templates are built from the survey of [Goldsmith et al. \(2008\)](#), re-

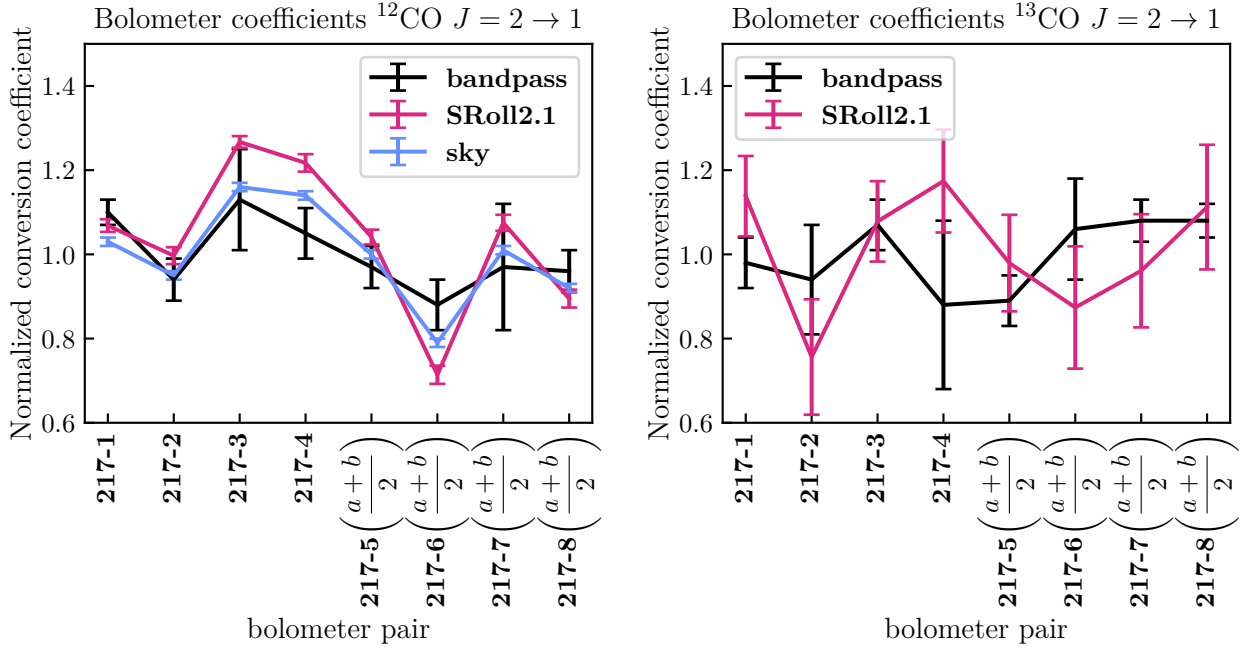


Fig. IV.17 CO RDCs for the 217GHz band, i.e. the $J = 2 \rightarrow 1$ CO line. Coefficients from PSBs are averaged over the pair, and all coefficients are normalized by the corresponding average $\overline{\delta L^{12/13CO} B_{CO}}$ over the full frequency band. Compared with the results from the Planck 2013 CO coefficients, cf. Fig.II.17. **Left:** ^{12}CO . **Right:** ^{13}CO .

gardless of the Sroll2.1 run computed. This means that in the case of the 217-143GHz Sroll2.1 run, the RDCs corresponding to the $J = 2 \rightarrow 1$ CO lines are **both determined and calibrated** on a $J = 1 \rightarrow 0$ survey. This calibration inadequacy impacts the average values of the ^{12}CO and ^{13}CO RDCs, which in turn impact the calibration of the later construction of the ^{12}CO and ^{13}CO maps. This issue of the $J = 2 \rightarrow 1$ lines calibration can equivalently be applied on the RDCs or on the maps, and I decided to calibrate the maps, which will be presented and carried out in subsection IV.2.2. However, I can use the calibration factor which I determine on the maps to be 1.881, and check the correction it implies on the RDC averages. If we assume a constant ratio $^{12}\text{CO}/^{13}\text{CO}$ for the 2 lines $J = 1 \rightarrow 0$ and $J = 2 \rightarrow 1$ we can then apply this calibration factor determined on the maps, to compare my ^{12}CO and ^{13}CO RDC averages with the ground-based values: $\overline{\delta L^{12CO} B_{217}} = 32.7 \pm 0.62 \mu\text{K}_{\text{CMB}}.\text{K}^{-1}.\text{km}^{-1}.\text{s}$ against a ground-based value of $45.0 \mu\text{K}_{\text{CMB}}.\text{K}^{-1}.\text{km}^{-1}.\text{s}$, and $\overline{\delta L^{13CO} B_{217}} = 28 \pm 3.7 \mu\text{K}_{\text{CMB}}.\text{K}^{-1}.\text{km}^{-1}.\text{s}$ against a ground-based value of $36.3 \mu\text{K}_{\text{CMB}}.\text{K}^{-1}.\text{km}^{-1}.\text{s}$. Even then, my average values are smaller, which may indicate the detected presence of molecular emission in the 143GHz band, as will be discussed in subsection IV.3.5.

d) The sky model consistency validation

Since I carried out 2 types of runs for Sroll2.1 with a frequency band in common, I can use the 2 separate results to check the consistency of my sky model. Indeed, if my sky model is representative enough of the data, and if my templates are accurate enough, the RDC

distributions along the 143GHz band of each foreground component should be the same for both the 100-143GHz and the 217-143GHz runs.

To this end, I plotted in Fig. IV.18 the RDC distributions along the 143GHz of the ^{12}CO , ^{13}CO , dust, DustVar, free-free, and synchrotron foreground components, for both the 100-143GHz and the 217-143GHz runs. In each case, the average over the distribution is subtracted to each RDC to ensure that all displayed distributions have a null average. With the exception of the DustVar RDC of bolometer 143-2b, all RDCs for all foregrounds are consistent between the 2 types of runs, confirming a satisfactory consistency of my sky model. Also, since the uncertainty on the DustVar RDC from the 217-143GHz run may be underestimated by a factor up to 2, as seen from Table. IV.2, even this single case of disagreement between the 2 run types may very well actually be a false negative.

Additionally, we can observe on the free-free RDC distribution a great difference of uncertainties bar sizes between the 100-143GHz run and the 217-143GHz one. This difference is consistent with the one observed from the difference of RDC uncertainty estimation methods displayed in Table. IV.2. However, the extremely close consistency in the RDC distributions between the 100-143GHz run and the 217-143GHz one would suggest that the uncertainty estimation from the simulation is *overestimated* for the free-free foreground emission. This is most probably due to the imperfect quality of the corresponding foreground emission models in the simulations, leading to an overestimation of the uncertainty on their associated RDCs.

e) The issue of the $J = 2 \rightarrow 1$ CO templates

As first discussed in subsection IV.1.4.c), the determination of the $J = 2 \rightarrow 1$ lines of ^{12}CO and ^{13}CO raises the issue of using a $J = 1 \rightarrow 0$ radio survey as the SRoll2.1 input ^{12}CO and ^{13}CO templates. Not only will the resulting maps have to be recalibrated, but it also requires checking that the spatial distribution of the $J = 1 \rightarrow 0$ lines is sufficiently representative of that of the $J = 2 \rightarrow 1$ to confidently trust the use of the survey from Goldsmith et al. (2008) as template.

While a high spatial correlation between the $J = 2 \rightarrow 1$ and $J = 1 \rightarrow 0$ lines is expected, for both the ^{12}CO and ^{13}CO , we need to verify that the spatial discrepancies do not introduce some *decorrelation* into the ^{12}CO and ^{13}CO RDCs determination. Thankfully, we expect that most of the cloud region can be considered at local thermodynamic equilibrium, with the exception of the outer regions of the cloud's envelope (Lequeux et al. (2002)). This means that, for most of the region covered by the templates, the lines ratio $\frac{J=2 \rightarrow 1}{J=1 \rightarrow 0}$ should only be a function of the temperature, and should then be mostly constant over the extension of the SRoll2.1 ^{12}CO and ^{13}CO templates.

In order to test the impact of the $J = 1 \rightarrow 0$ line as input template for the $J = 2 \rightarrow 1$ line, I computed 2 comparative runs:

1. A typical 217-143GHz SRoll2.1 data run,
2. A 217-143GHz SRoll2.1 data run in which the ^{12}CO foreground template was replaced by the same Taurus region (as was covered by Goldsmith et al. (2008)), but from the *Planck* Type 2 $J = 2 \rightarrow 1$ CO map (cf. Section II.4).

Both runs were computed using the parameter $\text{RSTEP} = 10$ to reduce the computational load. I built the new ^{12}CO template from the *Planck* Type 2 $J = 2 \rightarrow 1$ CO map by downscaling it to the template resolution of $N_{\text{side}} = 128$, and putting to zero all pixels outside of the region covered by the survey of Goldsmith et al. (2008). The resulting template is shown in Fig. IV.19 where it can be compared to the ^{12}CO template from Goldsmith et al.

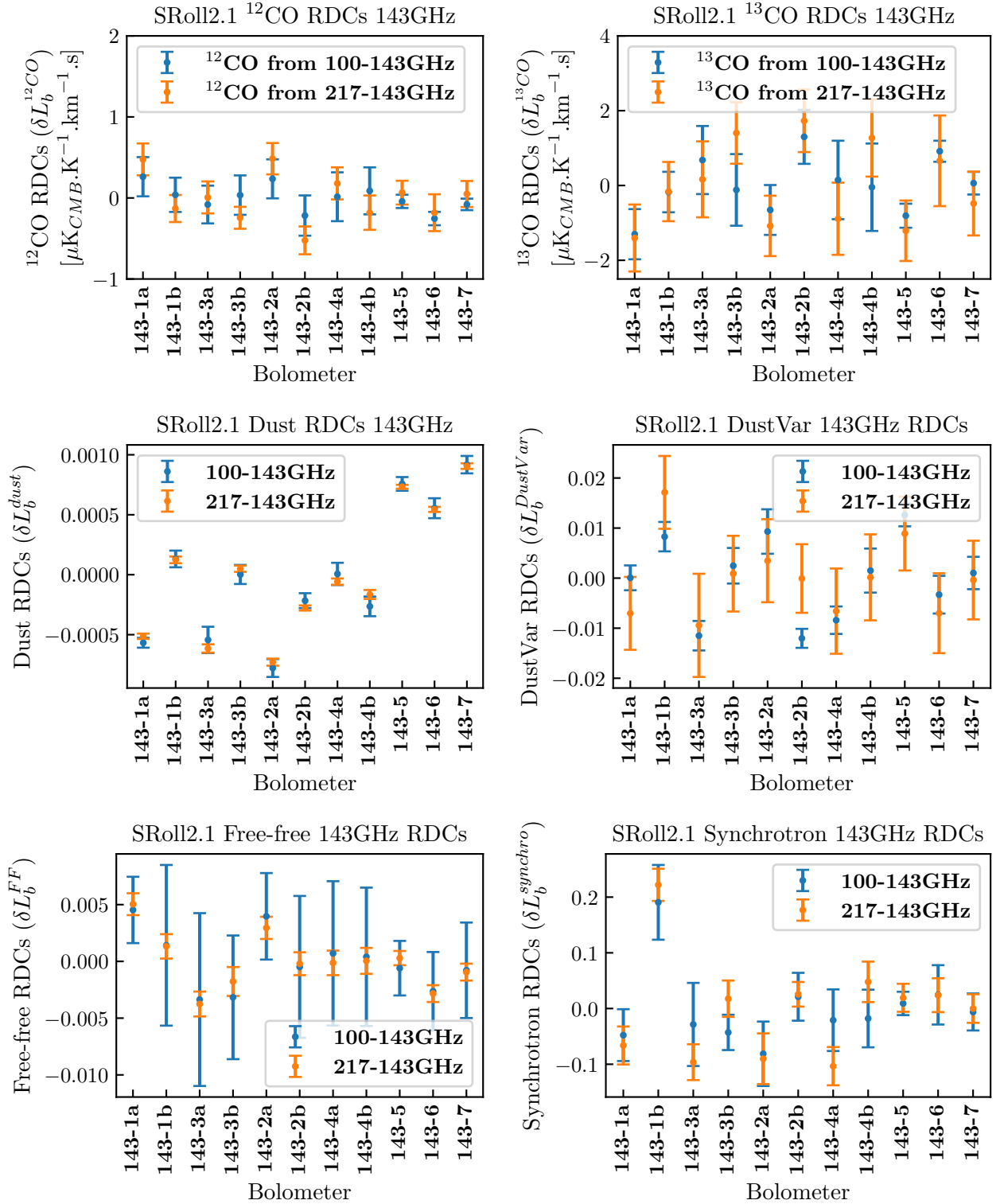


Fig. IV.18 RDC distributions along the 143GHz frequency band of the ^{12}CO (**Top left**), ^{13}CO (**Top right**), dust (**Middle left**), DustVar (**Middle right**), free-free (**Bottom left**), and synchrotron (**Bottom right**) foreground components, for both the 100-143GHz and the 217-143GHz SROLL2.1 runs. Averages were subtracted for all RDC distributions. We can see a good agreement between the 143GHz RDC determinations from the 2 SROLL2.1 run for all 4 foreground components, demonstrating good sky model consistency.

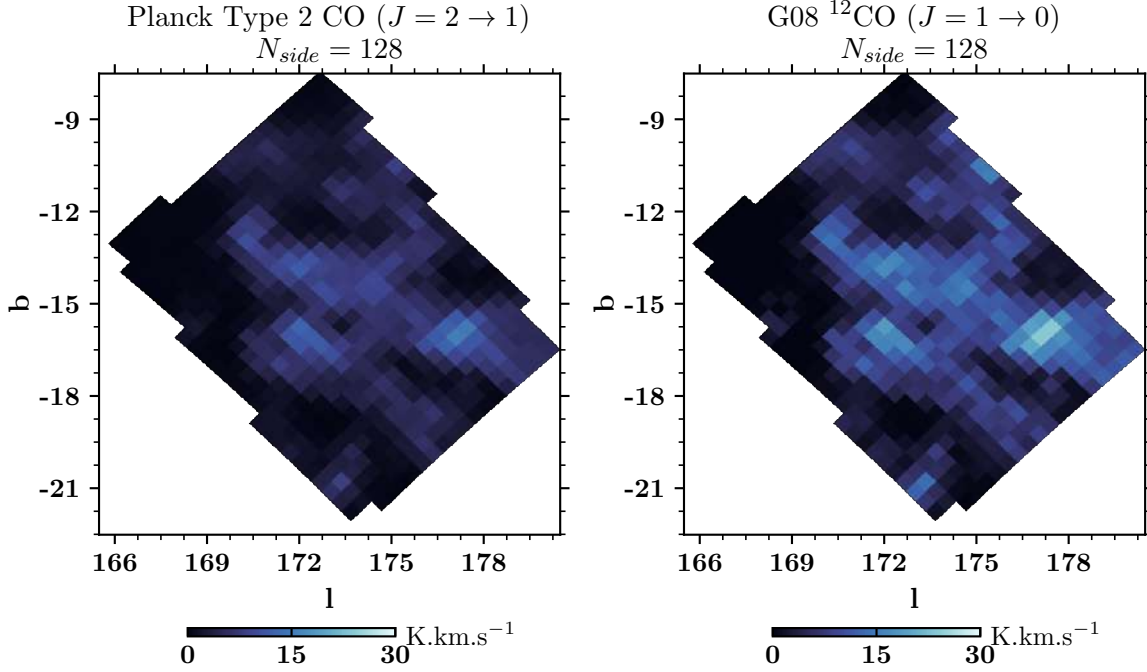


Fig. IV.19 **Left:** *Planck* Type 2 $J = 2 \rightarrow 1$ CO map of the Taurus region as surveyed by [Goldsmith et al. \(2008\)](#); **Right:** Integrated ^{12}CO map of the $J = 1 \rightarrow 0$ line emission from [Goldsmith et al. \(2008\)](#), projected in HEALPix. Both maps are at the SRoll2.1 template resolution of $N_{\text{side}} = 128$.

(2008). For this test, the ^{13}CO template remained unchanged as there is no map equivalent to the *Planck* Type 2 CO maps specifically mapping the ^{13}CO isotopologue.

From these 2 runs, the resulting ^{12}CO and ^{13}CO RDC distributions can be compared to ensure a good match. Since the later ^{12}CO and ^{13}CO maps are fully built from the corresponding RDCs, if the distributions from these 2 runs are compatible with one another, it means that the spatial distribution of the $J = 1 \rightarrow 0$ lines is *indeed* sufficiently representative of that of the $J = 2 \rightarrow 1$. The results are plotted in Fig. IV.20, where we can indeed observe a good consistency of the RDC distributions between the 2 runs, for both the ^{12}CO and the ^{13}CO , up to the level of uncertainty. The only exception is the ^{12}CO RDC of the 217-3 SWB, however I remind that my uncertainty estimation from the data can be underestimated, typically up to a factor ~ 1.5 for the ^{12}CO (cf. Table IV.2). Taking this factor of underestimation into account, these 2 values can actually coincide up to the uncertainty level.

To allow this comparison, the ^{12}CO RDC distribution from the SRoll2.1 run using the *Planck* Type 2 map as template was recalibrated to the level of the original ^{12}CO $J = 1 \rightarrow 0$ template, by multiplying them with a recalibration factor (determined to be 0.675, see below). This recalibration factor was computed from the affine fit of what I call a correlation plot, as defined below.

Correlation plot and affine fit A more quantitative evaluation of the maps and calibration consistency is computed by plotting a correlation graph of each corresponding map. The correlation plot is produced by binning the scatter plot of all the pixels from the map, each point's coordinates being defined as the value taken by the pixel in each considered map. An affine fit of the full (*non-binned*) cloud of points is then computed and

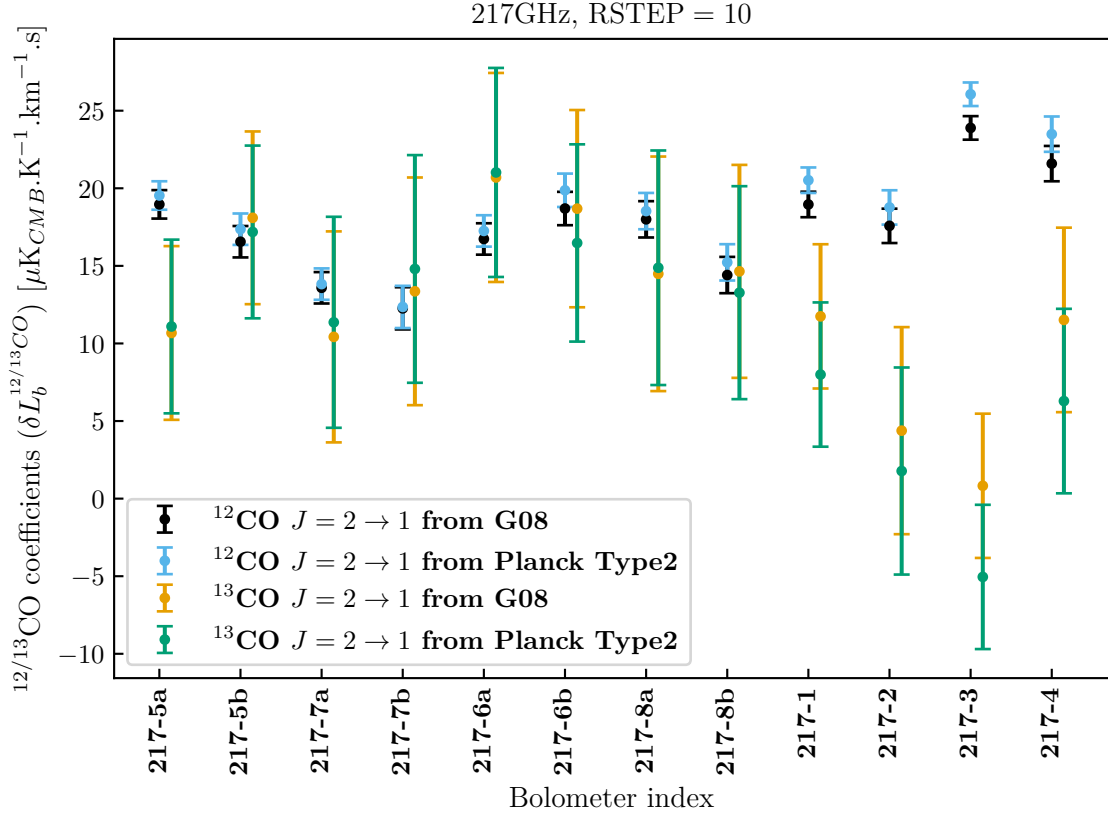


Fig. IV.20 Comparison of the SRoll2.1 ^{12}CO and ^{13}CO $J = 2 \rightarrow 1$ RDC distributions between a default run and one using the *Planck* Type 2 $J = 2 \rightarrow 1$ CO map as the ^{12}CO template. The distribution of the ^{12}CO RDCs from this second run was recalibrated to the calibration of the original ^{12}CO $J = 1 \rightarrow 0$ template from [Goldsmith et al. \(2008\)](#) using a recalibration factor, fitted on maps, of 0.675 (see Fig. IV.21).

plotted in red. The value Dist_{avg}^2 , given for each correlation plot, is the average squared-distance of each point to the fitted affine function. This gives an evaluation of the goodness of fit of the affine function, and will be used essentially to compare various correlation plots. As an example, if we consider the correlation plot between the *Planck* Type 2 $J = 2 \rightarrow 1$ and G08 ^{12}CO maps (seen in Fig. IV.21), respectively noted M^{Type2} and M^{G08} , and if we note F the corresponding affine fit of their correlation, then Dist_{avg}^2 is computed as:

$$\text{Dist}_{avg}^2 = \frac{1}{N_{pix}} \sum_{p=1}^{N_{pix}} (M_p^{\text{Type2}} - F(M_p^{\text{G08}}))^2.$$

Such correlation plots will be used later for the calibration of the Sroll2.1 $J = 2 \rightarrow 1$ ^{12}CO and ^{13}CO maps, as well as in all comparison of CO maps presented in Chapter V.

The aforementioned recalibration factor was fitted from the correlation plot between the *Planck* Type 2 $J = 2 \rightarrow 1$ CO map and the ^{12}CO $J = 1 \rightarrow 0$ map from [Goldsmith et al. \(2008\)](#), displayed in Fig. IV.21.

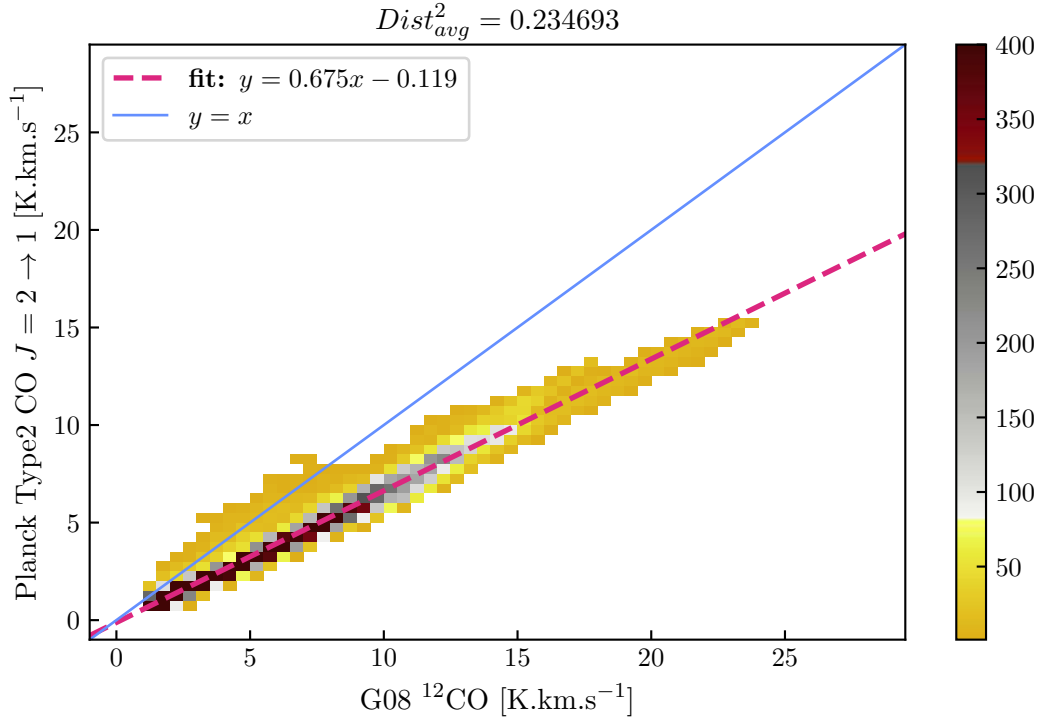


Fig. IV.21 Correlation plot between the Planck Type 2 $J = 2 \rightarrow 1$ CO map and the ^{12}CO $J = 1 \rightarrow 0$ map from Goldsmith *et al.* (2008). Both maps have been smoothed up to a Gaussian beam with FWHM = 30'. Plot computed using only pixels with an intensity superior to 1 K.km.s⁻¹ in the ^{12}CO G08 map. $Dist_{avg}^2$ corresponds to the average of squared distances of points with respect to the fit. **Red dashed line** is the best affine fit of the cloud of points; **Blue line** is the identity function.

IV.2 ^{12}CO and ^{13}CO maps computed from SRoll2.1 outputs

Now that all the relevant inner workings of SRoll2.1 as well as its outputs have been introduced, we are all set to build new full-sky ^{12}CO and ^{13}CO maps from it. I will first present the method I used to produce these new maps, separating them from the rest of the signal in the frequency maps output by SRoll2.1. After presenting the resulting maps, I will discuss the level of noise in the maps thus constructed, as well as how this noise is reduced by the use of the 143GHz frequency band.

IV.2.1 Method

I start by detailing the method I devised to build the ^{12}CO and ^{13}CO emission maps. First, I present the philosophy behind the method: what I exploit from the outputs of SRoll2.1 and the characteristics of the CO signal, what type and number of maps I build, and what are the limitations. I then detail the separation data model I use, and the full computation carried out to produce my final maps.

a) Separation philosophy

The ^{12}CO and ^{13}CO maps building procedure from SRoll2.1 is built upon the response *difference* between bolometers in the same frequency band to the ^{12}CO and ^{13}CO rotational

lines that are to be mapped. To this end I exploit a spectral property specific to the ^{12}CO and ^{13}CO emissions in the *Planck* data: the ^{12}CO and ^{13}CO signals being emitted as a rotational line are thereby strongly localised in frequency. Indeed, as can be seen in Figures II.3 and II.5, the spectral response of each bolometer along the same frequency band is fairly similar on average. Therefore, for broadband signals, i.e. all *non-CO* foreground emissions, the response difference between bolometers to a specific foreground corresponds to the difference integrated along the bandwidth as defined by Eq.III.5, thereby leading to similar bolometer responses. Indeed, once colour-corrected, the response difference between bolometers to a specific foreground is determined by the difference between bolometers of the spectral transmission averaged over their full bandwidth. In the case of the 100GHz and 217GHz bands, respectively averaging the spectral transmissions over the frequency ranges $[82\text{GHz}, 125\text{GHz}]$ and $[180\text{GHz}, 270\text{GHz}]$, it is useful to characterize the bolometers response differences along each band with 2 quantities: the standard deviation of the distribution of averaged spectral transmissions (i.e. the standard deviation σ_H of the distribution of $\overline{H}^{b\nu_{band}}$), and the maximal difference in the distribution, i.e. the difference between the maximum and the minimum of the distribution. These quantities are presented in Table.IV.3 in the "Avg" columns. Such an averaged spectral transmission can then be written, for a bolometer b from the 100GHz frequency band, as:

$$\overline{H}^{b100} = \frac{1}{125 - 82} \int_{82\text{GHz}}^{125\text{GHz}} H_{\nu}^b d\nu,$$

and for a bolometer b from the 217GHz frequency band, it can be written:

$$\overline{H}^{b217} = \frac{1}{270 - 180} \int_{180\text{GHz}}^{270\text{GHz}} H_{\nu}^b d\nu,$$

where H_{ν}^b is the spectral transmission of bolometer b .

However, at smaller frequency scale, each bolometer shows distinct transmission variations (cf. Figs.II.3 and II.5). Also, the ^{12}CO and ^{13}CO rotational lines are narrower than the spectral resolution of the detector's transmission measurement seen in Figs.II.3 and II.5: indeed, as introduced in I.4, the order of magnitude of the CO lines width is within $\sim 10^{6-7}\text{Hz}$, while the spectral resolution of the HFI bolometers transmission measurement, carried out on ground, is of the order of $\sim 10^8\text{Hz}$ (cf. [Planck Collaboration IX \(2014\)](#), [Planck Collaboration XIII \(2014\)](#)). Consequently, the ^{12}CO and ^{13}CO specific bolometer responses are extremely sensitive to the detector's spectral transmission's variations, leading to greater bolometers response difference for the CO emissions compared to the other foregrounds. In Table.IV.3, I displayed the standard deviation and maximal difference of the distributions of spectral transmissions at the ^{12}CO and ^{13}CO lines' frequencies, for both the 100GHz and 217GHz frequency bands.

Since I built my maps computing algorithm on exploiting the bolometers response *differences*, the most important characteristic to look for in the RDCs distributions is a good distinction between each RDC in the distribution, i.e. a high standard deviation of the RDC distribution, and a standard deviation greater than the average uncertainty along the considered RDC distribution. As previously introduced in subsection IV.1.4.b), the good estimators for this distinctive distribution spread are the standard deviation of the RDC distribution $\sigma_{\delta Lf}$, and the ratio $\sigma_{\delta Lf}/\overline{\Delta_{\delta Lf}}$, which is significantly greater than 1 for the ^{12}CO at both $J = 1 \rightarrow 0$ and $J = 2 \rightarrow 1$ lines (~ 10 , cf. Appendix.D), and the ^{13}CO for the $J = 1 \rightarrow 0$ line (> 3 , cf. Appendix.D). This ratio is low for the $J = 2 \rightarrow 1$ line of the ^{13}CO , as well as the corresponding $\sigma_{\delta L^{13\text{CO}}}$ leading to a more noisy map. This greater difference in our detectors' response, greater than the uncertainty on the said differences

Table IV.3: Characterisation of the spectral transmission differences between bolometers along the 100GHz and 217GHz bands. The *Avg* columns correspond to the distribution of the spectral transmission (ground-based measurement) averaged over the considered frequency band: $[82GHz, 125GHz]$ for the 100GHz band, and $[180GHz, 270GHz]$ for the 270GHz band. The ^{12}CO and ^{13}CO columns correspond respectively to the distribution of the spectral transmission at the ^{12}CO and ^{13}CO line frequencies present in the considered frequency band. The σ_H row gives the standard deviation of the considered distribution. The *max - min* row corresponds to the difference between the maximum and the minimum from the considered distribution.

	100GHz			217GHz		
	Avg ($\overline{H^b}^{100}$)	^{12}CO ($H_{115.271}^b$)	^{13}CO ($H_{110.201}^b$)	Avg ($\overline{H^b}^{217}$)	^{12}CO ($H_{230.542}^b$)	^{13}CO ($H_{220.402}^b$)
σ_H	0.06	0.10	0.07	0.06	0.10	0.10
max - min	0.18	0.26	0.21	0.21	0.32	0.36

determination, by at least an order of magnitude for the ^{12}CO as shown in the results presented in subsection IV.1.4.b) above and in Appendix.D, is precisely what allows us to perform a separation of the ^{12}CO and ^{13}CO emissions from only the output parameters of SRoll2.1. Since the maps are built by exploiting the response *difference* between bolometers, the bigger the spread of the response *difference* distribution, and especially the bigger the difference between extrema, the better the signal separation, and the less noisy the resulting maps.

In the present case, not only are the ^{12}CO and ^{13}CO response differences *intrinsically* greater than those of the *non-CO* foreground emissions, but a crucial procedure was also performed to ensure this hypothesis: the **realignment**. The cleaning step of SRoll2.1 (cf. Eq.IV.1) realigned all *non-CO* foreground emissions to their average response along the CO containing frequency band. Thereby, any residual response difference to the *non-CO* foreground emissions are only caused by errors on the difference response coefficients fitted by SRoll2.1 to compute the realignment (cf. subsection IV.1.4), and by imperfections in the foreground template maps. Therefore, up to this error level, it can be considered that the maps per bolometer output by SRoll2.1 have the same sky signal, and only differ from one another by their specific responses to the ^{12}CO and ^{13}CO lines. This means that a simple difference of 2 of said bolometer maps could yield a CO map, which should then be divided by the CO response difference between the 2 maps. In order to fully exploit all pairwise detector differences, as well as to allow the separation of the ^{12}CO and ^{13}CO isotopologues, a full χ^2 minimization is nonetheless computed in each pixel.

This method of separation significantly differs from most of those previously used in the *Planck* legacy component separation algorithms, which simultaneously used information at many frequency bands, and more notably used models of the foreground emission SEDs, to build simultaneously a map of each component present in the signal. My method only separates the ^{12}CO and ^{13}CO emissions from the rest of the signal, and to this end I only use the detectors' response differences fitted by SRoll2.1 with no additional *a priori* modelisation of the SED of the components comprising the full *Planck* data signal. This method resembles most the one described and used in the work of [Planck Collaboration XIII \(2014\)](#), where detector response coefficients are determined for the main components of the signal to then separate them, comprising that of the CO. However, in said work, the

determination of the coefficients is computed from the already projected frequency maps containing all the residues from the data analysis pipeline. Additionally, their frequency maps were produced *before* the development of the SRoll mapmaking algorithm and all the correction improvements it provides. Especially, the frequency maps from which the CO was computed did not undergo a realignment as done in SRoll2.1, and are subject to the bandpass mismatches as they were before their correction performed in the later *Planck* releases relying on the SRoll mapmaking algorithm. Additionally, the ^{12}CO and ^{13}CO lines were never separated in the *Planck* data until the recent work of Hurier (2019), which was computed on the CO maps resulting from the aforementioned work of Planck Collaboration XIII (2014), therefore suffering from the same limitations. I sadly could not access the data from the ^{12}CO and ^{13}CO separation performed in the work of Hurier (2019) and therefore could not compare it to my own results. Although, in my case, my coefficients determination is performed *at the same time* as that of all the other parameters aimed at cleaning the remaining instrumental systematic effects, resulting in an expected greater quality of the response coefficients' determination, and all the more since this determination is performed with all the additional signal *cleaning* brought by the SRoll2.1 solution, compared to both the *Planck* legacy CO maps and the work of Hurier (2019) based itself on said *Planck* legacy CO maps.

Why not more maps ? I previously mentioned, and will detail in the next subsection, that my map separation consists in extraction the ^{12}CO and ^{13}CO emissions from the rest of the sky signal, leading me to build 5 maps, 2 for the ^{12}CO and ^{13}CO emissions, and 3 for the I, Q, and U Stokes parameters of the remaining sky signal. I could try in my separation to additionally build maps of more components from the signal: more specifically maps of the components with response differences sufficiently *distinct* from one another, i.e. foreground components for which the bolometer responses are separated by substantially more than the uncertainty on their determination. The foreground components which may correspond to this description are the thermal dust and the added component DustVar, as can be seen from the results on the RDCs listed in Appendix.D. However, I chose to not build these maps for 2 main reasons:

1. Their response difference distribution is more noisy than that of the ^{12}CO and ^{13}CO emissions (cf. figures from subsection IV.1.4.b), and ratio values $\sigma_{\delta Lf}/\overline{\Delta_{\delta Lf}}$ from the tables of Appendix.D), which would result in noisy maps,
2. It would add 4 new maps to the separation: the 3 $[I^{dust}, Q^{dust}, U^{dust}]$ dust emission components, plus the intensity map of the DustVar component. Since, as will be detailed below, this map computation is performed in each pixel from the bolometers data, it would mean that in each pixel, we have to determine 9 unknown values from 19 (or 23 depending on the SRoll2.1 run) observations, which is statistically hazardous.

Though the first reason is not really an issue for all foregrounds, e.g. the dust displays a high value for the ratio $\sigma_{\delta Lf}/\overline{\Delta_{\delta Lf}}$ at all frequencies (cf. Appendix.D), the second reason is always a heavy limitation. For these 2 reasons, I chose, in the scope of my PhD thesis, to limit my construction to only the separation of the ^{12}CO and ^{13}CO components from the rest of the signal. This is also why all the **non-CO** foreground components were realigned in the SRoll2.1 version, but **not** the ^{12}CO and ^{13}CO emissions.

To compute the maps of other foregrounds using the same procedure, I would need to remove their *realignment* from the cleaning step of SRoll2.1 defined in Eq.IV.1. By choosing to realign the ^{12}CO and ^{13}CO foregrounds but **not** other foregrounds, for example

the dust and/or DustVar foregrounds, one could use the procedure described here to build the corresponding maps, provided that the associated $\sigma_{\delta L_f}/\overline{\Delta_{\delta L_f}}$ ratio is sufficiently high, and provided that the total number of maps to be determined remains as small as possible.

b) Separation model

The algorithm computing the ^{12}CO and ^{13}CO maps from the SRoll2.1 outputs produces 5 different output maps:

- 1 map $M^{12\text{CO}}$ of the ^{12}CO emission line intensity present in the SRoll2.1 run used ($J = 1 \rightarrow 0$ from a run on 100-143GHz, $J = 2 \rightarrow 1$ from a run on 217-143GHz),
- 1 map $M^{13\text{CO}}$ of the ^{13}CO emission line intensity present in the SRoll2.1 run used ($J = 1 \rightarrow 0$ from a run on 100-143GHz, $J = 2 \rightarrow 1$ from a run on 217-143GHz),
- 3 component maps $[I^{\text{other}}, Q^{\text{other}}, U^{\text{other}}]$ of the remaining signal after extraction of the ^{12}CO and ^{13}CO emissions, and averaged over all used bolometers.

The data model used to build the χ^2 minimization computing these 5 maps is defined as follows:

$$\begin{aligned} \tilde{S}_{b,p}^{\text{out}} = & I_p^{\text{other}} + \rho_b \cos(2\phi_{b,p}) Q_p^{\text{other}} + \rho_b \sin(2\phi_{b,p}) U_p^{\text{other}} \\ & + L_b^{12\text{CO}} M_p^{12\text{CO}} + L_b^{13\text{CO}} M_p^{13\text{CO}} + n_{b,p}, \end{aligned} \quad (\text{IV.8})$$

where we have:

- The cleaned maps *per bolometer* $\tilde{S}_{b,p}^{\text{out}}$ outputted by SRoll2.1,
- The corrected bolometer polarization efficiency ρ_b ,
- The detector polarization angle $\phi_{b,p}$ with respect to the north-south axis in the Galactic coordinate system, averaged in the pixel p , i.e. averaged over all rings r ,
- The ^{12}CO and ^{13}CO bolometer response coefficients $L_b^{12\text{CO}}$ and $L_b^{13\text{CO}}$, as fitted by SRoll2.1 (I use the $\delta L_b^{12\text{CO}}$ and $\delta L_b^{13\text{CO}}$ output by SRoll2.1, which correspond to the values of $L_b^{12\text{CO}}$ and $L_b^{13\text{CO}}$ as demonstrated by Eq.IV.5),
- The white noise $n_{b,p}$ in the pixel p of the bolometer b , of variance $1/w_{b,p}$ defined by $w_{b,p} = \frac{\mathfrak{h}\mathfrak{c}_{b,p}}{\sigma_{b,p}^2}$, noting as $\mathfrak{h}\mathfrak{c}_{b,p}$ the hitcount in pixel p of bolometer b summed over the rings falling in the said pixel p .

In the data model defined by Eq.IV.8, we use once again the hypothesis of the absence of CO line in the 143GHz band. In SRoll2.1, we used this hypothesis to define the *less strict* condition defined by Eq.IV.4, i.e. the closure condition of the ^{12}CO and ^{13}CO RDCs. It was safer numerically to use this hypothesis only to define the closure condition on the bolometer RDCs rather than strictly constraining all individual $\delta L_b^{12\text{CO}}$ and $\delta L_b^{13\text{CO}}$ along the 143GHz band. It ensured a safer convergence of the SRoll2.1 algorithm by conserving some leeway to account for the potential presence of significant non-CO molecular emissions in the 143GHz band, which would be imprinted in both a non-zero value of the averages $\overline{L^{12\text{CO}}\mathcal{B}^{143}}$ and $\overline{L^{13\text{CO}}\mathcal{B}^{143}}$, but also in the distribution of the coefficients $L_b^{12\text{CO}}$ and $L_b^{13\text{CO}}$ for $b \in \mathcal{B}_{143}$.

However, in the building computation of the CO maps, we use the full extent of the aforementioned physical hypothesis, which can be expressed as follows:

$$\forall b \in \mathcal{B}_{143}, L_b^{12\text{CO}} = 0 \text{ and } L_b^{13\text{CO}} = 0.$$

This interpretation is confirmed by the results of Figs. IV.12 and IV.13 where we can see that the distributions of the $\delta L_b^{12\text{CO}}$ and $\delta L_b^{13\text{CO}}$ along the 143GHz band spread mainly within the range of the uncertainty on their determination. Therefore, I used the signal from all bolometers to determine the remainder maps I^{other} , Q^{other} , and U^{other} , but only the signal from the bolometers in the set $\mathcal{B}_{\text{CO}}$ to determine the maps $M^{12\text{CO}}$ and $M^{13\text{CO}}$.

This method implies again the use of the bolometers from the 143GHz band as an *absolute* calibrator of the ^{12}CO and ^{13}CO maps. We will see in Section V.1, that this hypothesis is sufficiently correct since the calibration of my ^{12}CO and ^{13}CO maps is consistent with that of external CO observation surveys.

χ^2 minimization Using the model of Eq. IV.8 as well as the hypothesis discussed in the paragraph above, I can define the following χ^2 to be minimised, fitting the maps in each pixel:

$$\begin{aligned} \chi_p^2 &= \sum_{b \in \mathcal{B}} w_{b,p} \left(I_p^{\text{other}} + \rho_b \cos(2\phi_{b,p}) Q_p^{\text{other}} + \rho_b \sin(2\phi_{b,p}) U_p^{\text{other}} \right. \\ &\quad \left. + L_b^{12\text{CO}} M_p^{12\text{CO}} + L_b^{13\text{CO}} M_p^{13\text{CO}} - \tilde{S}_{b,p}^{\text{out}} \right)^2 \\ &= \sum_{b \in \mathcal{B}_{\text{CO}}} w_{b,p} \left(I_p^{\text{other}} + \rho_b \cos(2\phi_{b,p}) Q_p^{\text{other}} + \rho_b \sin(2\phi_{b,p}) U_p^{\text{other}} \right. \\ &\quad \left. + L_b^{12\text{CO}} M_p^{12\text{CO}} + L_b^{13\text{CO}} M_p^{13\text{CO}} - \tilde{S}_{b,p}^{\text{out}} \right)^2 \\ &\quad + \sum_{b \in \mathcal{B}_{143}} w_{b,p} \left(I_p^{\text{other}} + \rho_b \cos(2\phi_{b,p}) Q_p^{\text{other}} + \rho_b \sin(2\phi_{b,p}) U_p^{\text{other}} - \tilde{S}_{b,p}^{\text{out}} \right)^2. \end{aligned} \quad (\text{IV.9})$$

It is to note that the SNR differs greatly between the 5 maps I produce through this computation, with $\text{SNR}_I \sim \text{SNR}_{Q,U} \times 100$ and $\text{SNR}_I \sim \text{SNR}_{^{12}\text{CO}, ^{13}\text{CO}} \times 15$. With that in mind, I decided to perform the minimisation with each maps built at its own resolution, in order to maximise the resolution of the produced maps, while maintaining a reasonable level of noise in each one of them. These values were determined by keeping the highest HEALPix N_{side} resolution (cf. Gorski et al. (2005)) while ensuring the invertibility of the χ^2 matrix in all pixels. This leads to use the following resolutions :

- I^{other} is computed with $N_{\text{side}} = 2048$,
- Q^{other} and U^{other} are computed with $N_{\text{side}} = 256$,
- $M^{12\text{CO}}$ and $M^{13\text{CO}}$ are computed with $N_{\text{side}} = 1024$.

The computation of the χ^2 minimization defined by Eq. IV.9 leads to the following system of equations:

$$\begin{cases} \frac{\partial \chi^2}{\partial I^{\text{other}}} = 0 \\ \frac{\partial \chi^2}{\partial Q^{\text{other}}} = 0 \\ \frac{\partial \chi^2}{\partial U^{\text{other}}} = 0 \\ \frac{\partial \chi^2}{\partial M^{12\text{CO}}} = 0 \\ \frac{\partial \chi^2}{\partial M^{13\text{CO}}} = 0 \end{cases}$$

which can then be expressed in matrix form as $A \cdot X = V$, where X is the vector of values to be determined (i.e. the pixels of the 5 computed maps), and where the content of the matrix A and the vector V can be built from the output of the selected SRoll2.1 run. The full extent of the χ^2 computation, expressed in matrix form and in the general case, is detailed in Appendix.B.

However, since my algorithm builds maps at different resolutions, we actually need to compute a χ^2 for each pixel *at the maps lowest resolution*, i.e. at each pixel of the Q^{other} and U^{other} maps of resolution $N_{side} = 256$. Consequently, each χ^2 computes 1 pixel in the Q^{other} and U^{other} maps, 16 pixels in the M^{12CO} and M^{13CO} maps, and 64 pixels in the I^{other} map. The way in which this specific method of simultaneous computation at multiple resolutions impacts the matrix A and the vectors X and V is shown in Appendix.B.

It should be noted that the way the computation detailed in Appendix.B is carried out supposed that the detector polarization angle $\phi_{b,p}$ is stable inside the whole pixel p at the lowest resolution of $N_{side} = 256$. It is sufficiently correct to compute the sought maps, as can be seen from the resulting maps quality presented in Section V.1, but could be improved by better taking into account this variation: we could compute the average detector angle for the pixel p before any computation as well as perform some smoothing of the cross-terms to avoid them being impacted by the noise of the lowest SNR, i.e. the Q^{other} and U^{other} components in our case. But this would require some long engineering as well as a significant modification of the algorithm, leading to decide to put on hold this potential improvement of the method.

Finally, all that is left is to compute in each pixel of smallest resolution the corresponding inverse of the matrix A , and we can compute our maps with the expression: $X = A^{-1} \cdot V$. This matrix inverse A^{-1} is only defined if the available statistical information provided for the χ^2 is sufficient, and if not, the computation may result in some pixels' value being *undefined* in the resulting maps. Thankfully, and with the help of the addition of the bolometers from the 143GHz band, all pixels in my resulting maps are defined when using the aforementioned maps resolutions.

No polarization, yet As you may have noticed, from the presentation of my ^{12}CO and ^{13}CO signal separation method presented so far, and given the data model defined in Eq.IV.8, the ^{12}CO and ^{13}CO maps I built in the scope of this work are only the unpolarized intensity component of the respective lines emission signal. This is motivated by the low degree of polarization both expected and so far observed of the CO lines emission, as was discussed in subsection I.4.2, as well as for a reason invoked in the previous subsection: statistical limitations. Indeed, if the produced ^{12}CO and ^{13}CO maps were both polarized, it would mean that, in each pixel, the χ^2 minimization would have to fit 9 values from 19 measurements (or 23 depending on the line to be mapped), which would prove statistically hazardous. However, a computation that could be attempted, is the computation of a polarized CO map, where we no longer try to separate the ^{12}CO and ^{13}CO lines. This would only spawn a net addition of 1 map (adding the Q and U maps of CO, but removing the ^{13}CO map) to be determined compared to what is done in the present work. I have yet to attempt such a computation, but it could be a way to attempt a first full-sky determination of the CO emission polarization.

IV.2.2 Resulting maps

I gathered here the maps resulting from the computing method described in the subsection IV.2.1 above.

The ^{12}CO and ^{13}CO fullsky maps of the $J = 1 \rightarrow 0$ line, built from the 100-143GHz SRoll2.1 run, are displayed in Fig. IV.22. The associated I^{other} , Q^{other} , and U^{other} maps of the remaining signal from said 100-143GHz SRoll2.1 run are displayed in Fig. IV.23. The corresponding statistical error maps are presented in the following subsection IV.2.3. These 5 resulting maps are characterized by an angular resolution of $9'.65$, corresponding to the average FWHM of the bolometers' beam along the 100GHz frequency band (cf. subsection II.1.2.b)).

The ^{12}CO and ^{13}CO fullsky maps of the $J = 2 \rightarrow 1$ line, built from the 217-143GHz SRoll2.1 run, are displayed in Fig. IV.25. The associated I^{other} , Q^{other} , and U^{other} maps of the remaining signal from said 217-143GHz SRoll2.1 run are displayed in Fig. IV.26. The corresponding statistical error maps are presented in the following subsection IV.2.3. These 5 resulting maps are characterized by an angular resolution of $7'.25$, corresponding to the average FWHM of the bolometers' beam along the 143GHz frequency band (cf. subsection II.1.2.b)).

We can note that the polarization maps Q^{other} and U^{other} display a very small signal, especially in the 100-143GHz case (with signal ~ 30 times smaller than the intensity map I^{other}).

These maps will then be compared to external observations to qualify their quality and consistency with the current state of the art, as detailed in Section V.1.

The need for a $J = 2 \rightarrow 1$ recalibration As previously introduced in subsection IV.1.4.c), the $J = 2 \rightarrow 1$ ^{12}CO and ^{13}CO maps need to be *recalibrated*, since they were calibrated on the SRoll2.1 ^{12}CO and ^{13}CO input templates, i.e. the $J = 1 \rightarrow 0$ CO survey from Goldsmith et al. (2008).

The most straightforward way to compute such a *recalibration* is to base this new calibration on a separate survey of the $J = 2 \rightarrow 1$ CO line, with sufficient sky coverage, and to determine a correction factor for my maps' calibration from the affine fit of a correlation plot between my CO maps and said separate survey map. Such a high sky-coverage CO survey of CO $J = 2 \rightarrow 1$ emission is available in the form of the *Planck* legacy CO maps (as presented in Section II.4). A *completely* separate survey would have been more satisfactory, but I lacked the availability to such a CO survey, meeting all the requirements listed above. I therefore settled on using the Type 1 *Planck* legacy $J = 2 \rightarrow 1$ CO map, and not the Type 2, for 2 main reasons:

- ➔ I observe later, in Section V.1, that the calibration of the Type 1 $J = 1 \rightarrow 0$ map is always more consistent with outer CO radio-surveys than the Type 2 map, therefore I expect the same type of consistency for the $J = 2 \rightarrow 1$ line. This hypothesis is also supported by the results from Planck Collaboration XIII (2014), where the Type 1 map was proven consistent with external data, and the Type 2 of higher intensity compared to the Type 1 for both $J = 1 \rightarrow 0$ and $J = 2 \rightarrow 1$ lines,
- ➔ The Type 1 $J = 2 \rightarrow 1$ CO map has a very low ^{13}CO contamination, far lower than the Type 2 map ($X_{J=2 \rightarrow 1}^{\text{Type1}} = 0.01$ against $X_{J=2 \rightarrow 1}^{\text{Type2}} = 0.82$, see subsection II.4.3).

The correlation plot between the adequate combination of my SRoll2.1 $J = 2 \rightarrow 1$ ^{12}CO and ^{13}CO maps, i.e. $^{12}\text{CO} + 0.01 \times ^{13}\text{CO}$, and the Type 1 *Planck* legacy $J = 2 \rightarrow 1$ CO map is displayed in Fig. IV.24. From this plot, I can then use the fitted affine factor 1.881 to recalibrate my SRoll2.1 $J = 2 \rightarrow 1$ ^{12}CO map, and assuming an identical ratio $\frac{^{13}\text{CO}}{^{12}\text{CO}}$ for both lines $J = 1 \rightarrow 0$ and $J = 2 \rightarrow 1$, I use the same recalibration factor for the $J = 2 \rightarrow 1$ ^{13}CO map.

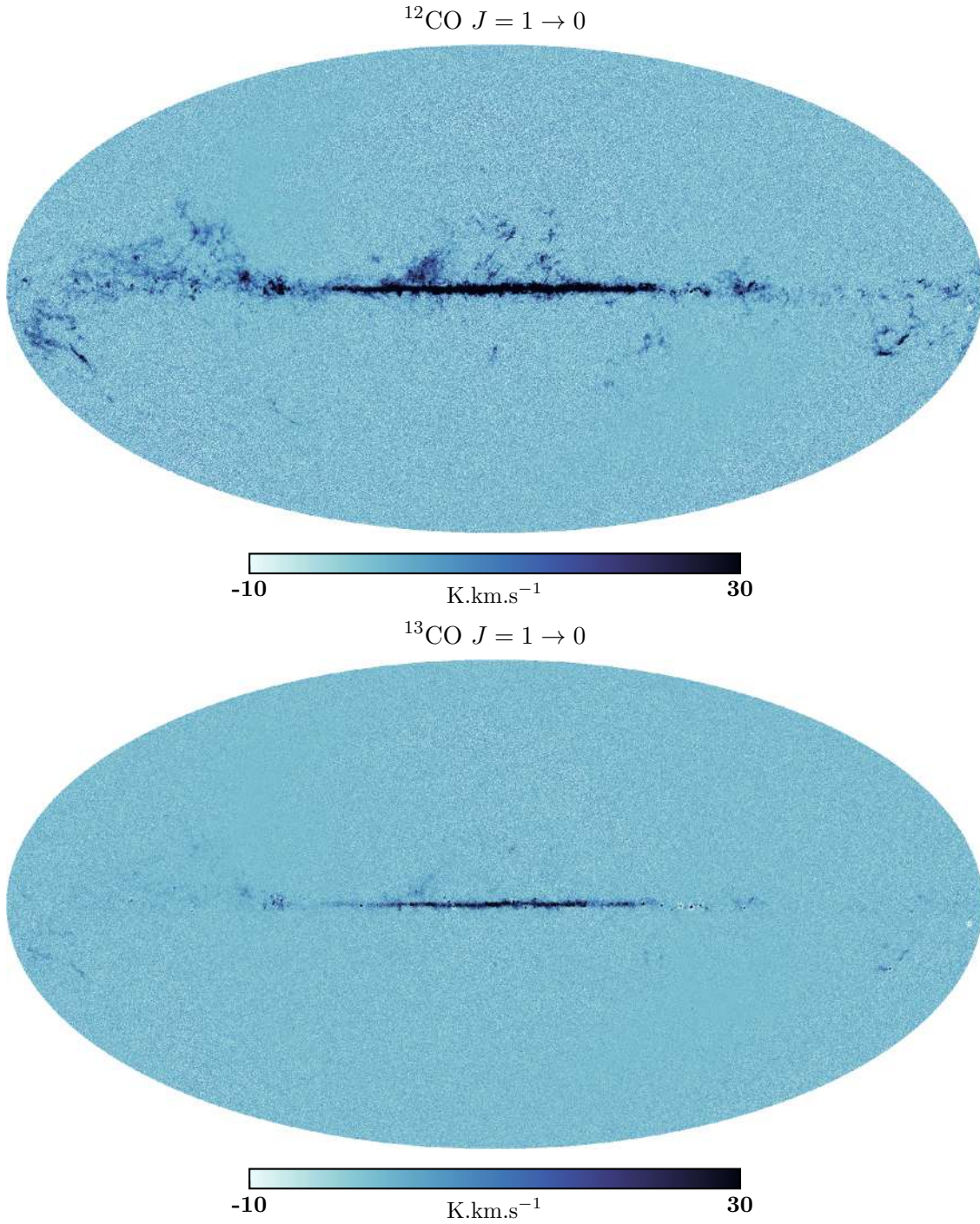


Fig. IV.22 Fullsky CO maps of the $J = 1 \rightarrow 0$ line, computed as described in subsection IV.2.1, at a resolution of $N_{side} = 1024$. **Top:** ^{12}CO , **Bottom:** ^{13}CO .

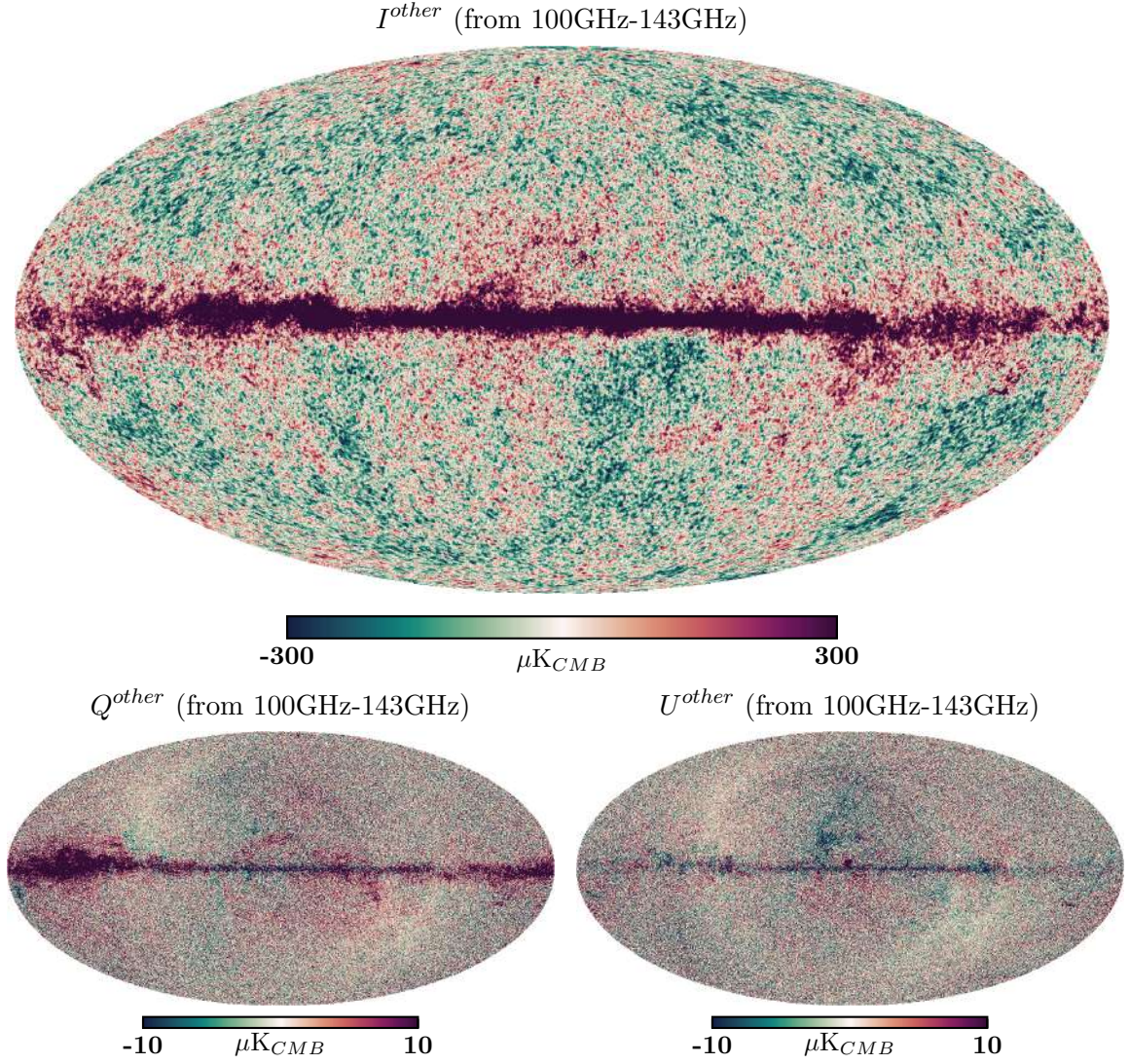


Fig. IV.23 Fullsky I^{other} , Q^{other} , and U^{other} maps resulting from the separation in the case of the $J = 1 \rightarrow 0$ CO line, i.e. computed from the output of the 100-143GHz *SROLL2.1* run. They are composed of all that is not CO. **Top:** I^{other} , **Bottom left:** Q^{other} , **Bottom right:** U^{other} .

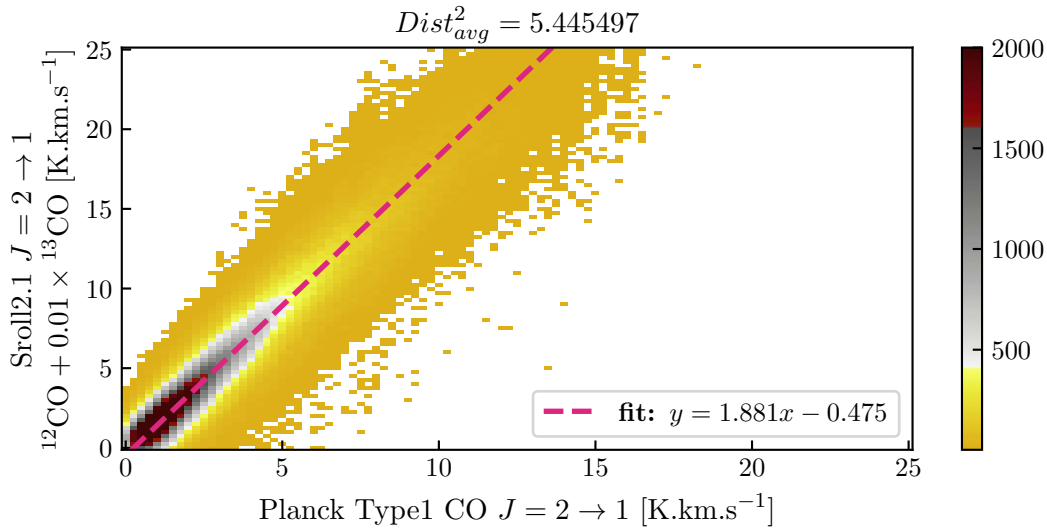


Fig. IV.24 Correlation plot between the combination of SRoll2.1 $J = 2 \rightarrow 1$ maps $^{12}CO + 0.01 \times ^{13}CO$ and Planck Type 1 $J = 2 \rightarrow 1$ CO map. Both maps have been set to a Gaussian beam with FWHM = $30'$. Plot computed using only pixels with an intensity superior to 0.5 K.km.s $^{-1}$ in the unsmoothed Type 1 Planck map. $Dist_{avg}^2$ corresponds to the average of squared distances of points with respect to the fit. **Red dashed line** is the best affine fit of the cloud of points.

Another recalibration method, based on physical and chemical arguments, could be to use a model of radiative transfer such as the RADEX code (van der Tak et al. (2007)), to estimate the line ratios $\frac{(J=2 \rightarrow 1)_{^{12}CO}}{(J=1 \rightarrow 0)_{^{12}CO}}$ and $\frac{(J=2 \rightarrow 1)_{^{13}CO}}{(J=1 \rightarrow 0)_{^{13}CO}}$. These ratios could be estimated using the RADEX code on parameters values characteristic of the Taurus region, and then used to recalibrate my SRoll2.1 $J = 2 \rightarrow 1$ ^{12}CO and ^{13}CO maps. I have yet to attempt this method, but believe it would be interesting to compare its results with the one from the recalibration on the Planck Type 1 map, since the underlying argument on which the 2 methods are based are radically different.

The power spectra of the ^{12}CO and ^{13}CO maps from Figs. IV.22 and IV.25, computed using a mask of sky fraction $f_{sky} = 90\%$ hiding the galactic disk, and log-smoothed with a multipole fraction of $\ell_{frac} = 15$ (cf. Appendix.C for spectra computations and masks definition), are plotted in Fig. IV.27. For comparison, in each case, I also plot the power spectrum of the difference map HM1-HM2 from the corresponding map built respectively from the subsurvey HM1 and HM2 (see Section II.3 for the subsurveys definition). The random noise in each map built from an HM1 subsurvey is of same level **but independent** from the one in map built from the HM2 subsurvey since they are built from the same quantity of data (half of the Planck mission) but mutually exclusive parts of the data. Therefore, the power spectrum of the difference map HM1-HM2 offers a power spectrum of a map of noise realisation for each corresponding map. This power spectra is divided by 4 to bring it back to the noise level of the original maps (see subsection IV.2.3 for more details). The HM1-HM2 difference maps for the $J = 1 \rightarrow 0$ and $J = 2 \rightarrow 1$ lines are respectively plotted in Figs. IV.28 and IV.29 of the next subsection. The last type of spectrum plotted in Fig. IV.27 is the cross-spectrum between the maps built from the subsurveys HM1*HM2. Since the random component of noise in each subsurvey map is independent, this cross-spectrum allows to search for the ^{12}CO and ^{13}CO signal from the maps even below the

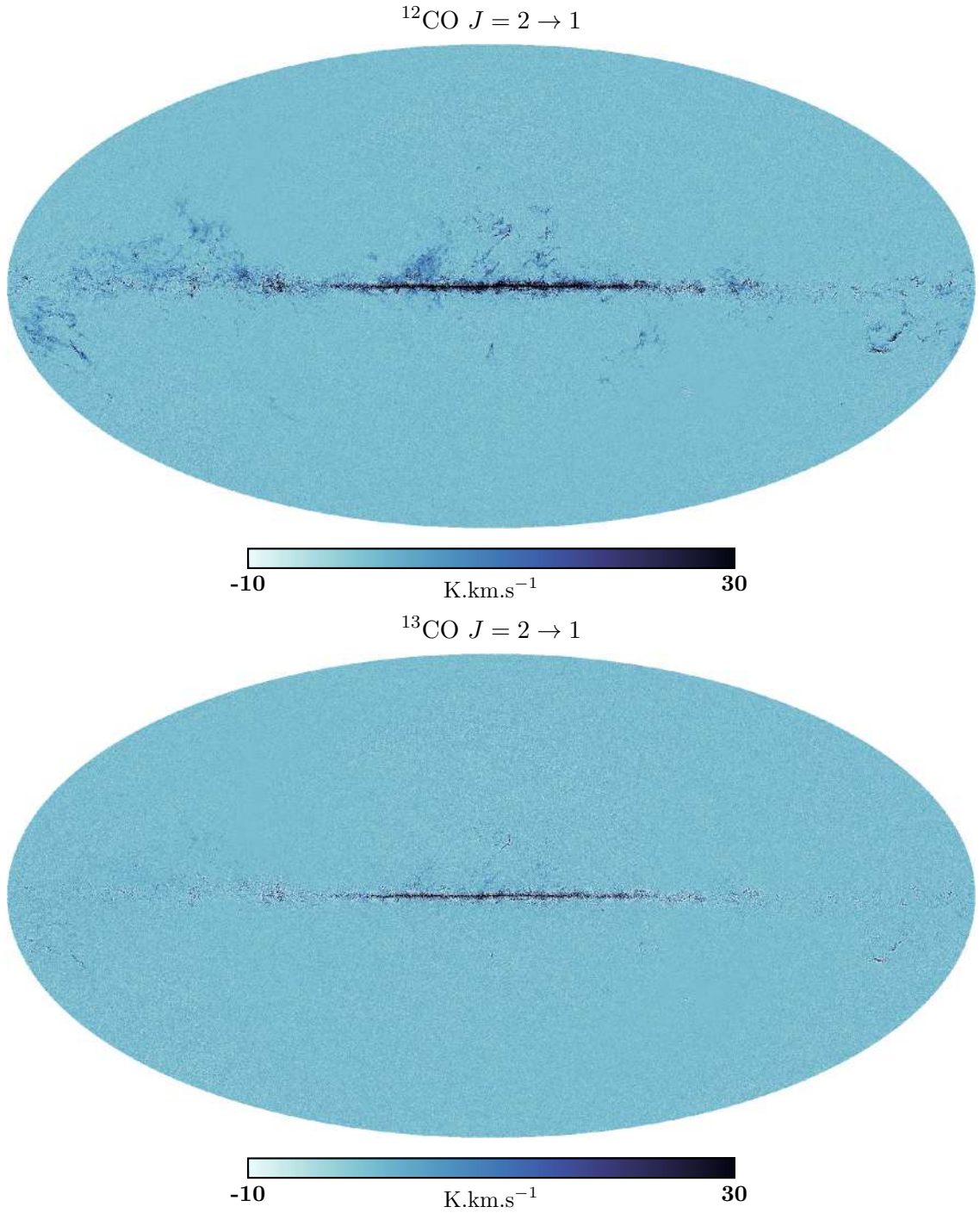


Fig. IV.25 Fullsky CO maps of the $J = 2 \rightarrow 1$ line, computed as described in subsection IV.2.1, at a resolution of $N_{\text{side}} = 1024$, and recalibrated using a factor $1/1.881$, fitted from the Planck legacy Type 1 map. **Top:** ^{12}CO , **Bottom:** ^{13}CO .

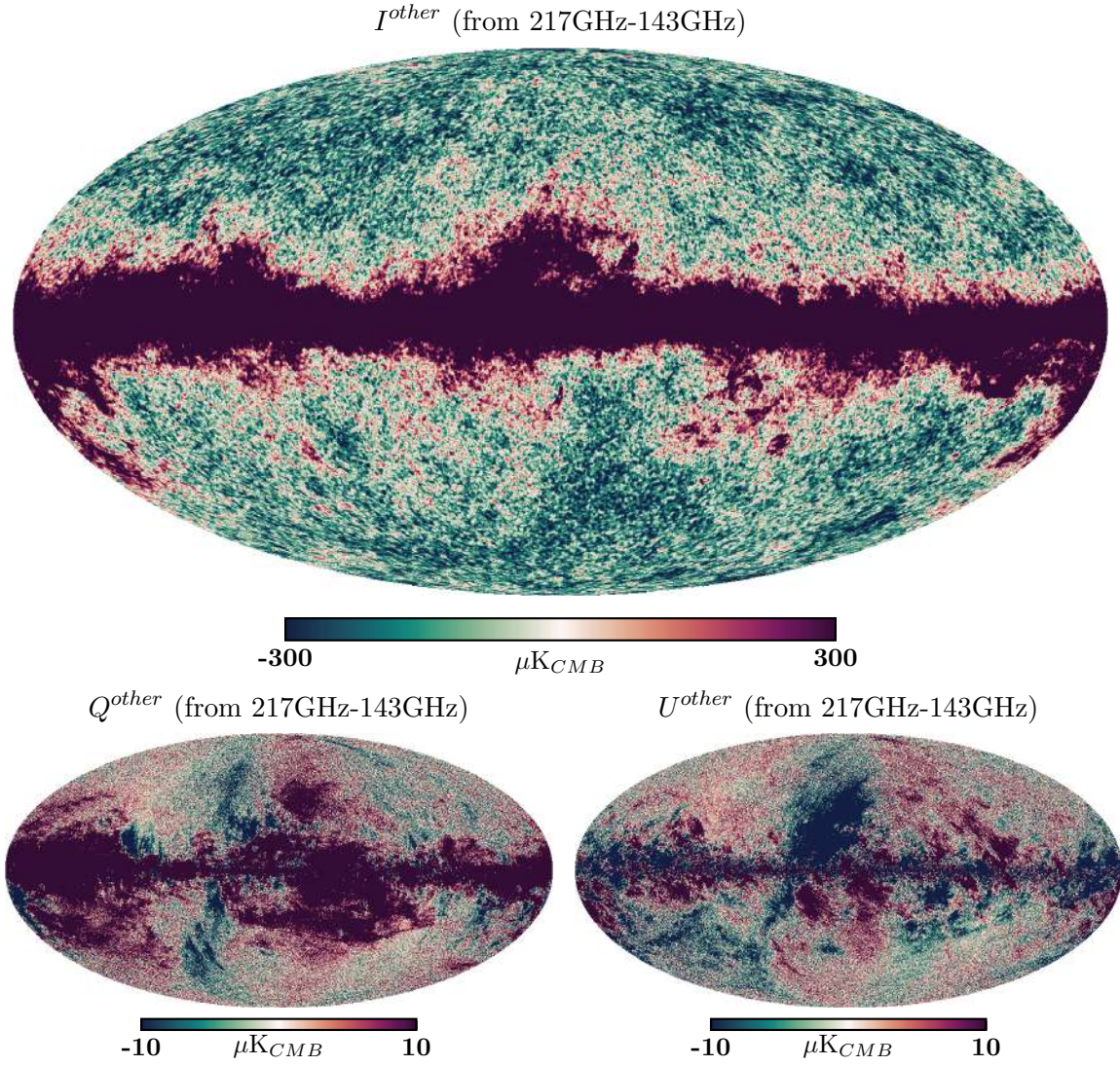


Fig. IV.26 Fullsky I^{other} , Q^{other} , and U^{other} maps resulting from the separation in the case of the $J = 2 \rightarrow 1$ CO line, i.e. computed from the output of the 217-143GHz SRoll2.1 run. They are composed of all that is not CO. **Top:** I^{other} , **Bottom left:** Q^{other} , **Bottom right:** U^{other} .

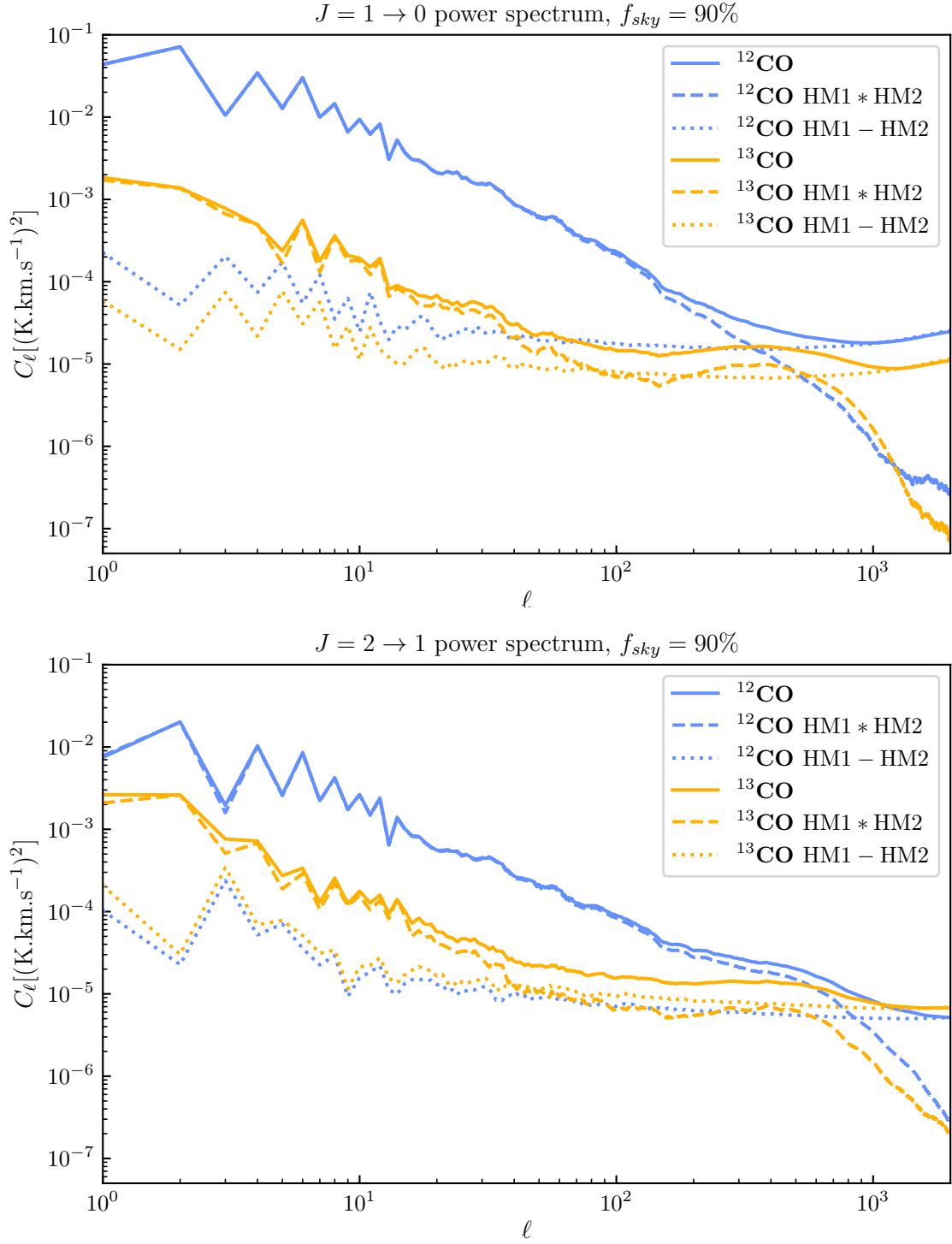


Fig. IV.27 ^{12}CO and ^{13}CO maps spectra, computed using a mask keeping a sky fraction $f_{sky} = 90\%$, hiding the galactic disk, and log-smoothed with a multipole fraction of $\ell_{frac} = 15$ (cf. Appendix.C). **Top:** $J = 1 \rightarrow 0$ line; **Bottom:** $J = 2 \rightarrow 1$ line. The plotted spectra in each case are the maps power spectrum, the cross-spectrum between the maps computed from the HM1 and HM2 subsurveys (see Section II.3), and the power spectrum of the difference maps built as HM1-HM2. The difference maps power spectra have been divided by 4 to be brought back to the original maps noise level and therefore correspond to the power spectrum of the noise in the corresponding maps.

noise level, as seen in the figure. This type of subsurvey cross-spectrum will be extensively exploited in the search for CO emission at high latitude and low column density in my ^{12}CO and ^{13}CO maps, as presented in Chapter V.

We can see in the power spectra of the $J = 1 \rightarrow 0$ CO maps that the noise level, given by the power spectrum of the difference map HM1-HM2, is lower for the ^{13}CO than for the ^{12}CO , by a factor between 2-3 in *power spectrum*. This will be observed and discussed further in the next subsection.

Another notable observation is the "bump" appearing in the power spectrum, as well as in the HM1 * HM2 cross-spectrum, of the ^{13}CO maps (more visible in the $J = 1 \rightarrow 0$ line than in the $J = 2 \rightarrow 1$ one), between multipoles $130 \lesssim \ell \lesssim 1000$. This is probably due to an effect in the SRoll2.1 maps stemming from the limit of using the 143GHz bolometers to build the ^{12}CO and ^{13}CO maps. This "bump" is further discussed in later Section V.2.

IV.2.3 Error maps and noise

As previously introduced for the computation of Fig.IV.27, and for the rest of this work, I need to have ways to estimate the noise level in my maps. This is useful when comparing my maps to external state-of-the-art CO observations to qualify my results, but it will prove mandatory when exploring the low column density regions of the high latitude while searching for CO signal outside of the dense molecular clouds. I will need such noise evaluations to estimate whether my search yields or not an actual detection.

To this end, I possess 2 sources of noise evaluation for my maps:

1. Difference maps of subsurveys, in my case I use the difference between maps built from the subsurveys HM1 and HM2, giving a difference map HM1-HM2 corresponding to a realisation of noise in the corresponding map with a level of noise equal to **twice** the level present in the full maps (each HM map has a noise level multiplied by a factor $\sqrt{2}$ because it uses half of the mission data, and the difference of these 2 maps adds another $\sqrt{2}$ factor),
2. The maps of **statistical error** from the χ^2 minimization computing the maps, as defined by Eq.IV.9, which represent the standard deviation of the maps estimate computed by the χ^2 minimization.

The difference maps HM1-HM2 of the ^{12}CO and ^{13}CO emissions are plotted, for the $J = 1 \rightarrow 0$ and $J = 2 \rightarrow 1$ lines, in the first row of, respectively, Figs.IV.28 and IV.29. The displayed difference maps have been divided by a factor of 2 to bring them back to the level of noise present in the corresponding original maps built from the full *Planck* mission data.

The *statistical error* maps for each SRoll2.1 runs, i.e. 100-143GHz and 217-143GHz, are respectively plotted in the 3 bottom rows of Figs.IV.28 and IV.29 for all 5 maps constructed from my CO maps building procedure. Note that these maps give only the *statistical error* for each map, i.e. the standard deviation of the corresponding map pixel value estimate computed by the χ^2 . Notably, these maps **do not** account for the uncertainty on the RDCs determination, as they were presented in subsection IV.1.4, and are listed in Appendix.D. Also, it only accounts for the random noise in the maps and not for systematic errors.

These statistical error maps show that, compared to the *Planck* legacy CO maps from [Planck Collaboration XIII \(2014\)](#), the noise in my maps is between that found in the Type 1 and Type 2 *Planck* maps, as shown in Fig.IV.30. For both the $J = 1 \rightarrow 0$ and $J = 2 \rightarrow 1$ lines, my ^{12}CO map statistical noise level is greater than that found in the Type 2 *Planck* maps (by about an order of magnitude), but also lower than the one from the Type 1 *Planck* maps (by a factor of about ~ 6), despite the additional separation of the ^{12}CO and ^{13}CO

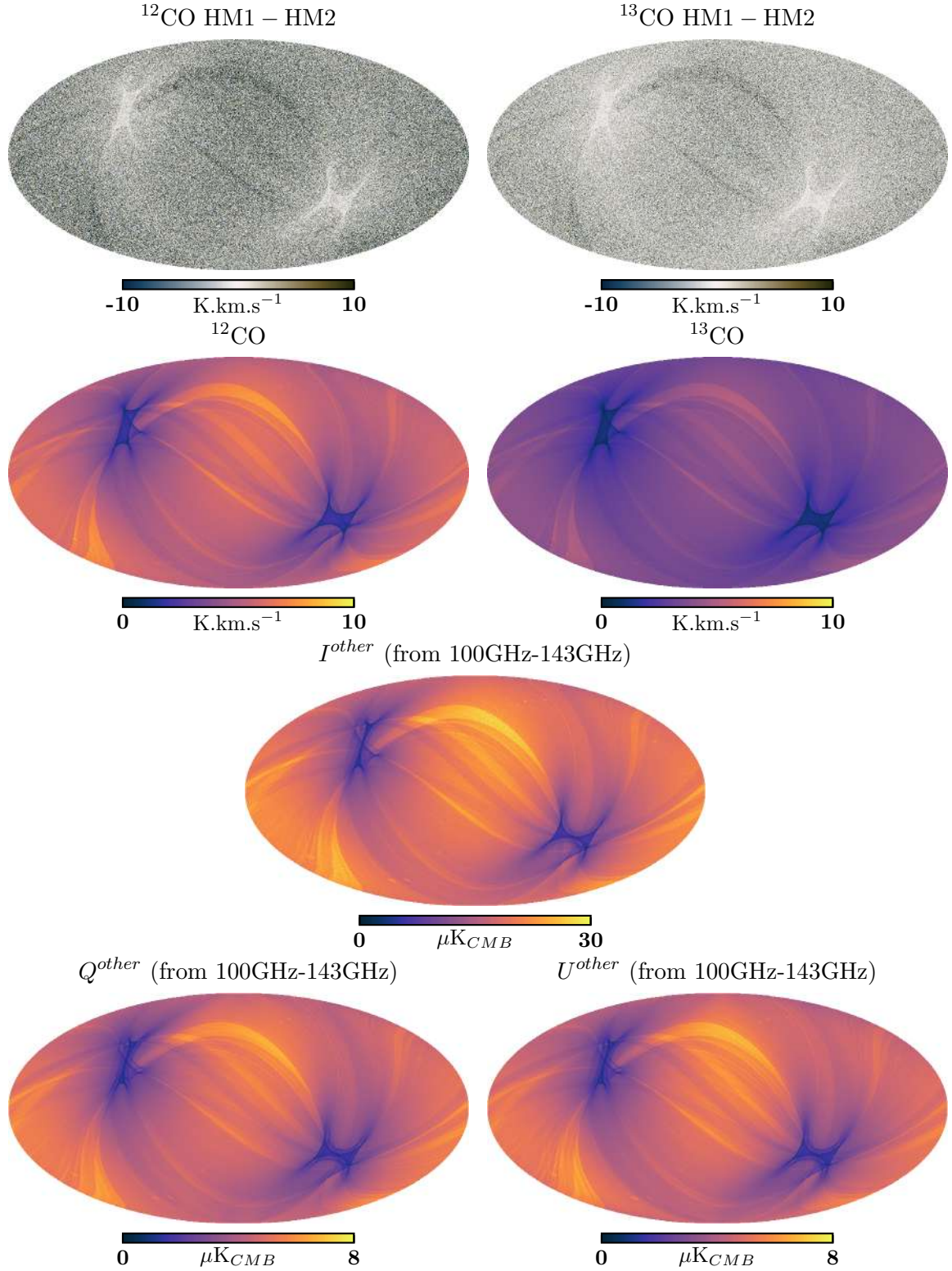


Fig. IV.28 Error maps from the *SROLL2.1* 100-143GHz run, containing the $J = 1 \rightarrow 0$ CO line. **First row:** difference maps HM1-HM2 of the ^{12}CO and ^{13}CO emissions, divided by 2 to bring them to the noise level in the corresponding maps from the full data; **Second row:** ^{12}CO and ^{13}CO statistical error maps; **Third row:** I^{other} statistical error map; **Fourth row:** Q^{other} and U^{other} statistical error maps. The statistical error maps are computed from the χ^2 minimization building the maps.

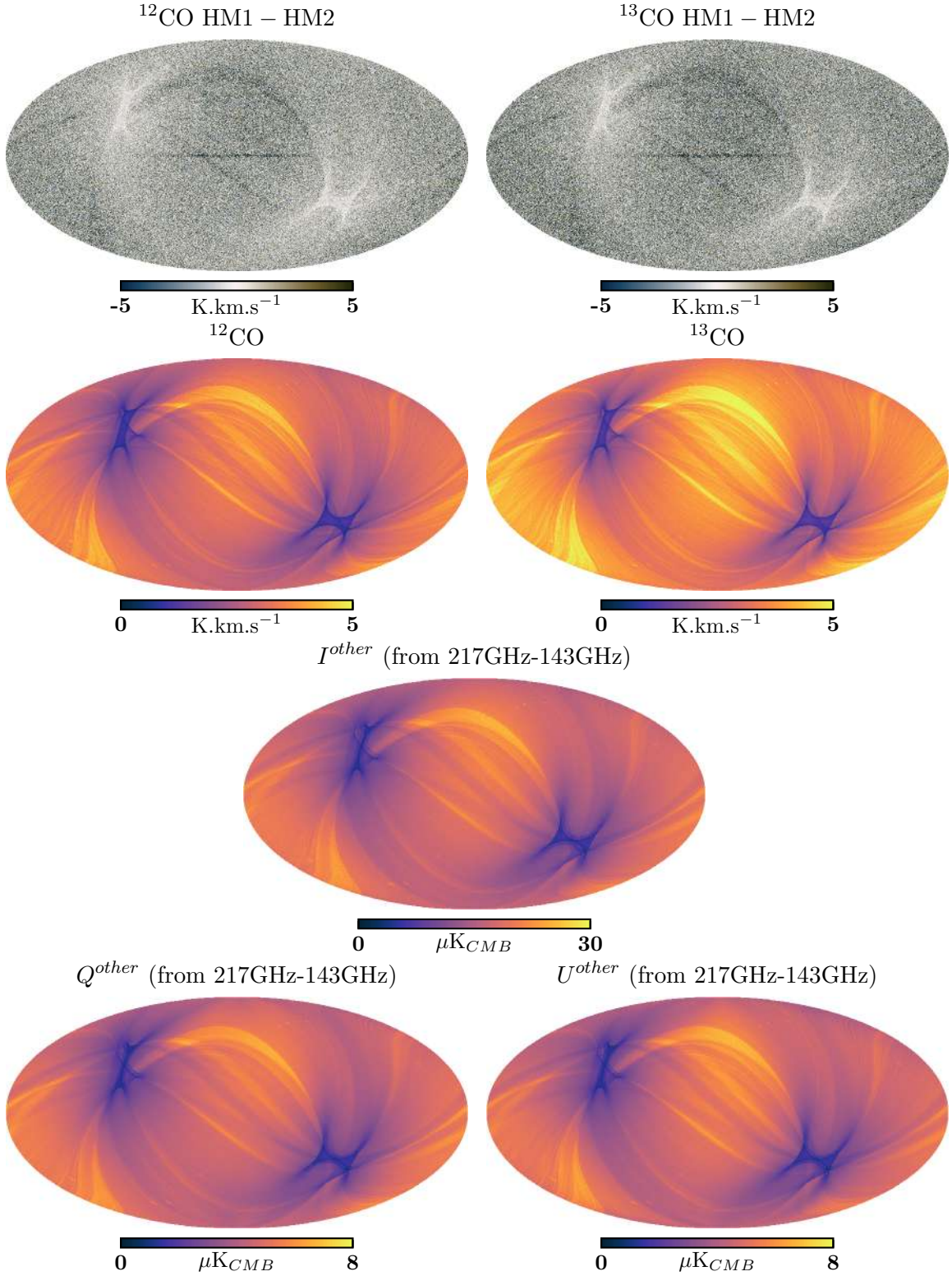


Fig. IV.29 Error maps from the SRoll2.1 217-143GHz run, containing the $J = 2 \rightarrow 1$ CO line. **First row:** difference maps HM1-HM2 of the ^{12}CO and ^{13}CO emissions, divided by 2 to bring them to the noise level in the corresponding maps from the full data; **Second row:** ^{12}CO and ^{13}CO statistical error maps; **Third row:** I^{other} statistical error map; **Fourth row:** Q^{other} and U^{other} statistical error maps. The statistical error maps are computed from the χ^2 minimization building the maps.

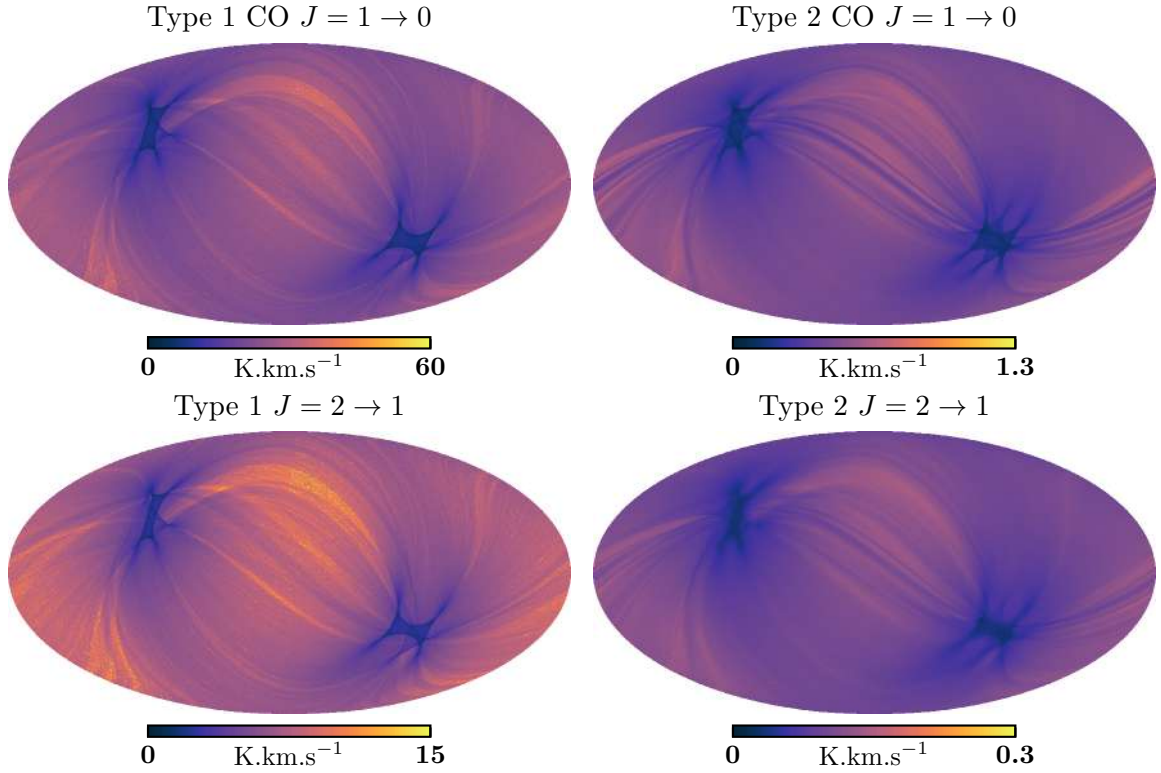


Fig. IV.30 Statistical error maps of the *Planck* CO maps. **Top:** Type 1 maps; **Bottom:** Type 2 maps. **Left:** $J = 1 \rightarrow 0$ line; **Right:** $J = 2 \rightarrow 1$ line. Adapted from Figs.3 and 4 of *Planck Collaboration XIII (2014)*.

isotopologues, as well as despite our quantity of frequency bands use and *a priori* hypotheses on the sky emissions kept to a minimum, bringing my method closer to the determination done in the Type 1 maps.

In Fig. IV.28, we can see that the statistical error in the $J = 1 \rightarrow 0$ ^{12}CO map is *greater* than that of the ^{13}CO one. This is because this statistical error is only affected by the spread of the RDC distribution used to determine the map, i.e. its standard deviation, which is, incidentally, greater for the ^{13}CO component than for the ^{12}CO . Indeed, as can be seen from the values in Table D.1, not only is the standard deviation of the ^{13}CO RDCs along the 100GHz band greater than that of the ^{12}CO , with $\sigma_{\delta L}^{13\text{CO}} = 3.7 \pm 0.41 \mu\text{K}_{\text{CMB}}.\text{K}^{-1}.\text{km}^{-1}.\text{s}$ against $\sigma_{\delta L}^{12\text{CO}} = 2.8 \pm 0.11 \mu\text{K}_{\text{CMB}}.\text{K}^{-1}.\text{km}^{-1}.\text{s}$, but the maximum distance between any pair of detectors along these RDC distributions is also greater for the ^{13}CO than for the ^{12}CO , with $(\max - \min)_{13\text{CO}} = 12.2 \mu\text{K}_{\text{CMB}}.\text{K}^{-1}.\text{km}^{-1}.\text{s}$ compared to $(\max - \min)_{12\text{CO}} = 7.7 \mu\text{K}_{\text{CMB}}.\text{K}^{-1}.\text{km}^{-1}.\text{s}$.

Conversely, in Fig. IV.29, the statistical error in the $J = 1 \rightarrow 0$ ^{12}CO map is *smaller* than that of the ^{13}CO one, though it can be explained by the same reasoning. Indeed, as can be seen from the values in Table D.4, not only is the standard deviation of the ^{12}CO RDCs along the 217GHz band greater than that of the ^{13}CO , with $\sigma_{\delta L}^{12\text{CO}} = 3.0 \pm 0.10 \mu\text{K}_{\text{CMB}}.\text{K}^{-1}.\text{km}^{-1}.\text{s}$ against $\sigma_{\delta L}^{13\text{CO}} = 2.7 \pm 0.58 \mu\text{K}_{\text{CMB}}.\text{K}^{-1}.\text{km}^{-1}.\text{s}$, but the maximum distance between any pair of detectors along these RDC distributions is also greater for the ^{12}CO than for the ^{13}CO , with $(\max - \min)_{12\text{CO}} = 10.4 \mu\text{K}_{\text{CMB}}.\text{K}^{-1}.\text{km}^{-1}.\text{s}$ compared to $(\max - \min)_{13\text{CO}} = 7 \mu\text{K}_{\text{CMB}}.\text{K}^{-1}.\text{km}^{-1}.\text{s}$.

a) Impact on noise of the use of the 143GHz band

The introduction of the error maps in the previous subsection provides us with the tools to evaluate and quantify the benefit, in terms of noise reduction, of adding the bolometers from the 143GHz to our SRoll2.1 sky model and to my maps building procedure, as was discussed in subsection IV.1.1.a).

I carried out this evaluation by plotting the results on error presented in Fig. IV.31. The 2 first rows compare the statistical error maps of the ^{12}CO and ^{13}CO emissions for 2 types of run: the default 100-143GHz SRoll2.1 run and maps computation procedure in the first row, and, in the second row, the same run performed **without the bolometers from the 143GHz band**, i.e. with only the 8 bolometers of the 100GHz band, leading to a construction very similar to the one of the Type 1 *Planck* CO maps, though still benefiting from all the reduced residuals from the SRoll2 mapmaking algorithm. This benefit can be seen through a reduced noise level compared to the Type 1 *Planck* legacy $J = 1 \rightarrow 0$ CO map, by a factor of about ~ 4 . I also computed the power spectra of the difference maps HM1-HM2 for both the ^{12}CO and ^{13}CO maps, for each type of run, using a fraction mask with $f_{sky} = 90\%$, and with a log-smooth of $\ell_{frac} = 15$ (cf. Appendix.C).

We can see that the run **without** the use of the bolometers from the 143GHz band leads to a level of noise almost 1 order of magnitude greater than the default result from a 100-143GHz SRoll2.1 run *in the power spectrum*. This means a **reduction of the maps noise level of more than $\gtrsim 3$** . This justifies the claim from subsection IV.1.1.a), stating the noise reducing effect of the addition of the bolometers from the 143GHz frequency band into the SRoll2.1 run structure and into my maps building procedure.

IV.3 Risks of systematic errors and biases

This section is dedicated to exploring, discussing, and estimating the main potential sources of systematic errors and biases produced by my method (either in SRoll2.1 or in the ^{12}CO and ^{13}CO maps computation), and affecting its results. First, I discuss and estimate the amount of dust $\rightarrow$ CO signal leakage, by producing maps of the effect for both isotopologues. Then, I show that the bolometers' monopole values (i.e. their zero-level), given as input to SRoll2.1, have a negligible impact on the foreground RDCs determination. Next, I discuss the effect of the bolometers' beam differences, which I planned to implement and characterize but did not have time to complete. Still, I believe this effect to be worth discussing, and the idea for its correction, which I did not complete, worth presenting. Afterwards, I evaluate the level of another potential type of signal leakage in my maps: the $^{12}\text{CO} \rightarrow ^{13}\text{CO}$ leakage. Finally, I discuss another source of potential bias, first mentioned in subsection IV.1.1.a), which is the effect of the presence of molecular emission, spatially correlated to the CO, in the 143GHz band.

IV.3.1 Leakage dust $\rightarrow$ CO

One potential source of error to watch out for in the method was already hinted in subsection III.2.1.a): it is the risk of signal leakage from the dust emission into the ^{12}CO and ^{13}CO maps. Such leakage can be generated by an imperfect separation of the dust and CO emissions as well as an imperfect realignment of the dust emission in SRoll2.1, and this potential effect needs to be checked.

To this end, I evaluated an upper limit to the amount of dust $\rightarrow$ CO leakage by building maps of the effect for both isotopologues, using a simulation run. To achieve this, I first compute 2 SRoll2.1 100-143GHz runs from the same simulation realization, but where I

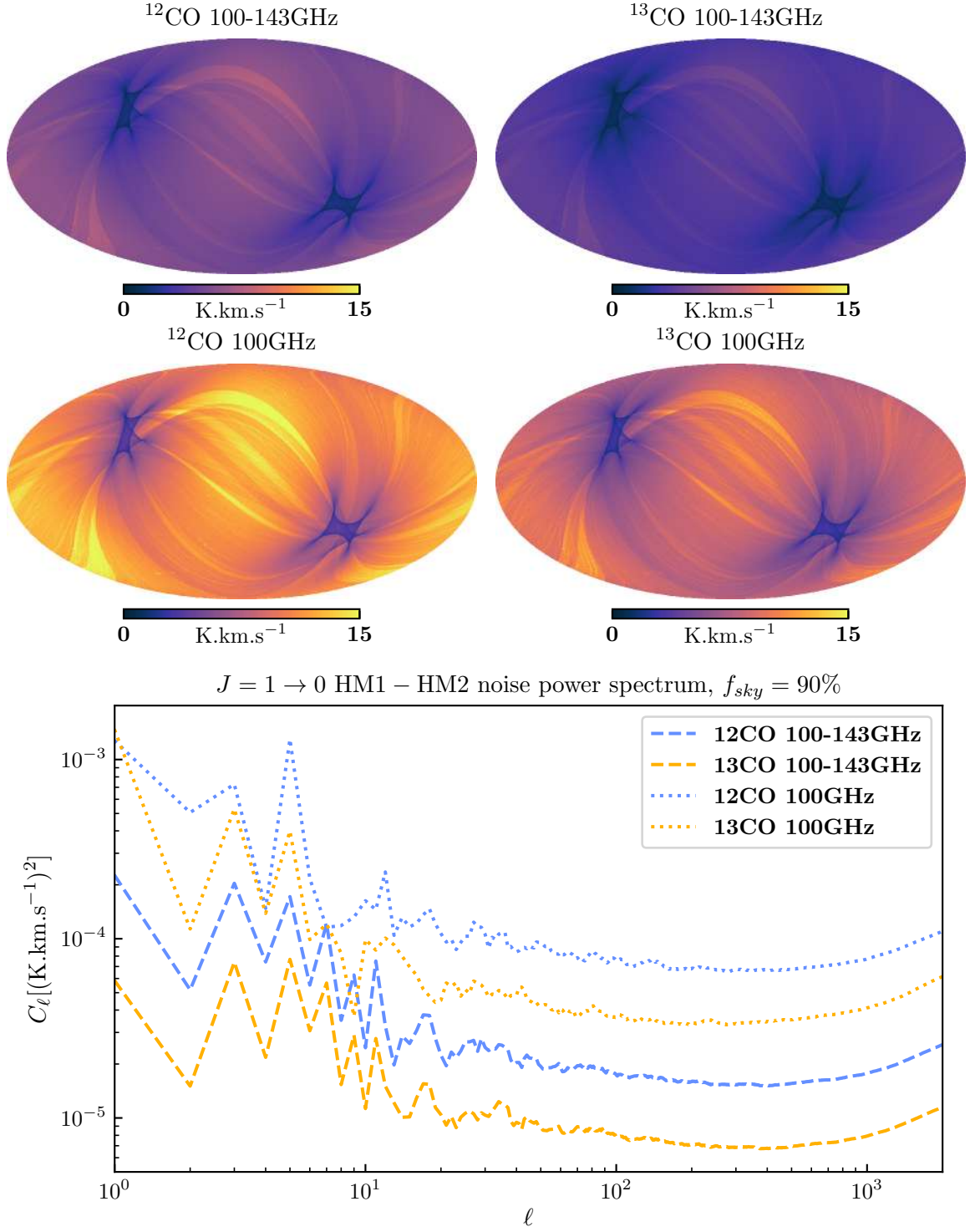


Fig. IV.31 Results of the noise reducing effect of adding the bolometers from the 143GHz frequency band into SRoll2.1, and to build the maps. **First row:** ^{12}CO and ^{13}CO statistical error maps from a 100-143GHz SRoll2.1 run; **Second row:** ^{12}CO and ^{13}CO statistical error maps from a SRoll2.1 run using **only** the 8 bolometers from the 100GHz frequency band. **Bottom:** power spectra of the difference map HM1-HM2 for the ^{12}CO and ^{13}CO emissions, in both cases of SRoll2.1 run corresponding to the statistical error maps of the first and second rows, using a mask with $f_{sky} = 90\%$ and log-smoothed with $\ell_{frac} = 15$.

added some dust signal in the input signal HPRs for *one* of the 2 runs. Using the same notations as introduced for the data model of SRoll2 and SRoll2.1, this addition of dust signal into the input simulation HPRs is computed as follows:

$$\begin{aligned} \forall b \in \mathcal{B}_{CO}, \quad g_b M_{b,r,p}^{simu+dust} &= g_b M_{b,r,p}^{simu} + (\delta L_b^{dust} - \overline{\delta L^{dust}}^{CO} + 0.2) A_{b,r,p} S_{b,p}^{dust \text{ add.}}, \\ \forall b \in \mathcal{B}_{143}, \quad g_b M_{b,r,p}^{simu+dust} &= g_b M_{b,r,p}^{simu} + (\delta L_b^{dust} - \overline{\delta L^{dust}}^{143} + 0.2) A_{b,r,p} S_{b,p}^{dust \text{ add.}}, \end{aligned} \quad (\text{IV.10})$$

where $g_b M_{b,r,p}^{simu}$ is the HPR of the reference simulation (the one used for the *untouched* run), δL_b^{dust} are the dust RDCs determined by the first reference SRoll2.1 run on the untouched simulation, $\overline{\delta L^{dust}}^{143}$ is the average value of the δL_b^{dust} along the 143GHz frequency band, i.e. $\overline{\delta L^{dust}}^{143} = \frac{1}{11} \sum_{b \in \mathcal{B}_{143}} \delta L_b^{dust}$, $\overline{\delta L^{dust}}^{CO}$ is the average value of the δL_b^{dust} along the 100GHz frequency band, i.e. $\overline{\delta L^{dust}}^{CO} = \frac{1}{8} \sum_{b \in \mathcal{B}_{CO}} \delta L_b^{dust}$, and $S^{dust \text{ add.}}$ is the sky emission map components of the added dust signal defined as:

$$S_{b,p}^{dust \text{ add.}} = \left[\frac{I_{\nu_b}^{MBB}}{I_{857}^{MBB}} I_p^{857\text{GHz}}, Q_p^{dust \text{ template}}, U_p^{dust \text{ template}} \right],$$

with I_{ν}^{MBB} taken as the dust MBB emission defined by Eq.II.3 (with dust temperature and spectral index as $T_{dust} = 19.5\text{K}$ and $\beta_{dust} = 1.55$, cf. Planck Collaboration IV (2020)) and I_{ν_b} corresponding to the I_{ν} value at the frequency of the bolometer b (i.e. I_{ν_b} is either I_{100} or I_{143}), $I_p^{857\text{GHz}}$ being the Planck legacy (2018 release) HFI 857GHz frequency map largely dominated by the dust emission (by an order $\sim 10^3$, cf Fig.I.10), and $Q_p^{dust \text{ template}}$ and $U_p^{dust \text{ template}}$ being respectively the Q and U polarization parts of the SRoll2.1 dust template as introduced in subsection IV.1.3.a). The 0.2 term appearing in Eq.IV.10 is used to add a sufficient amount of dust signal to be detectable in the HPRs while limiting the distortion this applies on the total sky signal. From these 2 SRoll2.1 runs, ^{12}CO and ^{13}CO maps are built using the method introduced in Section IV.2, and these 2 pairs of maps are therefore only differentiated by the presence (or absence) of the added dust signal into the input HPRs. I can then compute, from the difference between the results of these 2 runs, maps of the dust leakage into each of the ^{12}CO and ^{13}CO maps, which can be written:

$$\begin{aligned} M_p^{dust \rightarrow ^{12}\text{CO}} &= 5 \times (M_p^{^{12}\text{CO}, \text{simu}+dust} - M_p^{^{12}\text{CO}, \text{simu}}), \\ M_p^{dust \rightarrow ^{13}\text{CO}} &= 5 \times (M_p^{^{13}\text{CO}, \text{simu}+dust} - M_p^{^{13}\text{CO}, \text{simu}}), \end{aligned} \quad (\text{IV.11})$$

where $M^{^{12}\text{CO}, \text{simu}+dust}$ is the ^{12}CO map built from the run *containing* the added dust signal, $M^{^{12}\text{CO}, \text{simu}}$ is the ^{12}CO map from the reference simulation run *without* any HPR tampering, and using the same notations for the ^{13}CO . The factor of 5 is used to compensate for the 0.2 factor appearing in the dust signal addition in Eq.IV.10 above.

Overestimation I mentioned above that this method of evaluating the dust $\rightarrow$ CO leakage produces an *upper limit* of the effect. Indeed, I expect the leakage observed from the added dust signal to be *overestimated* compared to the one affecting the SRoll2.1 data runs. This overestimation is due to the nature of the dust signal added into the simulation HPRs, and has 2 main sources:

1. The dust emission map added into the HPRs is **not** the same as the dust model used to build the simulation sky model, and thereby does not rigorously correspond to the dust template used in the SRoll2.1 simulation runs,

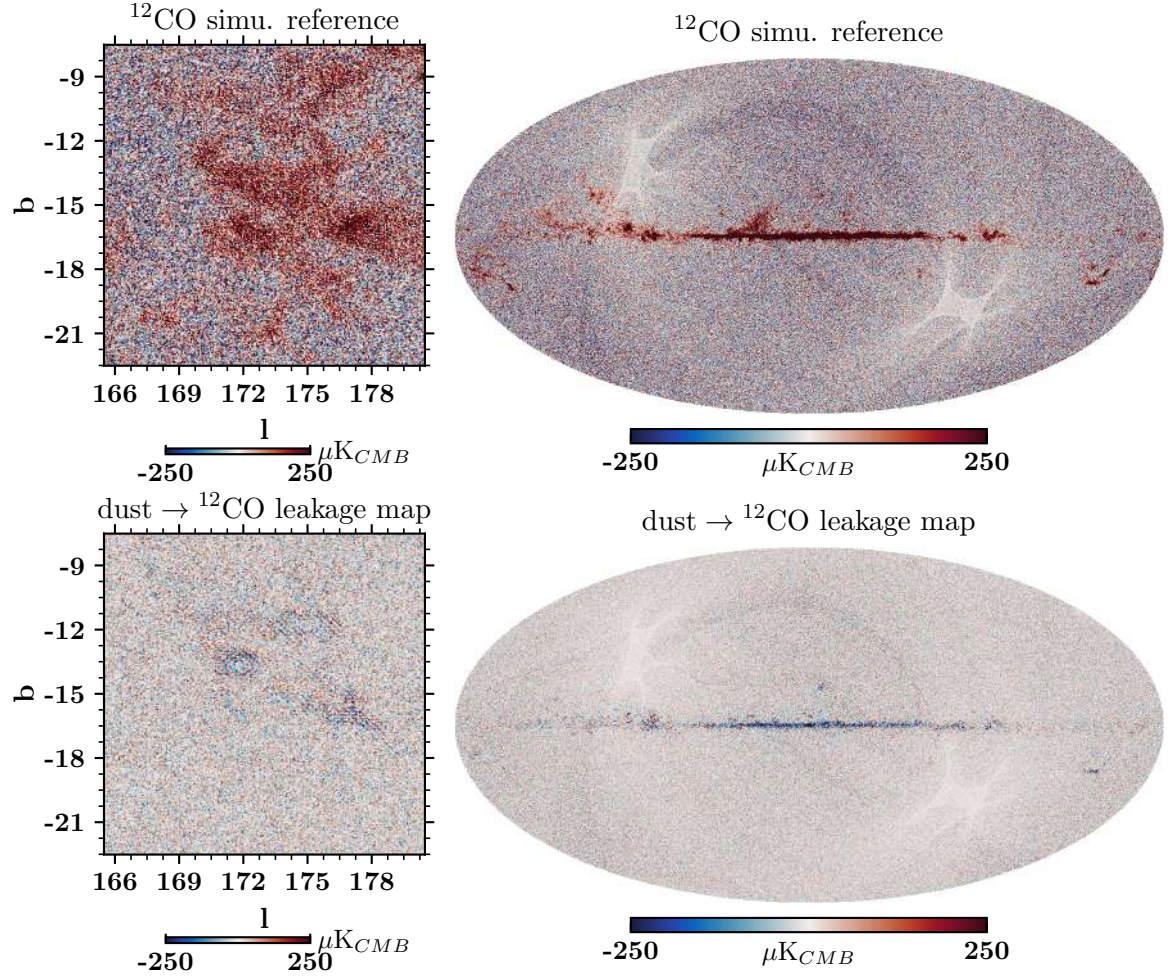


Fig. IV.32 dust $\rightarrow$ ^{12}CO leakage map computed as presented in Eq. IV.11 (**bottom row**), compared to the ^{12}CO map built from the SRoll2.1 run on the reference simulation (**top row**). **Left:** Taurus region; **Right:** fullsky map.

2. The intensity map of the added dust signal ($I_p^{857\text{GHz}}$) is not fully consistent with its polarized part ($Q_p^{\text{dust template}}$ and $U_p^{\text{dust template}}$): firstly because they were not built consistently from the same separation, and secondly because they are not at the same resolution with $I_p^{857\text{GHz}}$ taken at the full *Planck* resolution of $N_{\text{side}} = 2048$ and $Q_p^{\text{dust template}}$ and $U_p^{\text{dust template}}$ being at the SRoll2.1 template resolution of $N_{\text{side}} = 128$. Since the high polarization fraction of the dust emission is significant to separate it from the CO foreground, this inconsistency between the intensity and polarization parts of the added dust signal is expected to introduce decorrelation.

For these 2 reasons, I expect the added dust signal to leak more into the ^{12}CO and ^{13}CO maps than the dust signal from the data might.

Note that from the 2 aforementioned sources of overestimation, one is willingly introduced while the other is a collateral effect of the former. Indeed, to estimate the amount of dust $\rightarrow$ CO leakage from the dust emission present in the simulation sky model, I would need to compute the same leakage maps but by adding a dust signal *exactly equal* to the one present in the PSM. However, this would have implied an additional full SRoll2.1 run for which I no longer had the required amount of computing time at my disposal. I therefore had to choose the type of run I would be able to compute and settled for the one pre-

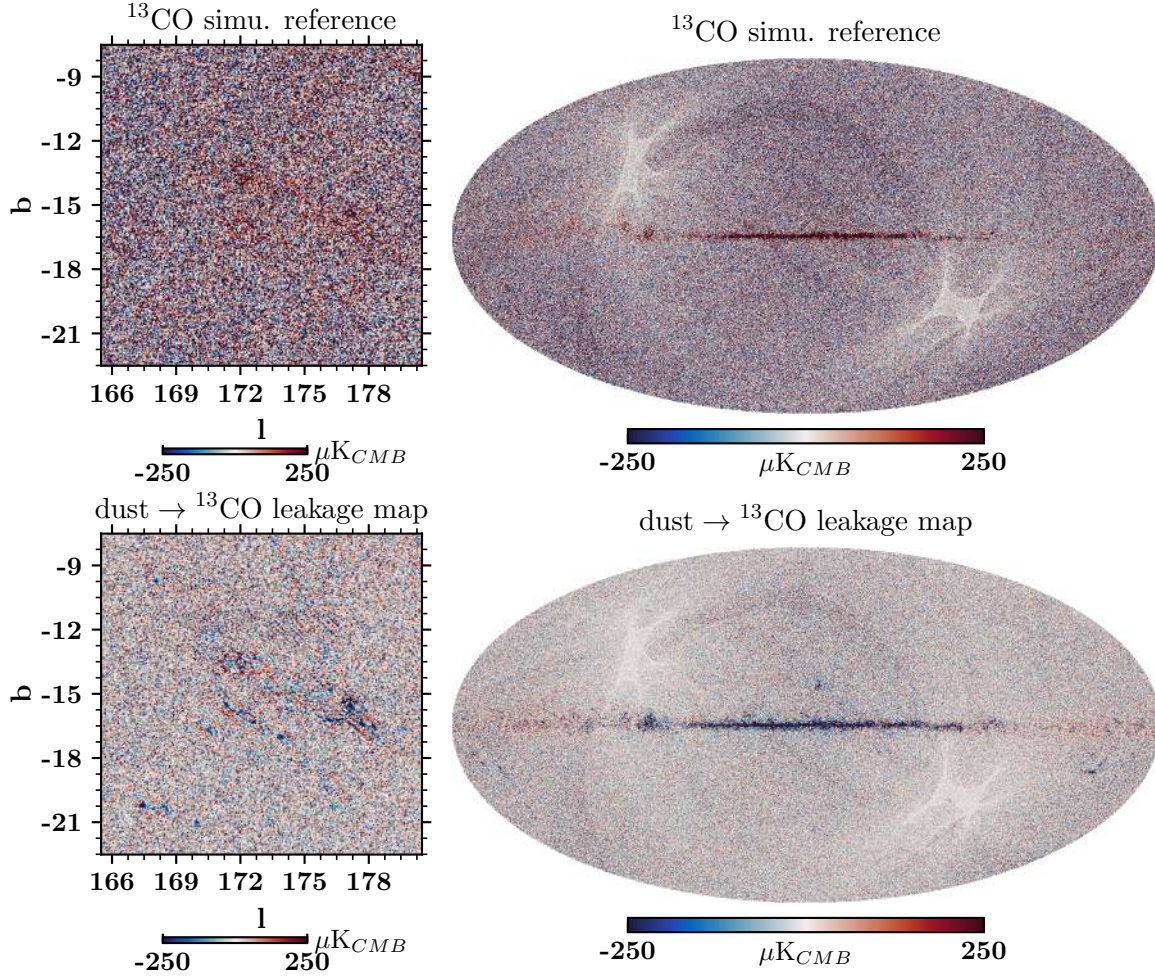


Fig. IV.33 dust $\rightarrow$ ^{13}CO leakage map computed as presented in Eq.IV.11 (**bottom row**), compared to the ^{13}CO map built from the SRoll2.1 run on the reference simulation (**top row**). **Left:** Taurus region; **Right:** fullsky map.

sented above to compute an *upper limit*: this is important because despite the quality of my SRoll2.1 dust template for the data, I know that this template is imperfect, and that any discrepancy between my template and the *actual* dust emission present in the *Planck* data may increase the dust $\rightarrow$ CO leakage. I therefore chose to add a dust signal *intentionally* taken different from the PSM dust model to estimate an *upper limit* of what can be expected to be observed in the data. As for the inconsistency between the intensity and polarization parts of the added dust signal, this is unfortunately due to the fact that all bolometers from the 857GHz band are SWBs and therefore no polarized parts of the 857GHz frequency map were produced.

I displayed the resulting upper limit maps of the dust $\rightarrow$ CO leakage in Figs.IV.32 and IV.33 respectively for the ^{12}CO and the ^{13}CO . In both cases, I plotted the leakage map computed as presented in Eq.IV.11, as well as the corresponding CO map built from the reference untouched simulation run. For the ^{12}CO , we can see in Fig.IV.32 a small leakage appearing in *negative* and only visible in regions of high CO intensity in the ^{12}CO map from the reference run. The average upper limit of dust $\rightarrow$ ^{12}CO leakage from this map (excluding the inner Galactic disk, i.e. excluding latitudes for $-1^\circ.5 < b < 1^\circ.5$) amounts to $\lesssim 3.1\%$ of the ^{12}CO signal, though it is actually less than that as the good agreement with external surveys will prove (see Section V.1).

A more significant leakage is visible for the ^{13}CO in Fig.IV.33. However, this leakage

appearing in the form of strong negative features, is only present along the sharp and peaked regions of higher intensity signal in the ^{13}CO reference map. This observation is consistent with the failing of my SRoll2.1 ^{12}CO and ^{13}CO maps to accurately recover the aforementioned regions of peaked signal at higher intensity, as will be extensively explored in Section V.1. An interpretation for the origin of this effect, as well as solutions, will also be discussed in Section V.2. The average upper limit of dust $\rightarrow ^{13}\text{CO}$ leakage from this map (excluding the inner Galactic disk, i.e. excluding latitudes for $-1^\circ.5 < b < 1^\circ.5$) amounts to $\lesssim 7.7\%$ of the ^{13}CO signal.

IV.3.2 Leakage $^{12}\text{CO} \rightarrow ^{13}\text{CO}$

The second type of signal leakage that ought to be monitored is the leakage between CO isotopologues. As previously discussed, the ^{12}CO and ^{13}CO emissions are highly spatially correlated, whereby any error on their separation, i.e. any error on their respective RDCs determination, can lead to signal leakage from one map to the other.

I evaluated the amount of $^{12}\text{CO} \rightarrow ^{13}\text{CO}$ leakage in the SRoll2.1 ^{12}CO and ^{13}CO 100-143GHz maps (i.e. mapping the $J = 1 \rightarrow 0$ line) by using the ^{13}CO map from Goldsmith et al. (2008) as a reference. To this end, I computed the optimal leakage factor $\mathcal{X}_{leak}^{^{12}\text{CO} \rightarrow ^{13}\text{CO}}$ to verify the following equation:

$$\forall p, M_p^{^{13}\text{CO}, G08} = M_p^{^{13}\text{CO}, SRoll2.1} - \mathcal{X}_{leak}^{^{12}\text{CO} \rightarrow ^{13}\text{CO}} M_p^{^{12}\text{CO}, SRoll2.1},$$

where $M_p^{^{13}\text{CO}, G08}$ is the ^{13}CO map from the survey of Goldsmith et al. (2008), and where $M_p^{^{12}\text{CO}, SRoll2.1}$ and $M_p^{^{13}\text{CO}, SRoll2.1}$ are respectively the ^{12}CO and ^{13}CO SRoll2.1 100-143GHz maps.

I fitted the value of $\mathcal{X}_{leak}^{^{12}\text{CO} \rightarrow ^{13}\text{CO}}$ through a χ^2 minimization computed on the Taurus region covered in the work of Goldsmith et al. (2008). The χ^2 to be minimized is then the following:

$$\chi^2 = \sum_p \left(M_p^{^{13}\text{CO}, SRoll2.1} - \mathcal{X}_{leak}^{^{12}\text{CO} \rightarrow ^{13}\text{CO}} M_p^{^{12}\text{CO}, SRoll2.1} - M_p^{^{13}\text{CO}, G08} \right)^2.$$

Minimizing this quantity boils down to solving the equation $\frac{\partial \chi^2}{\partial \mathcal{X}_{leak}^{^{12}\text{CO} \rightarrow ^{13}\text{CO}}} = 0$, which leads to the following expression of $\mathcal{X}_{leak}^{^{12}\text{CO} \rightarrow ^{13}\text{CO}}$:

$$\begin{aligned} \frac{\partial \chi^2}{\partial \mathcal{X}_{leak}^{^{12}\text{CO} \rightarrow ^{13}\text{CO}}} &= 0 \\ \Leftrightarrow \sum_p M_p^{^{12}\text{CO}, SRoll2.1} \left(M_p^{^{13}\text{CO}, SRoll2.1} - \mathcal{X}_{leak}^{^{12}\text{CO} \rightarrow ^{13}\text{CO}} M_p^{^{12}\text{CO}, SRoll2.1} - M_p^{^{13}\text{CO}, G08} \right) &= 0 \\ \Leftrightarrow \sum_p \left(M_p^{^{12}\text{CO}, SRoll2.1} M_p^{^{13}\text{CO}, SRoll2.1} - M_p^{^{12}\text{CO}, SRoll2.1} M_p^{^{13}\text{CO}, G08} \right. \\ &\quad \left. - \mathcal{X}_{leak}^{^{12}\text{CO} \rightarrow ^{13}\text{CO}} (M_p^{^{12}\text{CO}, SRoll2.1})^2 \right) = 0 \\ \Leftrightarrow \mathcal{X}_{leak}^{^{12}\text{CO} \rightarrow ^{13}\text{CO}} &= \frac{\sum_p \left(M_p^{^{12}\text{CO}, SRoll2.1} M_p^{^{13}\text{CO}, SRoll2.1} - M_p^{^{12}\text{CO}, SRoll2.1} M_p^{^{13}\text{CO}, G08} \right)}{\sum_p (M_p^{^{12}\text{CO}, SRoll2.1})^2}. \end{aligned} \tag{IV.12}$$

The above determination of $\mathcal{X}_{leak}^{^{12}\text{CO} \rightarrow ^{13}\text{CO}}$ is always computed on maps at the same resolution of $N_{side} = 1024$.

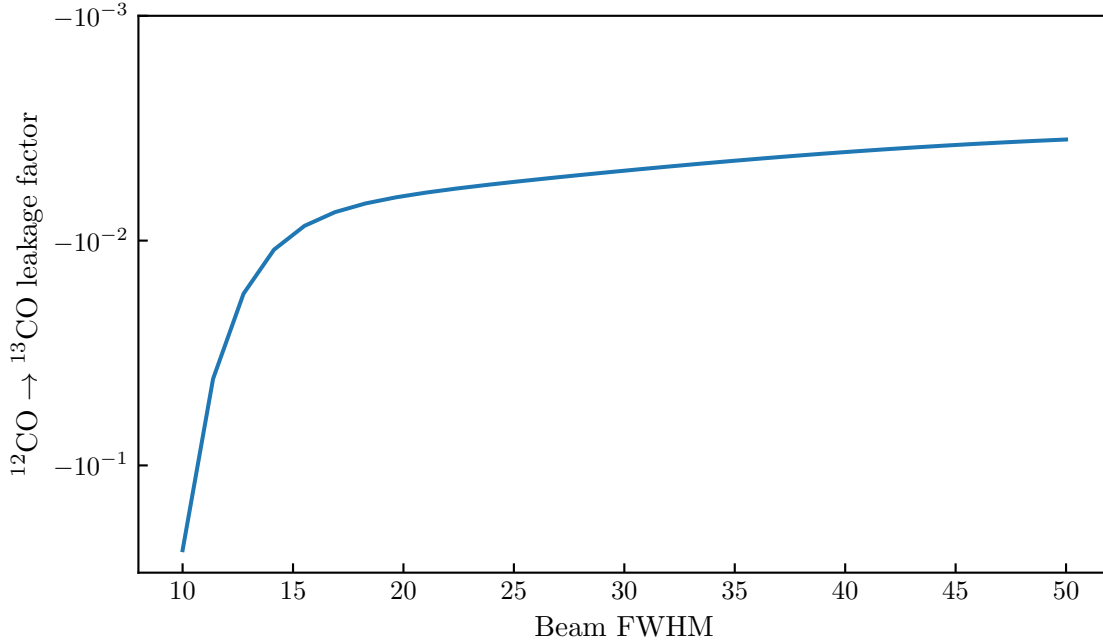


Fig. IV.34 $\mathcal{X}_{leak}^{^{12}\text{CO} \rightarrow ^{13}\text{CO}}$ leakage factor computed using the expression of Eq. IV.12, as a function of the smoothing applied on maps. Each determination of $\mathcal{X}_{leak}^{^{12}\text{CO} \rightarrow ^{13}\text{CO}}$ is computed on the Taurus region as surveyed by Goldsmith et al. (2008), where all maps were smoothed up to a Gaussian beam with FWHM given in abscissa. The first value (at FWHM = 10') corresponds to unsmoothed SRoll2.1 maps.

However, since the level of noise is very different between the SRoll2.1 and the G08 maps, this computation is dominated by the noise if the maps are not smoothed beforehand to reduce the leakage factor sensitivity to the noise variations. Since I do not know in advance any optimal smoothing value for this determination, I do not simply compute *one* value of $\mathcal{X}_{leak}^{^{12}\text{CO} \rightarrow ^{13}\text{CO}}$, but I compute the evolution of this leakage factor as a function of the smoothing applied on maps: I compute $\mathcal{X}_{leak}^{^{12}\text{CO} \rightarrow ^{13}\text{CO}}(FWHM)$ where $FWHM$ is the FWHM of the Gaussian beam up to which the maps have been smoothed before computing the leakage factor using Eq. IV.12 above. The first computed value is then $FWHM = 10'$, which corresponds to unsmoothed SRoll2.1 maps (see subsection IV.2.2 above).

I plotted the result in Fig. IV.34, where I displayed the value of $\mathcal{X}_{leak}^{^{12}\text{CO} \rightarrow ^{13}\text{CO}}(FWHM)$. We can see that the leakage factor $\mathcal{X}_{leak}^{^{12}\text{CO} \rightarrow ^{13}\text{CO}}$ first displays a notably high value when fitted on *unsmoothed* maps (higher than $> 10\%$). Then, its value very quickly goes below 1%, when maps are smoothed up to a Gaussian beam of only $FWHM \simeq 14'$. For smoothing values beyond $FWHM > 18'$, the leakage factor is almost constant at a value of about $\mathcal{X}_{leak}^{^{12}\text{CO} \rightarrow ^{13}\text{CO}} \sim -6 \times 10^{-3}$. My interpretation is that, without smoothing, the determination of the leakage factor is dominated by the noise. It should be noted that the values of $\mathcal{X}_{leak}^{^{12}\text{CO} \rightarrow ^{13}\text{CO}}(FWHM)$ are all *negative*, which denotes a signal leakage from the ^{13}CO map into the ^{12}CO one. This may be caused not by an *actual* $^{13}\text{CO} \rightarrow ^{12}\text{CO}$ signal leakage direction, but by the impact of the dust $\rightarrow$ CO leakage which creates sharp signal dips in the map at higher intensities, as will be further discussed in Section V.2. The main result of this test is that, whatever the direction of the leakage might be (either $^{13}\text{CO} \rightarrow ^{12}\text{CO}$ or $^{12}\text{CO} \rightarrow ^{13}\text{CO}$), this effect corresponds to less than $< 1\%$ of the ^{12}CO map (as determined

from even minimal smoothing).

IV.3.3 Bolometers monopole effect

Another potential source of systematic error is the effect of the bolometers' monopole accuracy on the RDCs' determination. Indeed, as was introduced in subsection III.2.1.a), the monopoles used as input in SRoll2 and SRoll2.1, and subtracted from the respective bolometers' signal, are the ones determined in the *Planck* 2015 release ([Planck Collaboration VIII \(2016\)](#), [Planck Collaboration XI \(2014\)](#)), by fitting the dust emission intensity as a function of the gas column density N_{H} , and assuming a dust intensity of zero at $N_{\text{H}} = 0$. These monopoles' determination has been improved in the work of [Delouis, J.-M. et al. \(2021\)](#), by factors of 2 to 3 with respect to the values obtained in the previous *Planck* legacy estimations.

However, even though I did not use these improved values of the bolometers' zero-level from [Delouis, J.-M. et al. \(2021\)](#), my new ^{12}CO and ^{13}CO maps are determined from response *differences* between bolometers, which are fitted in SRoll2.1 by exploiting bolometers signal *differences*. The fact that every decisive step in the determination of the SRoll2.1 ^{12}CO and ^{13}CO RDCs, and the following computation of the ^{12}CO and ^{13}CO maps, are based upon a model fitting from *differential values* of the data make it unlikely for any of these computation steps to be significantly impacted by the bolometers' monopoles.

Nevertheless, to ensure the validity of the aforementioned hypothesis, I tested the impact of the bolometers' monopoles on the determination of the ^{12}CO , ^{13}CO , and dust RDCs. Since the ^{12}CO and ^{13}CO maps are built from the fitted RDCs, the quality of the RDCs determination is equivalent to the quality of the maps. My test consists in the comparison of the ^{12}CO , ^{13}CO , and dust RDC determination from 2 SRoll2.1 runs:

- A *default* SRoll2.1 100-143GHz data run,
- A SRoll2.1 100-143GHz data run with the input bolometers' monopole values **multiplied by a factor 10** for all bolometers.

In both cases, the run is carried out on one tenth of the input data, i.e. $\text{RSTEP} = 10$, in order to reduce the computational load of the runs, leading to uncertainty on the RDCs' determination being multiplied by a factor $\sqrt{\text{RSTEP}} = \sqrt{10}$.

I displayed the results from this test in Fig. IV.35, where I plotted the absolute value of the difference between the ^{12}CO , ^{13}CO , and dust RDCs from either type of run, i.e. $|\text{RDC}_{\text{monopoles} \times 10} - \text{RDC}_{\text{ref}}|$, as well as the relative shift of the ^{12}CO and ^{13}CO RDCs along the 100GHz band induced by the monopoles being multiplied by 10, i.e. I plotted the values of $\left| \frac{\text{RDC}_{\text{monopoles} \times 10} - \text{RDC}_{\text{ref}}}{\text{RDC}_{\text{ref}}} \right|$. We can see that for the 3 foregrounds ^{12}CO , ^{13}CO , and dust, the difference on the RDC determination induced by the drastic change in monopoles' value is comparable to the corresponding RDC uncertainty, as listed in Appendix.D. Especially, the ^{12}CO and ^{13}CO RDCs, which are the backbone of the corresponding maps computation, display relative variation due to this monopole increase of at most $< 0.1\%$ for the ^{12}CO , and at most $< 0.5\%$ for the ^{13}CO . This suffices to consider the impact of the bolometers' monopole value on the SRoll2.1 ^{12}CO and ^{13}CO RDCs, and equivalently, their effect on the corresponding maps, to be negligible.

Note that no uncertainty bar is given for Fig. IV.35. This is because this test reaches the limits of the significance for my uncertainty estimation: indeed, adding the effect of the probable uncertainty overestimation from the simulations (see subsection IV.1.4.a)) with the multiplicative factor $\sqrt{10}$ on the uncertainty, due to the test being computed with $\text{RSTEP} = 10$, leads to enormous uncertainty bars. This estimated uncertainty is also not

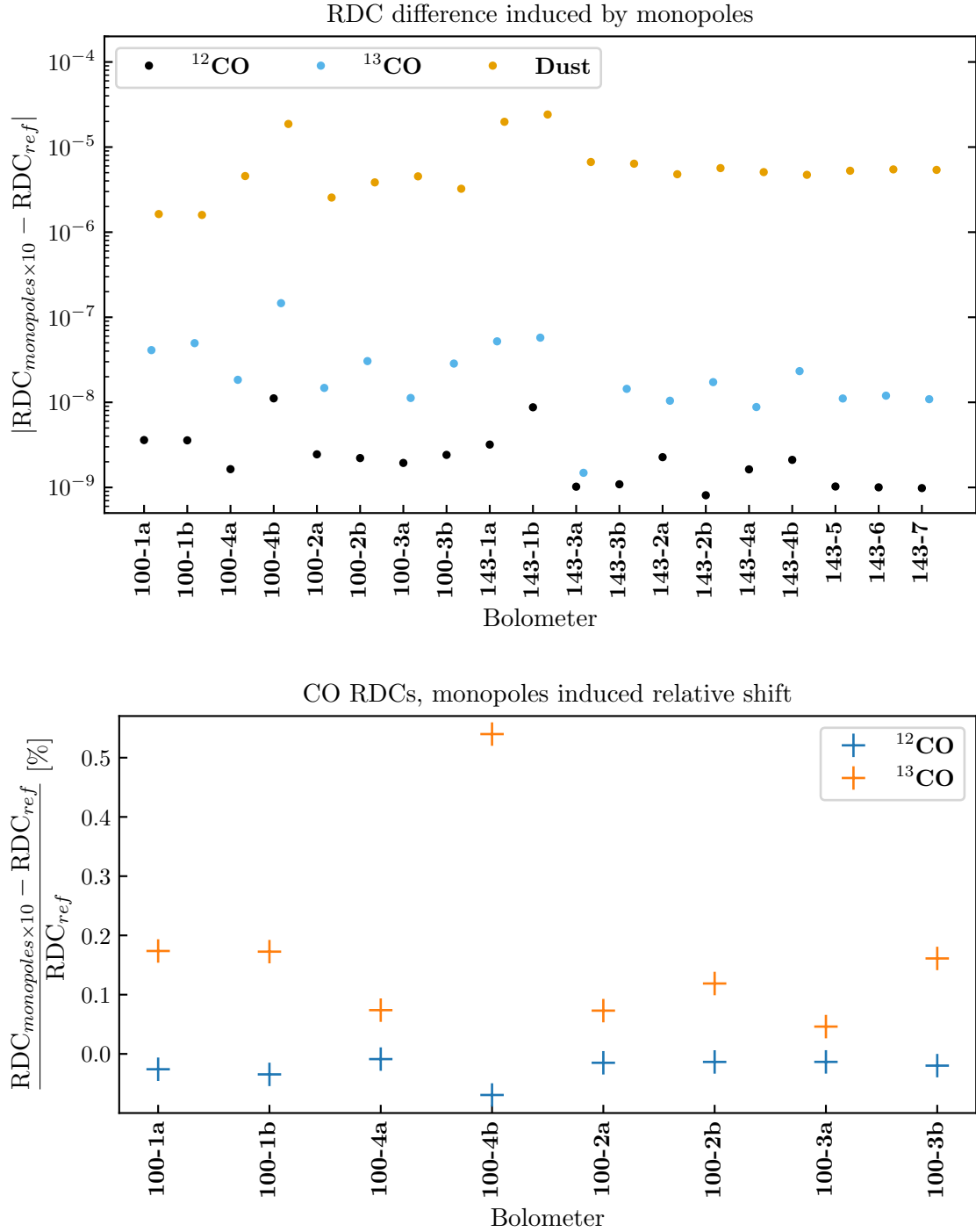


Fig. IV.35 Variation of ^{12}CO , ^{13}CO , and dust RDCs when the bolometers' monopoles used as SRoll2.1 input are multiplied by 10 in a 100-143GHz run. **Top:** Difference between RDCs from modified monopoles and the reference RDCs determined from the default SRoll2.1 run; **Bottom:** 100GHz band relative difference between RDCs from modified monopoles and the reference RDCs (i.e. divided by the value of the respective reference RDC).

consistent with the observed variation of the residual $|\text{RDC}_{\text{monopoles} \times 10} - \text{RDC}_{\text{ref}}|$ along the bolometers, which is very stably distributed around zero. I believe that the uncertainty estimation is not relevant for this test and therefore I do not show it. Nevertheless, for full disclosure, I reported the same figure **with** the uncertainty bars in Fig.D.5 of Appendix.D.

IV.3.4 Beam effect

There is another systematic effect which I planned to check for, as well as started the implementation of a potential correction, but which I unfortunately did not have time to complete: the beam differences effect. Indeed, as was presented in subsection II.1.2.b), not only the various bolometers display significant beam shape discrepancies at all frequencies, and also along each frequency band (see Figs.II.6 and II.7), but the average beam size also varies significantly from one frequency band to the other (cf. Table.II.1). These beam differences are directly *imprinted* into the timeline data, and therefore into the SRoll2.1 input HPRs, which in turn could impact the quality of the RDCs determination in SRoll2.1 by introducing strong decorrelation.

The beam differences may also be the cause of the "bump" in the power spectrum of the $^{13}\text{CO } J = 1 \rightarrow 0$ map, as was seen in Fig.IV.27 and will be further discussed in subsection V.2.4.

As previously mentioned in subsection III.2.1.a), in order to mitigate this effect, the input template maps of the various foreground emissions were taken in SRoll2 at a low resolution of $N_{\text{side}} = 128$. At this resolution, the angular size of the pixels is $27''.5$ (cf. Table.A.1), which is way more than any of the HFI bolometers' beam size (almost 3 times bigger than the biggest bolometer beam size of $9''.65$ for the 100GHz band). This way, any effect of the beam differences is blurred since their scale is subpixel. However, the downside of this solution is the degraded quality of the input foreground templates' resolution. Increasing the foreground templates' resolution would improve the overall RDCs determination, improve the separation of the spatially correlated foregrounds (e.g. separation of dust and CO, as well as separation of ^{12}CO and ^{13}CO), and improve the quality of the realignment which is computed using the input template maps. Yet, increasing the foreground templates' resolution requires to correct the effect of the beam differences in the data.

A potential first solution to correct this effect is to build an estimate from the simulations, and remove this estimate from the data. To this end, one could produce 2 types of TOI simulations, following the HFI simulation pipeline introduced in Section III.3, but **without** adding the noise realization: one using the full bolometers scanning beam (as shown in Fig.III.15), and a second one using a *Gaussian beam* of same FWHM value for all bolometers. From these 2 types of TOIs, the corresponding HPRs can be produced, from which can in turn be computed an HPR of the *scanning beam effect correction*, which could then be determined as follows:

$$g_b M_{b,r,p}^{\text{beam corr}} = g_b M_{b,r,p}^{\text{gauss}} - g_b M_{b,r,p}^{\text{scan}},$$

where $g_b M_{b,r,p}^{\text{gauss}}$ is the simulation HPR computed using a Gaussian beam, $g_b M_{b,r,p}^{\text{scan}}$ is the simulation HPR computed using the corresponding scanning beam, and both types of HPR are computed *without noise*.

The *scanning beam effect correction* HPRs computed from the above equation therefore correspond to an HPR evaluating the effect specifically induced by the scanning beam compared to a reference Gaussian beam. Adding these correction HPRs to each corresponding input data HPRs of SRoll2.1 would be a way to bring the input signal of all bolometers to the same reference Gaussian beam, and therefore to correct the effect of the beam differences.

Unfortunately, I did not have time to finish implementing this method, let alone characterize its results, but I think that it could be a first step to correcting the beam difference effect, allowing a generalization of using the SRoll algorithm on multiple frequency bands at the same time. This would then allow the increase of the input foreground templates' resolution, which I believe to be the most significant way to improve the SRoll2.1 method. I also expect an improvement of the templates resolution to be essential to make an iterative solution on SRoll possible, as presented in Section IV.4.

IV.3.5 CO correlated molecular emission in the 143GHz band

I will discuss one last potential source of bias in my method, which was already mentioned before: the effect of the presence of molecular emission in the 143GHz band, spatially correlated to the ^{12}CO and ^{13}CO signals. First, I list the various molecular species which display line emission in the 143GHz band and are expected to be formed and excited in the same type of physical conditions as the CO. Then, I discuss the ways in which such emissions may impact the SRoll2.1 RDCs determination as well as the ^{12}CO and ^{13}CO maps computation defined in Section IV.2.

a) Origin & nature

While the significance and detectability of molecular emission in the 143GHz frequency band is everything *but* obvious, the presence of molecular species emitting spectral lines at frequencies within this band is a certainty. Not only the existence of molecular spectral lines at such frequencies has been known and classified for decades, with more than 630 such spectral lines already listed in 1968 as can be seen from pages 502 to 516 of Cord et al. (1968), but some of these have also been observed in dense cloud regions.

In Lang (1980), they listed 19 spectral lines from molecular species at frequencies within the HFI 143GHz band, all observed in dense cloud regions (mainly observed in the Orion A region). I reported the molecular species at the origin of these spectral lines, as well as their emission frequencies, in Table IV.4. While the 3 lines of highest intensity observed in Orion A displayed intensities from $\sim 50\%$ to $\sim 66\%$ of the observed ^{13}CO intensity from the same region, it should be noted that all 19 spectral lines are expected to be spatially correlated (at least partially) to the CO emission in the bolometers' data from the 143GHz band. The summed intensity over these 19 lines is then probably indiscriminately considered as *one and the same* by SRoll2 when trying to fit RDCs for the ^{12}CO and ^{13}CO emissions for the bolometers at the 143GHz band. As discussed below, given that these lines are distributed along the 143GHz band spectral transmission, this most probably would result in a low distribution *distinctness* of the ^{12}CO and ^{13}CO RDCs along the 143GHz band, but an increased non-zero *true* values of the averages $\overline{L^{12}\text{CO}}^{\mathcal{B}_{143}}$ and $\overline{L^{13}\text{CO}}^{\mathcal{B}_{143}}$. This is illustrated in Fig. IV.36, where I reported all 19 spectral lines over the bolometers' bandpasses along the 143GHz frequency band.

b) Impact on the SRoll2.1 RDCs and CO maps

The various molecular emissions present in the 143GHz, if sufficiently bright and sufficiently spatially correlated to the CO emission, would be detected by the SRoll2.1 ^{12}CO and ^{13}CO RDCs along the 143GHz band. More precisely, it can be *imprinted* into 2 characteristics of the ^{12}CO and ^{13}CO RDC distributions along the 143GHz band, each one leading to a distinct impact on the maps produced by my separation defined in Section IV.2. Thus, a detection of molecular emissions from the 143GHz band can read as follows:

Table IV.4: List of molecular spectral lines observed in dense cloud regions, at frequencies within the HFI 143GHz band. With the exception of the SO spectral line observed in NGC 2024, as well as the OCS and second SiO lines observed in Sagittarius B2, all other lines were observed in the Orion A region. Taken from [Lang \(1980\)](#).

These spectral lines are also compared to the bolometers' bandpasses along the 143GHz band in Fig. IV.36.

Molecule	Frequency [GHz]
SiO	129.3613; 130.2684
SO	138.1786
H ₂ CO	140.839529; 145.602971; 150.498359
DCN	144.826810; 144.826841
CH ₃ OH	145.09375; 145.09747; 145.10323; 145.12441 145.12637; 145.13188; 145.13346; 169.33534
OCS	145.94679
CS	146.96916
H ₂ S	168.762762

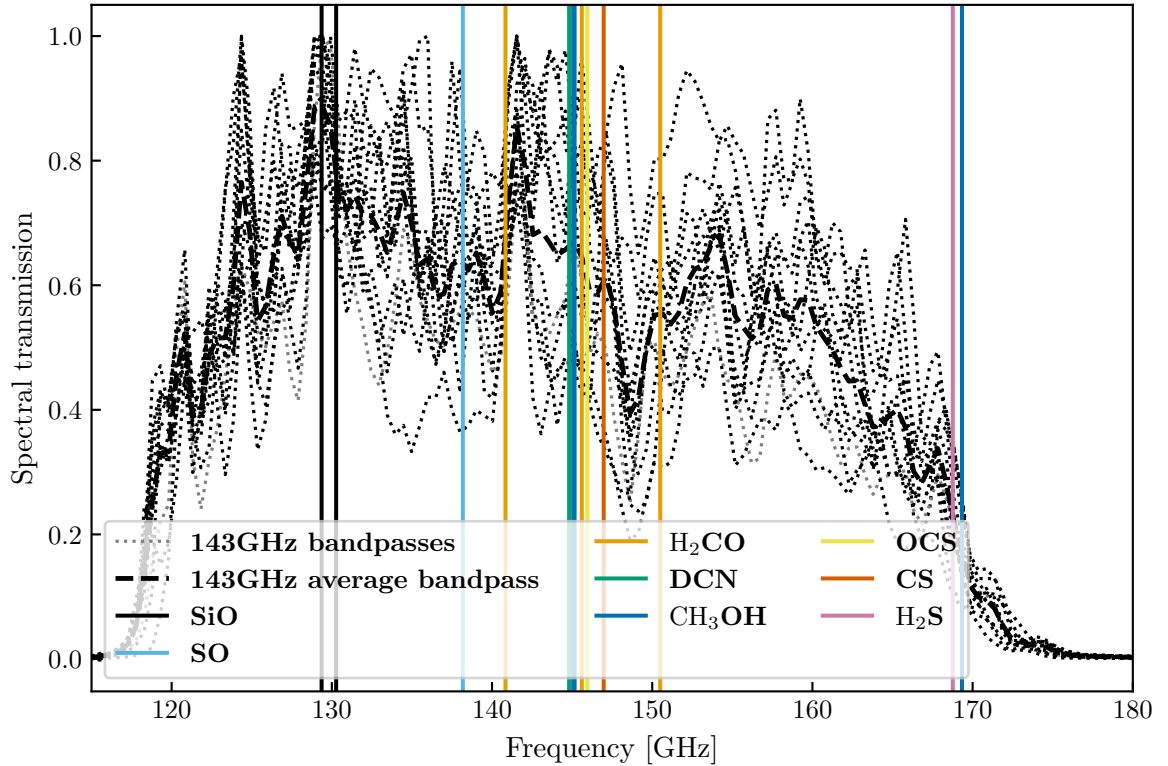


Fig. IV.36 Distribution of all 19 spectral lines (all **vertical lines**) listed in Table IV.4 along the HFI 143GHz frequency bandpasses. Each individual bolometers' bandpass are plotted in **dotted lines**, while the average spectral transmission along the 143GHz band is plotted as the **dashed line**.

1. A non-zero *true* value of the average ^{12}CO or ^{13}CO RDCs along the 143GHz band, i.e. $\overline{L^{12}\text{CO}}^{\mathcal{B}_{143}} > 0$ or $\overline{L^{13}\text{CO}}^{\mathcal{B}_{143}} > 0$. I call "*true* value" the physical value of either $\overline{L^{12}\text{CO}}^{\mathcal{B}_{143}}$ or $\overline{L^{13}\text{CO}}^{\mathcal{B}_{143}}$, which we can **not** access since SRoll2.1 is based on the hypothesis that these 2 averages are null (see Eq.IV.4),
2. A significant distinctness of the ^{12}CO or ^{13}CO RDC distributions along the 143GHz band, i.e. $\left. \frac{\sigma_{\delta L^{12}\text{CO}}}{\Delta_{\delta L^{12}\text{CO}}} \right|_{143} \gg 1$ or $\left. \frac{\sigma_{\delta L^{13}\text{CO}}}{\Delta_{\delta L^{13}\text{CO}}} \right|_{143} \gg 1$.

These 2 features are almost independent, and therefore one can appear without the other. I will discuss below how each of them is spawned, and what would be the consequences on the produced maps.

Non-zero averages' *true* value The *true* values of $\overline{L^{12}\text{CO}}^{\mathcal{B}_{143}}$ and $\overline{L^{13}\text{CO}}^{\mathcal{B}_{143}}$ are directly linked to the intensity of the molecular emissions in the 143GHz band **and** their level of spatial correlation respectively with the ^{12}CO and ^{13}CO input templates (which is probably higher with the ^{13}CO than with the ^{12}CO). The higher the intensity of the molecular emissions in the 143GHz band, the higher the *true* values of $\overline{L^{12}\text{CO}}^{\mathcal{B}_{143}}$ and $\overline{L^{13}\text{CO}}^{\mathcal{B}_{143}}$, and vice versa. Similarly, the more spatially correlated these molecular emissions are with the ^{12}CO and ^{13}CO input templates, the higher the *true* values of respectively $\overline{L^{12}\text{CO}}^{\mathcal{B}_{143}}$ and $\overline{L^{13}\text{CO}}^{\mathcal{B}_{143}}$, and vice versa.

As mentioned above, even if the *true* values of $\overline{L^{12}\text{CO}}^{\mathcal{B}_{143}}$ or $\overline{L^{13}\text{CO}}^{\mathcal{B}_{143}}$ are greater than zero, these values can *not* be accessed by SRoll2.1 since they are used to define the ^{12}CO and ^{13}CO RDCs closure conditions defined by Eq.IV.3. Therefore, since SRoll2.1 *forces* the averages $\overline{L^{12}\text{CO}}^{\mathcal{B}_{143}}$ and $\overline{L^{13}\text{CO}}^{\mathcal{B}_{143}}$ to converge towards zero, the immediate consequence of their non-zero *true* value would be an underestimation of the averages $\overline{L^{12}\text{CO}}^{\mathcal{B}_{\text{CO}}}$ and $\overline{L^{13}\text{CO}}^{\mathcal{B}_{\text{CO}}}$ determined by SRoll2.1 (see Eq.IV.5). Thankfully, given the way I compute the new $M^{12\text{CO}}$ and $M^{13\text{CO}}$ maps, I do not actually need to know the value of the averages $\overline{L^{12}\text{CO}}^{\mathcal{B}_{\text{CO}}}$ and $\overline{L^{13}\text{CO}}^{\mathcal{B}_{\text{CO}}}$, but only those of $\delta L^{12\text{CO}}^{\mathcal{B}_{\text{CO}}}$ and $\delta L^{13\text{CO}}^{\mathcal{B}_{\text{CO}}}$, respectively equal to $\overline{L^{12}\text{CO}}^{\mathcal{B}_{\text{CO}}} - \overline{L^{12}\text{CO}}^{\mathcal{B}_{143}}$ and $\overline{L^{13}\text{CO}}^{\mathcal{B}_{\text{CO}}} - \overline{L^{13}\text{CO}}^{\mathcal{B}_{143}}$ (see Eq.IV.5). Indeed, let's consider an alteration of the ^{12}CO and ^{13}CO RDCs corresponding to a non-zero *true* values of $\overline{L^{12}\text{CO}}^{\mathcal{B}_{143}}$ and $\overline{L^{13}\text{CO}}^{\mathcal{B}_{143}}$ before computing my maps. This is equivalent to applying the following transformation:

$$\forall b \in \mathfrak{B}, \mathcal{L}_b^{12\text{CO}} = L_b^{12\text{CO}} + K_{12} \text{ and } \mathcal{L}_b^{13\text{CO}} = L_b^{13\text{CO}} + K_{13}, \quad (\text{IV.13})$$

where K_{12} and K_{13} are constants of any value and correspond respectively to the *true* values of $\overline{L^{12}\text{CO}}^{\mathcal{B}_{143}}$ and $\overline{L^{13}\text{CO}}^{\mathcal{B}_{143}}$. If I was now to use these altered response coefficients to build my new maps, the data model of Eq.IV.8 would become:

$$\begin{aligned} \tilde{S}_{b,p}^{\text{out}} &= I_p^{\text{other}} + \rho_b \cos(2\phi_{b,p}) Q_p^{\text{other}} + \rho_b \sin(2\phi_{b,p}) U_p^{\text{other}} \\ &\quad + \mathcal{L}_b^{12\text{CO}} M_p^{12\text{CO}} + \mathcal{L}_b^{13\text{CO}} M_p^{13\text{CO}} + n_{b,p} \\ &= \left(I_p^{\text{other}} + K_{12} M_p^{12\text{CO}} + K_{13} M_p^{13\text{CO}} \right) + \rho_b \cos(2\phi_{b,p}) Q_p^{\text{other}} \\ &\quad + \rho_b \sin(2\phi_{b,p}) U_p^{\text{other}} + L_b^{12\text{CO}} M_p^{12\text{CO}} + L_b^{13\text{CO}} M_p^{13\text{CO}} + n_{b,p}. \end{aligned} \quad (\text{IV.14})$$

As we can see in the above equation, since the constants K_{12} and K_{13} do not depend on b , the above data model of Eq.IV.14 with altered averages is equivalent to the data model

of Eq. IV.8 but where the map $K_{12}M_p^{12CO} + K_{13}M_p^{13CO}$ has been added to the remainder intensity map I_p^{other} , not impacting in any way the values of the maps M^{12CO} and M^{13CO} . Consequently, we can conclude that the value of the averages $\overline{L^{12CO}}^{B_{CO}}$ and $\overline{L^{13CO}}^{B_{CO}}$ do not impact the determination of the maps M^{12CO} and M^{13CO} , but only affect the intensity component of the remainder maps, i.e. they only affect I_p^{other} by imprinting it with a level of the maps M^{12CO} and M^{13CO} corresponding to the *true* values of, respectively, $\overline{L^{12CO}}^{B_{143}}$ and $\overline{L^{13CO}}^{B_{143}}$.

This result can be tested by *artificially* modifying the ^{12}CO and ^{13}CO RDCs according to Eq. IV.13 before computing my maps. For ease of computation, I carried out this test on maps built from the 100GHz bolometers only, but still using RDCs resulting from a standard SRoll2.1 100-143GHz run. I used the following values to compute the RDCs' modification of Eq. IV.13: $K_{12} = 100 \times \overline{L^{12CO}}^{B_{CO}}$ and $K_{13} = 100 \times \overline{L^{13CO}}^{B_{CO}}$. I plotted the result in Fig. IV.37, where I displayed the I^{other} , ^{12}CO , and ^{13}CO difference maps between the ones resulting from the run using altered RDCs and the ones from the *unaltered* RDCs, which can be written:

$$\begin{aligned} M_p^{12CO, diff} &= M_p^{12CO, alt} - M_p^{12CO, ref}, \\ M_p^{13CO, diff} &= M_p^{13CO, alt} - M_p^{13CO, ref}, \\ I_p^{other, diff} &= I_p^{other, alt} - I_p^{other, ref}, \end{aligned} \quad (\text{IV.15})$$

where $M^{12CO, alt}$ is the ^{12}CO map computed from the altered ^{12}CO RDCs as defined in Eq. IV.13, $M^{12CO, ref}$ is the ^{12}CO map computed from the *unaltered* RDCs, and the same notations are used for the ^{13}CO and I^{other} maps. Note that, in this test, I *artificially* altered the values of the RDCs without changing the actual content of the frequency maps from the outputs of SRoll2.1, i.e. $\tilde{S}_{b,p}^{out}$ is **not** modified in any way. Therefore, using the data model of the maps computation in each case, given by Eq. IV.8 for the reference case and by Eq. IV.14 for the altered case, I get the following relation for the present test:

$$\begin{aligned} I_p^{other, ref} &= I_p^{other, alt} + K_{12}M_p^{12CO} + K_{13}M_p^{13CO} \\ \Leftrightarrow I_p^{other, diff} &= I_p^{other, alt} - I_p^{other, ref} = -(K_{12}M_p^{12CO} + K_{13}M_p^{13CO}). \end{aligned}$$

To confirm the validity of the relation above, I also plotted the map of the sum $I^{other, diff} + K_{12}M^{12CO} + K_{13}M^{13CO}$. As can be seen in Fig. IV.37, the interpretation of the effect of an RDC average modification, as defined by Eq. IV.13, is fully confirmed by the difference maps $M^{12CO, diff}$ and $M^{13CO, diff}$, as well as the sum map $I^{other, diff} + K_{12}M^{12CO} + K_{13}M^{13CO}$, all being compatible with the **null map**. Also note that the values of K_{12} and K_{13} taken for my test are 100 times the default averages values $\overline{L^{12CO}}^{B_{CO}}$ and $\overline{L^{13CO}}^{B_{CO}}$, which leads to a noise level in the sum map $I^{other, diff} + K_{12}M^{12CO} + K_{13}M^{13CO}$ to be equal to 100 times that of the sum map $M^{12CO} + M^{13CO}$.

Furthermore, as discussed in subsection IV.1.4.c), the SRoll2.1 ^{12}CO and ^{13}CO RDC averages along the 100GHz band are slightly overestimated compared to the ground-based measurement, with $\overline{L^{12CO}}^{B_{100}} = 14.8 \pm 0.10 \mu\text{K}_{CMB} \cdot \text{K}^{-1} \cdot \text{km}^{-1} \cdot \text{s}$ against a ground-based value of $14.2 \mu\text{K}_{CMB} \cdot \text{K}^{-1} \cdot \text{km}^{-1} \cdot \text{s}$, and $\overline{L^{13CO}}^{B_{100}} = 24.3 \pm 0.37 \mu\text{K}_{CMB} \cdot \text{K}^{-1} \cdot \text{km}^{-1} \cdot \text{s}$ against a ground-based value of $16.2 \mu\text{K}_{CMB} \cdot \text{K}^{-1} \cdot \text{km}^{-1} \cdot \text{s}$ (cf. Planck Collaboration XIII (2014)). If the ground-based measurements are correct, this would suggest that the *mean intensity* of the molecular emission in the 143GHz band is negligible compared to my uncertainty and errors in the 100-143GHz SRoll2.1 run.

However, even after correcting for the recalibration computed in subsection IV.2.2, the

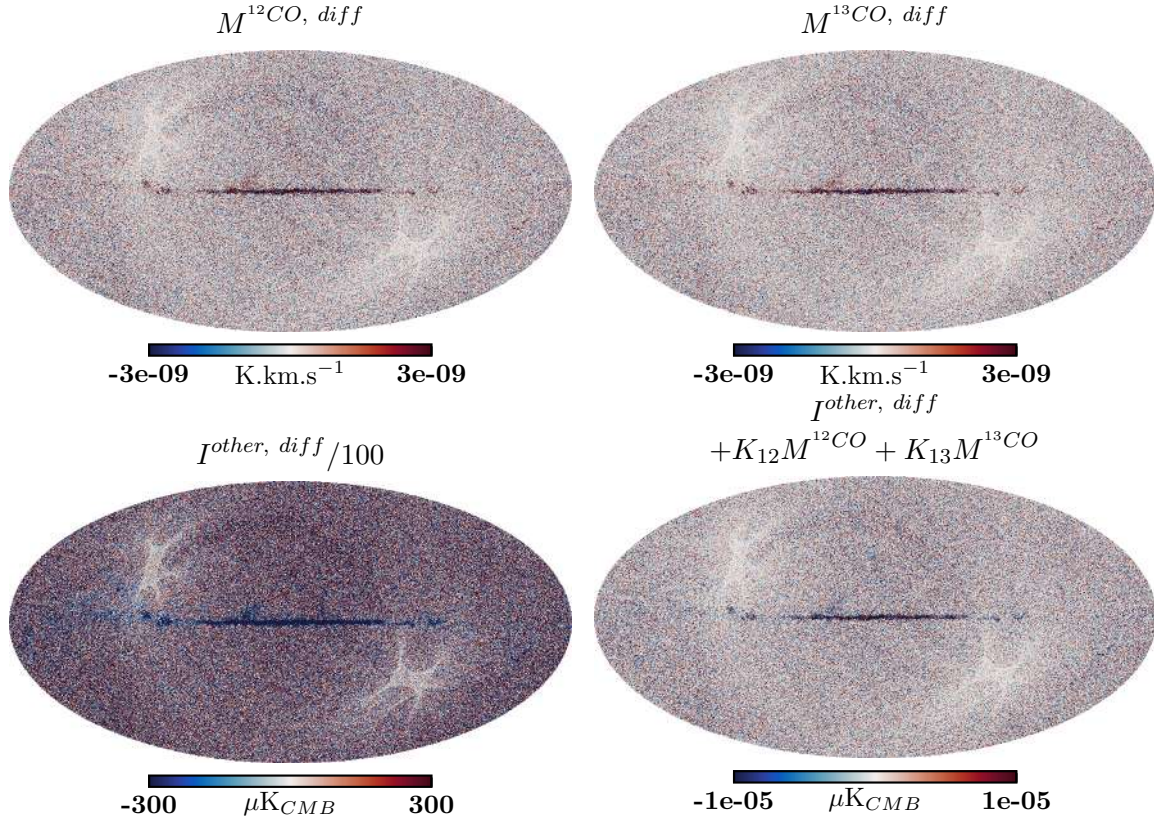


Fig. IV.37 Difference maps corresponding to the effect of altering the ^{12}CO and ^{13}CO RDCs according to Eq.IV.13, using alteration parameters equal to $K_{12} = 100 \times \overline{L^{12}\text{CO}}^{\mathcal{B}_{\text{CO}}}$ and $K_{13} = 100 \times \overline{L^{13}\text{CO}}^{\mathcal{B}_{\text{CO}}}$. Maps were computed using only bolometers from the 100GHz band. **Top left:** ^{12}CO difference map (see Eq.IV.15); **Top right:** ^{13}CO difference map (see Eq.IV.15); **Bottom left:** I^{other} difference map divided by 100 (see Eq.IV.15); **Bottom right:** sum map $I^{\text{other}, \text{diff}} + K_{12}M^{12}\text{CO} + K_{13}M^{13}\text{CO}$.

SRoll2.1 ^{12}CO and ^{13}CO RDC averages along the 217GHz band are still underestimated compared to the ground-based measurement, with $\overline{L^{12}\text{CO}}^{\mathcal{B}_{217}} = 32.7 \pm 0.62 \mu\text{K}_{\text{CMB}} \cdot \text{K}^{-1} \cdot \text{km}^{-1} \cdot \text{s}$ against a ground-based value of $45.0 \mu\text{K}_{\text{CMB}} \cdot \text{K}^{-1} \cdot \text{km}^{-1} \cdot \text{s}$, and $\overline{L^{13}\text{CO}}^{\mathcal{B}_{217}} = 28 \pm 3.7 \mu\text{K}_{\text{CMB}} \cdot \text{K}^{-1} \cdot \text{km}^{-1} \cdot \text{s}$ against a ground-based value of $36.3 \mu\text{K}_{\text{CMB}} \cdot \text{K}^{-1} \cdot \text{km}^{-1} \cdot \text{s}$. If the ground-based measurements are correct, such an underestimation of the averages $\overline{L^{12}\text{CO}}^{\mathcal{B}_{217}}$ and $\overline{L^{13}\text{CO}}^{\mathcal{B}_{217}}$ could suggest a detection of the molecular emission from the 143GHz band, which would then be imprinted in the I^{other} map as discussed above. Nevertheless, their summed intensity cannot reliably be estimated from the amount of the underestimation, since this recalibration is computed on the 217-143GHz SRoll2.1 maps impacted by the effect discussed in Section V.2 (see also subsection V.1.4), which means that the recalibration factor used may still be underestimated.

^{12}CO and ^{13}CO RDC distributions *distinctness* along the 143GHz band The distinctness of the ^{12}CO and ^{13}CO RDC distributions along the 143GHz band depends on the response differences between bolometers to the **summed** molecular emissions present in the frequency band. These response differences mainly depend on the bolometers transmission differences at the frequencies of the molecular lines present in the 143GHz band (i.e. the

differences due to the transmission "wiggles" at each present molecular line, as seen in Figs.II.4 and IV.36). However, since we expect multiple molecular lines to be present in the band, as presented in previous subsection IV.3.5.a), the ^{12}CO and ^{13}CO RDCs along the 143GHz band correspond to the bolometers' transmission differences **summed over all molecular lines** present in the band (see Fig.IV.36). For this reason, the distinctness of the ^{12}CO and ^{13}CO RDCs along the 143GHz band is expected to be reduced.

Additionally, this distributions' distinctness also depends on the intensity of the molecular lines in the 143GHz band, since a greater intensity of the lines would also increase the standard deviation of the ^{12}CO and ^{13}CO RDCs along the 143GHz band. However, because the RDCs correspond to a **sum** of response differences, it is entirely possible for the intensity of the lines to be sufficient to induce a distinctively non-zero averages' *true* value, while still displaying *indistinct* ^{12}CO and ^{13}CO RDC distributions, i.e. displaying ratio values

$$\left. \frac{\sigma_{\delta L^{12}\text{CO}}}{\Delta_{\delta L^{12}\text{CO}}} \right|_{143} \lesssim 1 \text{ and } \left. \frac{\sigma_{\delta L^{13}\text{CO}}}{\Delta_{\delta L^{13}\text{CO}}} \right|_{143} \lesssim 1.$$

Using the information provided by these ratios along the 143GHz band, i.e. $\left. \frac{\sigma_{\delta L^{12}\text{CO}}}{\Delta_{\delta L^{12}\text{CO}}} \right|_{143} = 0.755$ and $\left. \frac{\sigma_{\delta L^{13}\text{CO}}}{\Delta_{\delta L^{13}\text{CO}}} \right|_{143} = 1.06$ as estimated from the 100-143GHz SRoll2.1 run, we can see that the ^{12}CO and ^{13}CO foreground RDC distributions are compatible with a negligibility of the molecular emissions present in the 143GHz band. Indeed, this ratio is, along the 143GHz band, of the order of $\lesssim 1$ for both the ^{12}CO and ^{13}CO foregrounds, meaning that the spread of the RDC distributions in these cases are of the order of the level of uncertainty (see Fig.IV.18). However, the value of this ratio being still slightly greater than 1 for the ^{13}CO could be caused by the molecular emission at 143GHz, though it is too close to 1 (only 1.06) to allow any firm conclusion.

If this distinctness is high enough, one could build a map of the molecular emissions from the 143GHz band using the same separation method as introduced in Section IV.2, but by using only one set of CO RDCs (either the ^{12}CO RDCs or the ^{13}CO RDCs), and using only the bolometers from the 143GHz band. Of course, to carry out such a computation, one should use the RDCs determined by SRoll2.1 for the 143GHz band bolometers and **not** set them to zero. However, when attempting such a computation with my ^{13}CO SRoll2.1 RDCs along the 143GHz band, the computation fails and all pixels display non-invertible matrices, even when lowering the output map resolution down to $N_{\text{side}} = 128$.

IV.4 Attempts at an iterative solution on SRoll2.1

The original design of the SRoll2.1 version first included an iterative scheme to improve the ^{12}CO and ^{13}CO input templates. Then, the idea was to build final ^{12}CO and ^{13}CO maps from a SRoll2.1 run using the templates towards which the iterations would have converged. However, when attempting said iterations, I observed an intensity drift of the templates. Because of the highly complex nature of the method, it was complicated to isolate the potential sources of this effect to identify their main contributors. In order to simplify this exploration while limiting my consumption of computation hours, I built a "toy model" of the SRoll2.1 solution to test some potential sources of the aforementioned intensity drift.

In this section, I will present the philosophy and structure of the intended iterative scheme on SRoll2.1. Then, I will summarize the main observation from the first attempts at this iterative method: the maps' intensity drift. Finally, I will present my toy model used to investigate the potential sources of the intensity drift, and the main results it yielded.

In the end, given the numerous potential sources for the intensity drift, and the amount

of investigative work needed to attempt a correction for each of them, I decided to limit the SRoll2.1 version to a single iteration, leading to the method presented in Section IV.1.

IV.4.1 Philosophy and method of iteration

The idea of the iterative approach to SRoll2.1 was inspired by the test from the work of Delouis et al. (2019) discussed in subsection III.2.4, where SRoll2 is applied on a 2-components model sky but starting from *degraded* input templates. At each step, new maps of the 2 components comprising the model sky are computed from the RDCs determined by SRoll2 and used as new input templates for the next SRoll2 run. The templates were observed to converge towards stable template maps.

Similarly, the SRoll2.1 iterative scheme was developed to apply the same idea of iterative construction of the templates, but applied on the full data signal, and by fitting only the ^{12}CO and ^{13}CO input templates. The general structure of the process was to run the SRoll2.1 algorithm (as it is presented in Section IV.1), and produce ^{12}CO and ^{13}CO maps using the separation method presented in Section IV.2. Then, these maps were to be used as ^{12}CO and ^{13}CO input templates for a new SRoll2.1 run, keeping all other parameters and inputs of this new run *unchanged* with respect to the previous one. This defined an iteration on SRoll2.1, and from any successive run, new maps could be produced to then be used as new input templates. The hypothesis, which remained to be verified, was that the ^{12}CO and ^{13}CO maps produced at each iteration should converge towards new and improved fullsky maps of the isotopologues' emissions.

I detailed the steps of the looping structure on SRoll2.1 in the schematic of Fig. IV.38. Indeed, a few additional precautions had to be taken to be able to use the produced maps as new input templates. The complete looping structure I devised unfolds as follow:

1. The first iteration, which I call **loop 0**, consists in what I have presented so far in Sections IV.1 and IV.2: use the G08 maps as ^{12}CO and ^{13}CO SRoll2.1 input templates, using the SRoll2.1 mask defined in subsection IV.1.2.b), and compute ^{12}CO and ^{13}CO fullsky maps from the SRoll2.1 run's outputs,
2. In order to use the ^{12}CO and ^{13}CO maps computed in the **loop 0** as new SRoll2.1 input templates for the next loop, called **loop 1**, I have to make them fit to be used as templates by:
 - Setting to **zero** all pixels in the map with a *negative* value. Since the CO emission is expressed in units of brightness temperature, it cannot be of negative value, meaning that negative-valued pixels from my maps are considered to be only noise, and therefore put to zero,
 - Setting to **zero** all pixels in the map at high Galactic latitude, i.e. all pixels verifying either $b > 35^\circ$ or $b < -35^\circ$. Since these high latitudes are expected to essentially be dominated by noise, they are put to zero,
 - Setting to **zero** all pixels displaying an integrated intensity lower than $< 1\text{K.km.s}^{-1}$ in the D22 ^{12}CO map. The objective is to ensure that any signal remaining in my map is *definitely* a CO signal and to avoid the risk of leaving any noise in the map which may be fitted by the next SRoll2.1 run as signal.

The mask corresponding to a typical signal selection as defined above is displayed in Fig. IV.38. Finally, the maps resulting from this signal selection are downgraded to the SRoll2.1 input resolution of $N_{\text{side}} = 128$, and they are ready to be used as input templates for the next SRoll2.1 run,

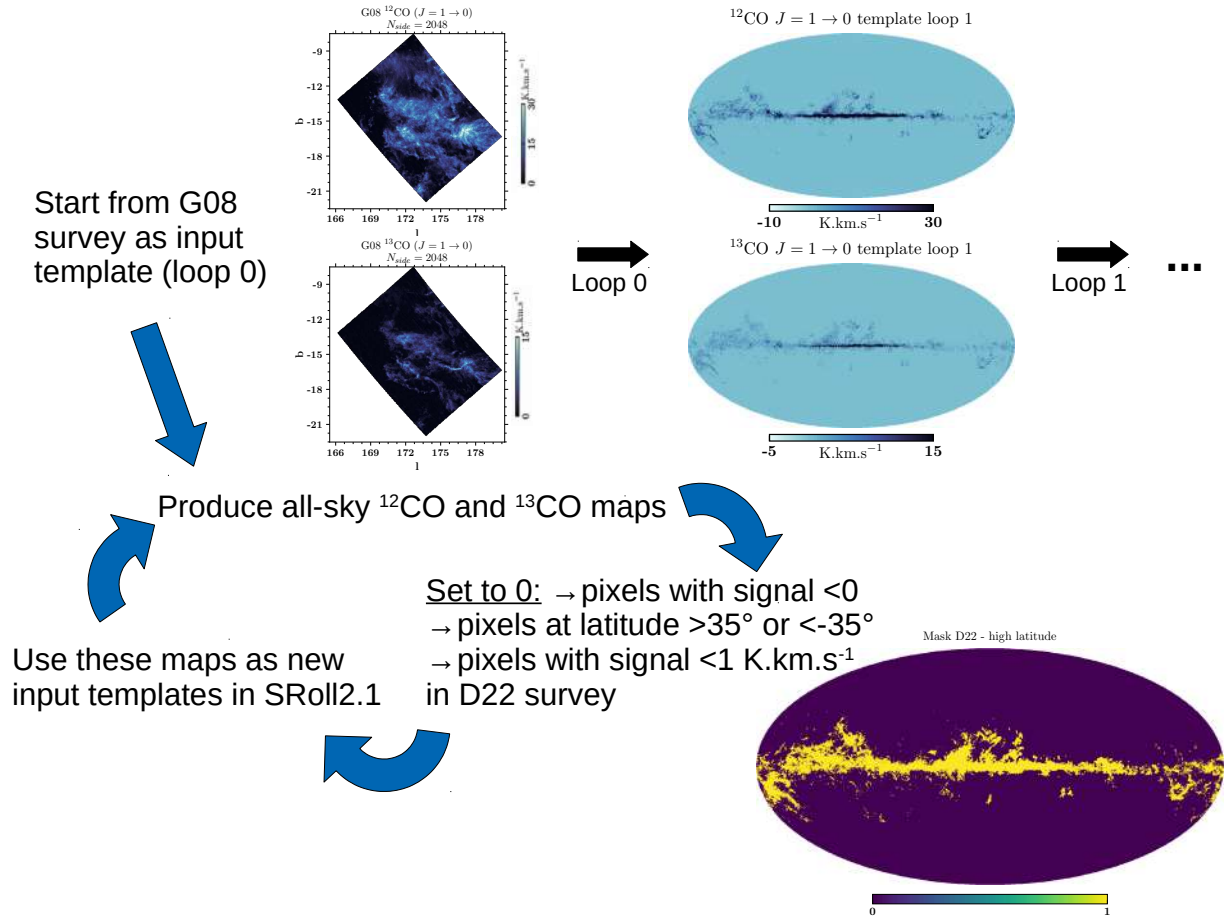


Fig. IV.38 Schematic of the iterative scheme on SROLL2.1, aiming to improve the ^{12}CO and ^{13}CO input templates at each iteration. The first loop (**loop 0**) is computed using the G08 maps as input templates. The signal from the new ^{12}CO and ^{13}CO maps built from a previous loop is selected by setting to zero any pixel not deemed certain to contain CO signal (negative-valued pixels, high latitude pixels with $b > 35^\circ$ or $b < -35^\circ$, pixels with intensity smaller than $< 1 \text{ K.km.s}^{-1}$ in the D22 map). The mask in the schematic corresponds to such a signal selection on the maps. The maps are then rescaled to a resolution of $N_{\text{side}} = 128$ before being used as new ^{12}CO and ^{13}CO input templates for a next SROLL2.1 run.

3. The last modification applied on the **loop 1**, and remaining for any successive loops, is that of the SROLL2.1 input *Galactic mask*. Indeed, starting from **loop 1** and ongoing for any successive loops, the new SROLL2.1 ^{12}CO and ^{13}CO input template are no longer the G08 maps, limited to the Taurus region, but *fullsky* maps of the 2 emissions. Therefore, the discussion presented in subsection IV.1.2.b) no longer holds, i.e. we no longer need to mask *all* CO emission except the Taurus region. Incidentally, the input Galactic mask used for *any loop of iteration number greater than 0* is the *Planck* legacy SROLL mask, as displayed in the left column of Fig. IV.6,
4. **Loop 1** can now be carried out by computing a SROLL2.1 run using the Galactic mask defined in the previous step, and using the ^{12}CO and ^{13}CO input templates defined by the step 2 above built from the results of **loop 0**. From the outputs of this new SROLL2.1 run, new ^{12}CO and ^{13}CO maps are computed and correspond to the results of **Loop 1**,

5. Any additional successive loop can be computed by applying the steps 2 and 4 on the results from its preceding loop.

The first results I obtained using the hereby defined iterative scheme on SRoll2.1 are presented in the next subsection.

IV.4.2 First results and intensity drift

I attempted the iterative scheme defined in the previous subsection on both the 100-143GHz and the 217-143GHz SRoll2.1 runs. In each case, I computed iterations up to the loop 3. The results are summarized in the 2 paragraphs below, for each frequency pairing. In both cases an intensity drift was observed instead of a straightforward convergence of the maps.

Iterations on the 100-143GHz run I mainly used 2 types of plots to assess the evolution of the ^{12}CO and ^{13}CO maps from one loop to another: first the correlation plots between each successive loop, which I display in Fig.IV.39 for the ^{12}CO and in Fig.IV.40 for the ^{13}CO , and the power spectra of the successively computed maps, plotted in Fig.IV.41.

Notably, the ^{12}CO map appears to converge towards a stable map, with a goodness-of-fit increasing between each successive loop, and with an affine fit factor converging towards 1. This can also be seen in the successive power spectra of the ^{12}CO maps, with small modifications between loops, yet visibly converging at high ℓ (from loop 2).

However, the ^{13}CO map is not stable in this iterative scheme. We can see in Fig.IV.40 that the affine fit factor is *always* greater than 1, around a value of ~ 1.27 , which means that the overall map intensity is multiplied by this factor ~ 1.27 at each loop. Also, the goodness-of-fit slightly degrades after each successive loop, but less significantly than the intensity drift. This is also seen in Fig.IV.41, where the power spectra of the ^{13}CO map from each successive loop is almost *unaltered* in shape with the exception of a multiplicative factor observed on the whole spectrum between successive loops, i.e. an intensity drift.

Iterations on the 217-143GHz run The first tests on the 217-143GHz run were computed without separating the ^{12}CO and ^{13}CO isotopologues. This means that SRoll2.1 was only provided with a ^{12}CO input template, fitted only ^{12}CO RDCs, and I computed only a ^{12}CO fullsky map. This is due to initial difficulties with the isotopologues' separation for the $J = 2 \rightarrow 1$ line. When I understood and addressed this issue, I had already observed the intensity drift on the ^{13}CO map from the 100-143GHz run, and checked the behaviour on the 217-143GHz run before implementing the corrected separation of the ^{13}CO . This is why the correlation plots between each successive loop and the power spectra of the successively computed maps, respectively plotted in Fig.IV.42 and Fig.IV.43, are only displayed for the ^{12}CO map.

The main result of iterating on the 217-143GHz run is again the observation of maps' intensity drift. We can see from Fig.IV.43 a dramatic increase of the goodness-of-fit between each loops, **but** also an affine fit factor systematically greater than 1 and with a fairly stable value of ~ 1.22 . This means that the spatial distribution of the signal in the map converges, but the overall map's calibration is multiplied at each loop by the aforementioned ~ 1.22 factor. This could be due to the non-separation of the ^{12}CO and ^{13}CO in this case, and if the separation was done, the maps might exhibit the same behaviour as in the 100-143GHz case, i.e. the convergence of the ^{12}CO map and the divergence of the ^{13}CO map. This is again observed in Fig.IV.42 where the power spectrum of the ^{12}CO map is very stable in shape from one loop to the other, but is multiplied by a constant factor at each loop, i.e. displays an intensity drift of the map.

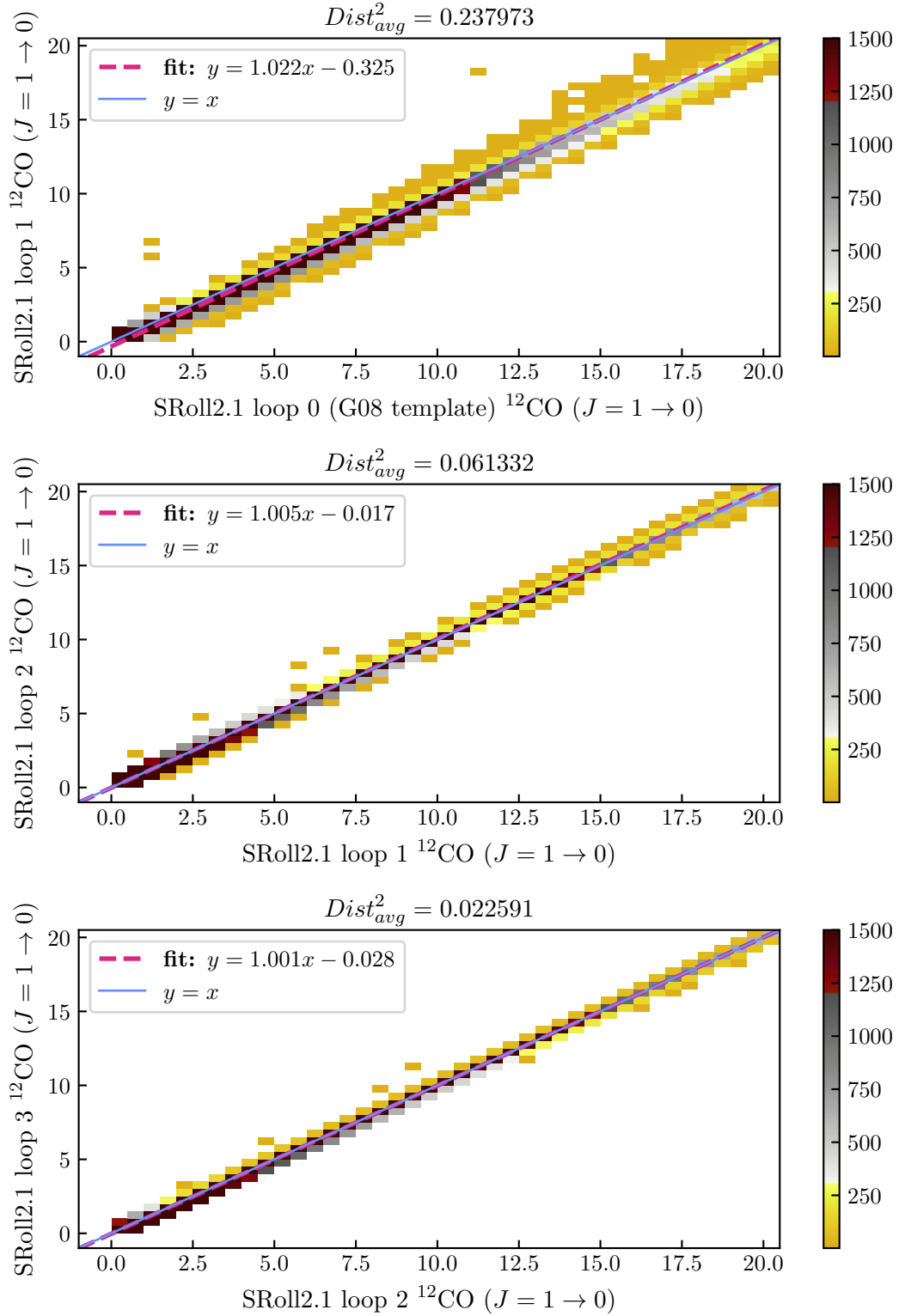


Fig. IV.39 Correlation plots between the ^{12}CO maps resulting from successive loops in the SROLL2.1 iterative scheme applied on the 100-143GHz run. All maps have been set to a Gaussian beam with $\text{FWHM} = 30'$. Plots computed using only pixels with a positive intensity value in both maps. $Dist_{avg}^2$ corresponds to the average of squared distances of points with respect to the fit. **Top:** loop 1 Vs loop 0; **Middle:** loop 2 Vs loop 1; **Bottom:** loop 3 Vs loop 2. **Red dashed line** is the best affine fit of the cloud of points; **Blue line** is the identity function.

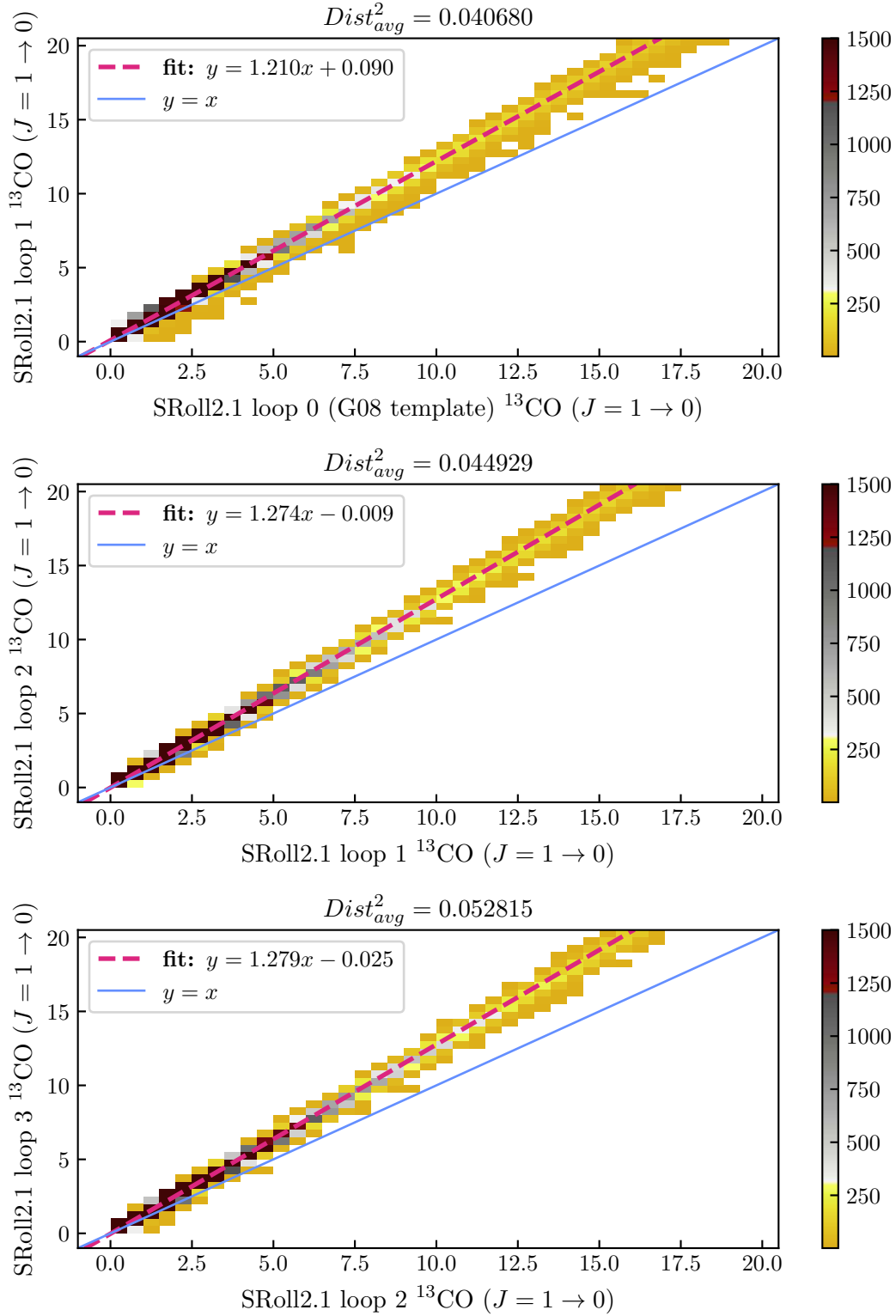


Fig. IV.40 Correlation plots between the ^{13}CO maps resulting from successive loops in the SRoll2.1 iterative scheme applied on the 100-143GHz run. All maps have been set to a Gaussian beam with FWHM = 30'. Plots computed using only pixels with a positive intensity value in both maps. $Dist_{avg}^2$ corresponds to the average of squared distances of points with respect to the fit. **Top:** loop 1 Vs loop 0; **Middle:** loop 2 Vs loop 1; **Bottom:** loop 3 Vs loop 2. **Red dashed line** is the best affine fit of the cloud of points; **Blue line** is the identity function.

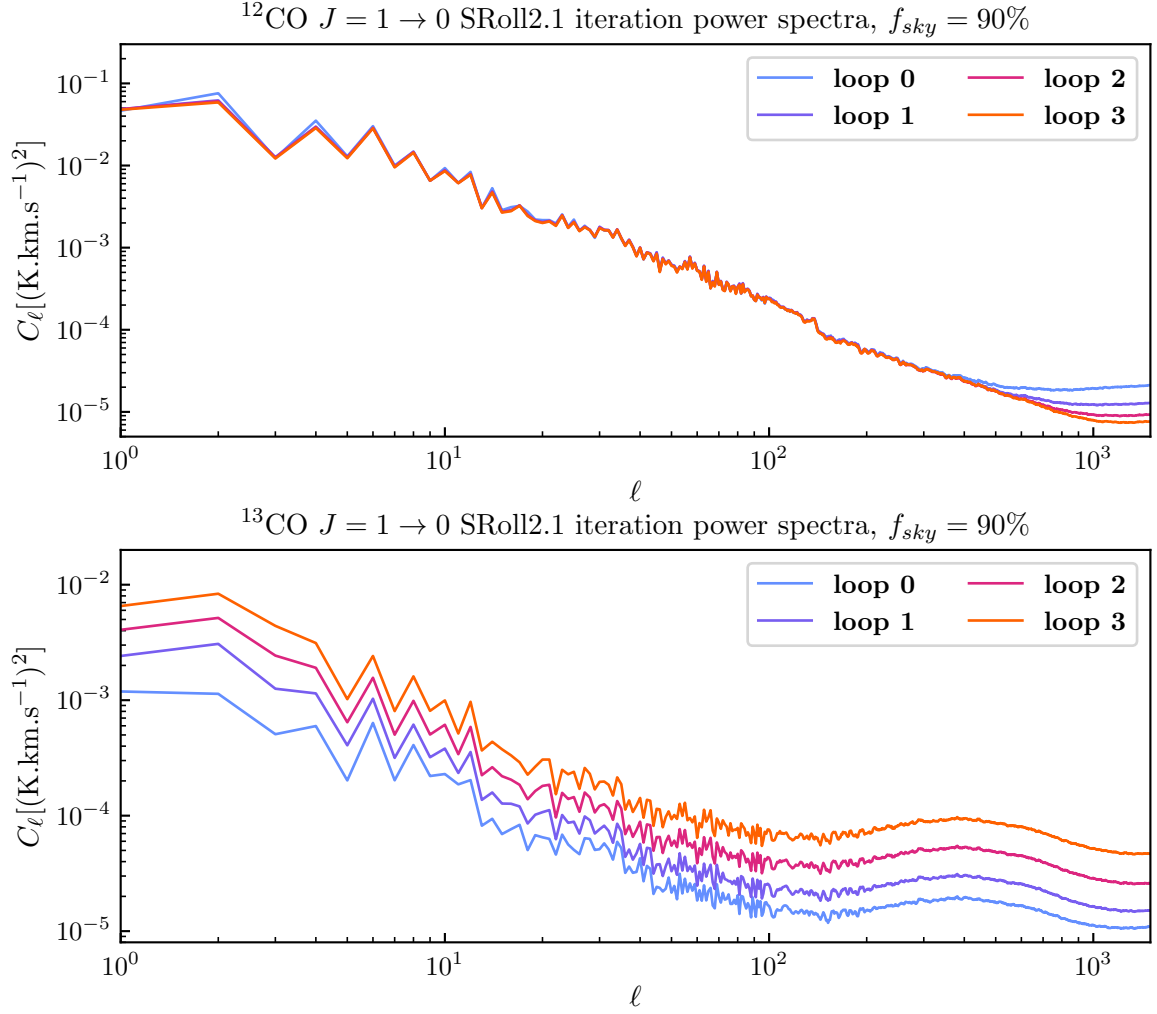


Fig. IV.41 Power spectra of ^{12}CO and ^{13}CO maps from successive loops in the SRoll2.1 iterative scheme applied on the 100-143GHz run, computed using a mask keeping a sky fraction $f_{\text{sky}} = 90\%$, hiding the galactic disk, and log-smoothed with a multipole fraction of $\ell_{\text{frac}} = 100$ (cf. Appendix.C). **Top:** ^{12}CO ; **Bottom:** ^{13}CO .

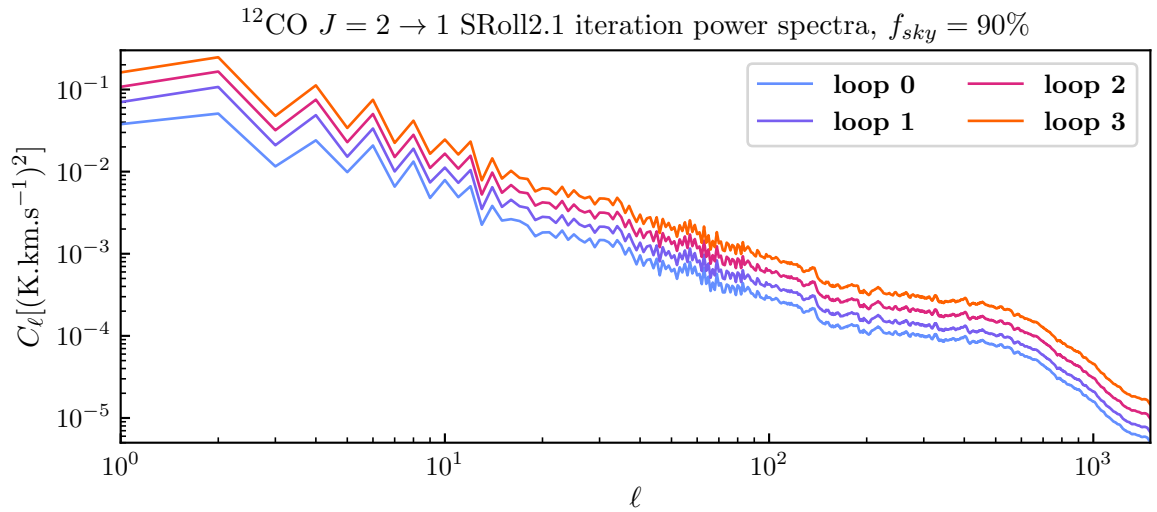


Fig. IV.42 Power spectra of ^{12}CO maps from successive loops in the SRoll2.1 iterative scheme applied on the 217-143GHz run, computed using a mask keeping a sky fraction $f_{\text{sky}} = 90\%$, hiding the galactic disk, and log-smoothed with a multipole fraction of $\ell_{\text{frac}} = 100$ (cf. Appendix.C).

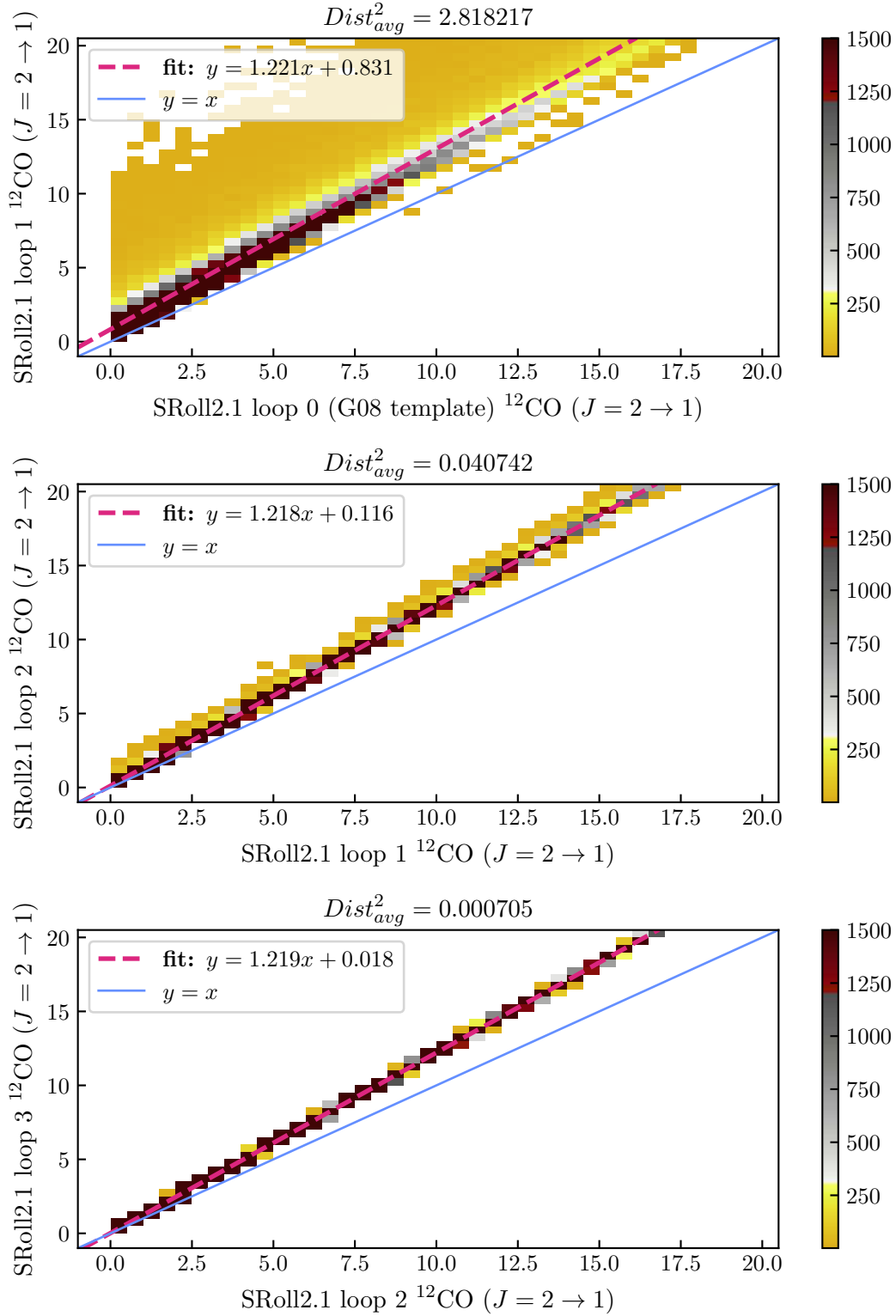


Fig. IV.43 Correlation plots between the ^{12}CO maps resulting from successive loops in the SRoll2.1 iterative scheme applied on the 217-143GHz run. All maps have been set to a Gaussian beam with FWHM = 30'. Plots computed using only pixels with a positive intensity value in both maps. $Dist_{avg}^2$ corresponds to the average of squared distances of points with respect to the fit. **Top:** loop 1 Vs loop 0; **Middle:** loop 2 Vs loop 1; **Bottom:** loop 3 Vs loop 2. **Red dashed line** is the best affine fit of the cloud of points; **Blue line** is the identity function.

I tried to understand this intensity drift behaviour, which led me to build a very simplified "toy model" of the SRoll2.1 iterative scheme, of which the structure and results are presented in the next subsection.

IV.4.3 Toy model exploration

In order to help isolate the potential sources of the map intensity drift effect, and allow their exploration without having to consume precious computational hours, I built a simplified model of both the SRoll2.1 iterative scheme, as well as the sky on which it is applied. I call it a "toy model" given its technical simplicity, allowing a cheap and simple exploration of the behaviour of the iterative scheme in a controlled environment.

I will first present the nature and definition of the model sky on which the toy model is applied, as well as the structure of the toy model computation. Then, I will present the main sources of the intensity drift I tested using the hereby defined toy model.

a) Toy model definition

The toy model is entirely built around maps instead of HPR data, drastically reducing the size of handled data. The idea is to build maps of a simple model sky seen by bolometers with different responses to each sky signal components, similar to the conditions of the *Planck* mission data. Then, from this model sky seen by multiple bolometers, simulate the fitting procedure of SRoll2.1, followed by the maps building defined in this work. I will first detail here the way I built the model sky for the toy model, then how I model the SRoll2.1 and maps building procedures on this model sky.

Toy model's sky emission The sky emission is modelled as a sum of 3 *unpolarized* intensity components made to model the 2 main foreground emissions separated in SRoll2.1: the thermal dust, made of 1 intensity map and a spatial SED variation map, and the CO emission. The 3 maps used to build the toy model's sky emission are then the following:

- For the CO component, I use the SRoll2.1 simulation template of the CO emission, i.e. the map of the PSM CO emission,
- For the dust component, I use the *Planck* 2018 release version of the HFI 857GHz frequency map, largely dominated by dust emission,
- For the dust SED spatial variation component, I use the SRoll2.1 DustVar input template map, as defined in subsection IV.1.1.b).

Since I will use the toy model to mimic a SRoll2.1 100-143GHz run, building the sky emission maps for each bolometer from the RDCs determined from such a run, I need to bring the dust intensity map to the intensity level of the dust in the 100GHz frequency band. This is done by recalibrating the *Planck* 857GHz map as follows:

$$M_p^{TM, dust} = \frac{I_{100}^{MBB}}{I_{857}^{MBB}} I_p^{857GHz},$$

where $M_p^{TM, dust}$ is the dust emission map used for the toy model, I_ν^{MBB} is taken as the dust MBB emission defined by Eq.II.3 (with dust temperature and spectral index as $T_{dust} = 19.5K$ and $\beta_{dust} = 1.55$, cf. [Planck Collaboration IV \(2020\)](#)), and I_p^{857GHz} is the *Planck* legacy (2018 release) HFI 857GHz frequency map. Note that, with the exception of the last test exploring the impact of the input templates' resolution, all maps of the components for the toy model

sky emission are taken at a resolution of $N_{side} = 128$, again reducing the size of handled data and reducing the computation time of the various tests.

From these 3 component maps, a sky emission map is constructed *for each bolometer* of the 100GHz and 143GHz bands by computing a linear combination of said components, defined as follow:

$$M_{b,p}^{TM, sky} = L_b^{TM, dust} M_p^{TM, dust} + L_b^{TM, DustVar} M_p^{TM, DustVar} + L_b^{TM, CO} M_p^{TM, CO}, \quad (IV.16)$$

where $M^{TM, DustVar}$ is the DustVar emission map used for the toy model, $M^{TM, CO}$ is the CO emission map used for the toy model, and $L_b^{TM, dust}$, $L_b^{TM, DustVar}$, and $L_b^{TM, CO}$ are the response coefficients to respectively the dust, DustVar, and CO components of the toy model's sky emission for the bolometer b , taken as:

$$\begin{aligned} L_b^{TM, dust} &= \delta L_b^{dust} + 1, \\ L_b^{TM, DustVar} &= \delta L_b^{DustVar}, \\ L_b^{TM, CO} &= \frac{\delta L_b^{CO}}{\delta L^{CO} B_{CO}}, \end{aligned}$$

where δL_b^{dust} , $\delta L_b^{DustVar}$, and δL_b^{CO} are the RDCs resulting from the SRoll2.1 100-143GHz data run, as presented in Appendix.D. The "+1" term in the dust coefficients is used to build a sky emission with a non-zero average dust emission in the bolometers from the 100GHz frequency band, though this value is of negligible effect of the determined CO maps as demonstrated in subsection IV.3.5. The normalization by the average RDC for the CO is computed to have unitless coefficients for all components. This normalization is necessary for the CO RDCs only, since the only RDCs determined by SRoll2.1 which are *not unitless* are those of the ^{12}CO and ^{13}CO .

Toy model's computation structure Now that I have a sky emission map for each of the 19 bolometers considered in the toy model (simulating a SRoll2.1 100-143GHz run), I can use them to simulate first a SRoll2.1 run, by fitting RDCs for the 3 sky components of the model sky for each bolometer, and then use them to realign the sky emission maps and construct a map of the CO component. To simulate a SRoll2.1 run on the sky emission maps from the bolometers, I compute a χ^2 minimization to determine the aforementioned RDCs, using the following data model:

$$\delta M_{b,p}^{TM, sky} = \delta L_b^{TM, dust} C_p^{TM, dust} + \delta L_b^{TM, DustVar} C_p^{TM, DustVar} + \delta L_b^{TM, CO} C_p^{TM, CO},$$

where $\delta M_b^{TM, sky}$ is the difference map between the sky emission map of bolometer b and the average sky emission map from all bolometers, i.e. $\delta M_{b,p}^{TM, sky} = M_{b,p}^{TM, sky} - \frac{1}{19} \sum_{b \in \mathfrak{B}} M_{b,p}^{TM, sky}$, $C^{TM, dust}$, $C^{TM, DustVar}$, and $C^{TM, CO}$ are template maps of respectively the dust, DustVar, and CO components of the sky emission model, which vary depending on the test considered, and $\delta L_b^{TM, dust}$, $\delta L_b^{TM, DustVar}$, and $\delta L_b^{TM, CO}$ are the RDCs to be fitted. Note that, similarly to what is done in SRoll2.1, all sky emission maps and template maps used to compute the minimization are masked using the same mask as defined for SRoll2.1, and noted M^{mask} . The χ^2 thereby defined to fit the toy model RDCs is defined as:

$$\begin{aligned} \chi^2 = \sum_b \sum_p M_p^{mask} & \left(\delta L_b^{TM, dust} C_p^{TM, dust} + \delta L_b^{TM, DustVar} C_p^{TM, DustVar} \right. \\ & \left. + \delta L_b^{TM, CO} C_p^{TM, CO} - \delta M_{b,p}^{TM, sky} \right)^2. \end{aligned}$$

Similarly to what is done in SRoll2.1, I also ensure the conditions $\sum_{b \in \mathcal{B}_{143}} \delta L_b^{TM, CO} = 0$, $\sum_{b \in \mathcal{B}_{CO}} \delta L_b^{TM, dust} = 0$, and $\sum_{b \in \mathcal{B}_{CO}} \delta L_b^{TM, DustVar} = 0$. From this determination of the toy model RDCs, I can compute the realignment for each bolometers' sky emission maps, which can be written:

$$M_{b,p}^{TM, realign} = M_{b,p}^{TM, sky} - \delta L_b^{TM, dust} C_p^{TM, dust} - \delta L_b^{TM, DustVar} C_p^{TM, DustVar}.$$

Finally, I can compute a map of the CO component by minimizing another χ^2 defined as:

$$\begin{aligned} \chi^2 = & \sum_{b \in \mathcal{B}_{CO}} \left(I_p^{TM, other} + \delta L_b^{TM, CO} \mathcal{M}_p^{TM, CO} - M_{b,p}^{TM, realign} \right)^2 \\ & + \sum_{b \in \mathcal{B}_{143}} \left(I_p^{TM, other} - M_{b,p}^{TM, realign} \right)^2, \end{aligned}$$

where $\mathcal{M}^{TM, CO}$ is the newly determined map of the CO component, and $I^{TM, other}$ is the fitted map of the remaining sky emission averaged over all bolometers and cleared of the CO component.

From the toy model hereby defined, I can simulate a simplified version of a SRoll2.1 100-143GHz run, followed by a subsequent CO map computation using its output realigned maps. Using this, I can now simulate the iteration scheme on SRoll2.1 as introduced above, by following the exact same steps as defined in subsection IV.4.1. Notably, the change of mask between loop 0 and loop 1, and the signal selection on the output CO map to make it usable as an input template for the next loop.

Using this toy model, I simulate the iteration scheme on SRoll2.1 to explore 3 different potential sources that may produce the CO maps' intensity drift observed in the data. In each of these 3 tests, I use the toy model to compute the simulation of 10 successive loops. Also, I display the results from these tests by presenting 3 types of plots:

- The residual CO maps of the loops 0 and 9 (i.e. the first and last CO maps' computations), computed as the difference map between the computed CO map at the considered loop and the original CO map of the sky emission model, i.e. $M_p^{res, CO} = \mathcal{M}_p^{TM, CO} - M_p^{TM, CO}$,
- The plot of the evolution of the average error on the CO RDCs with each loop, computed as:

$$\overline{\delta L^{err, CO}}^{\mathcal{B}_{CO}} = \frac{1}{8} \sum_{b \in \mathcal{B}_{CO}} \left(\delta L_b^{TM, CO} - L_b^{TM, CO} \right), \quad (\text{IV.17})$$

- The plot of the evolution of the standard deviation of the error on the CO RDCs with each loop, computed as:

$$\sigma_{err}^{CO} = \sqrt{\frac{1}{8} \sum_{b \in \mathcal{B}_{CO}} \left(\delta L_b^{TM, CO} - L_b^{TM, CO} - \overline{\delta L^{err, CO}}^{\mathcal{B}_{CO}} \right)^2}. \quad (\text{IV.18})$$

b) Impact of noise

The first effect I tested was the impact of only noise in the data. I simulated this effect by adding an independent realization of Gaussian noise map to each bolometer's sky emission

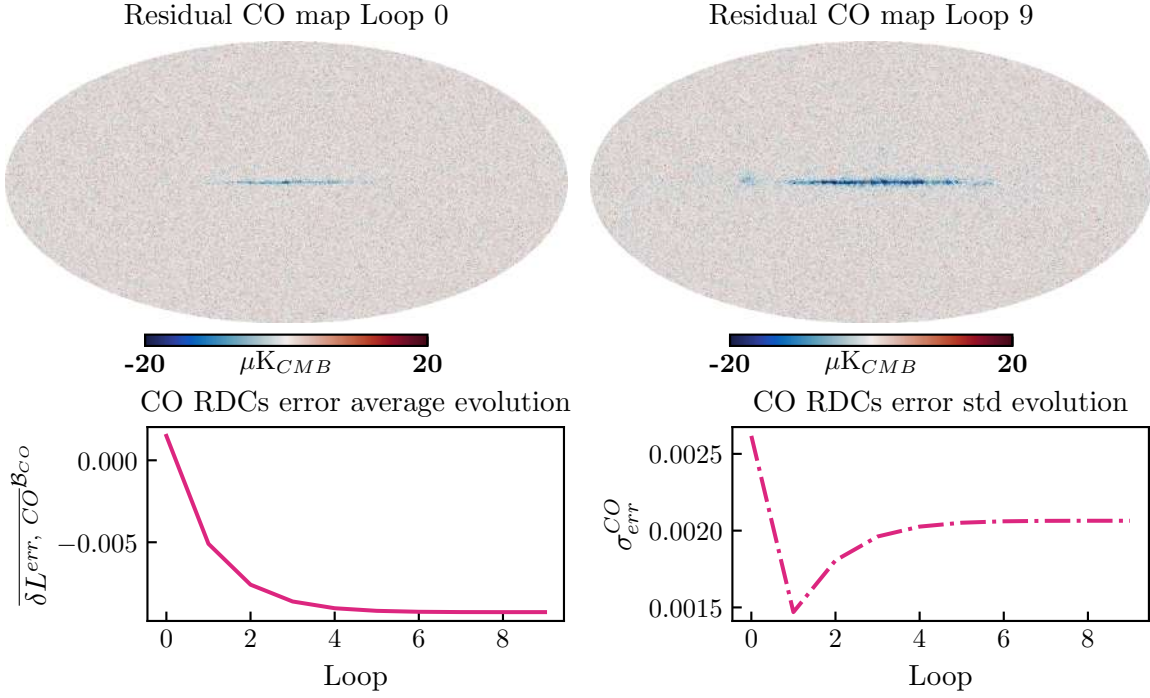


Fig. IV.44 Results of a toy model test to estimate the effect of the presence of noise in the data on the iterative scheme. An independent map of Gaussian noise with a standard deviation of $5\mu\text{K}_{\text{CMB}}$ is added to each bolometers' sky emission map. **Top left:** CO residual map of loop 0; **Top right:** CO residual map of loop 9; **Bottom left:** evolution of the average error on the CO RDCs (see Eq.IV.17); **Bottom right:** evolution of the standard deviation of the error on the CO RDCs (see Eq.IV.18).

map. This corresponds to transforming above Eq.IV.16 into:

$$M_{b,p}^{TM, sky} = L_b^{TM, dust} M_p^{TM, dust} + L_b^{TM, DustVar} M_p^{TM, DustVar} + L_b^{TM, CO} M_p^{TM, CO} + M_{b,p}^{noise},$$

where M_b^{noise} is a map of zero-mean Gaussian noise with an arbitrary standard deviation of $5\mu\text{K}_{\text{CMB}}$, with one independent realization per bolometer b . The value $5\mu\text{K}_{\text{CMB}}$ is a fairly small noise, given this value is almost equal to the standard deviation of the signal in the map $M_p^{TM, CO}$ divided by 20, accounting for the 20.5% of the pixels in said map which are equal to zero. Since I only want to look at the effect of the noise in the maps, I take the input templates to be the exact sky components map, i.e. $C_p^{TM, dust} = M_p^{TM, dust}$, $C_p^{TM, DustVar} = M_p^{TM, DustVar}$, and $C_p^{TM, CO} = M_p^{TM, CO}$.

I plotted the results from this test in Fig.IV.44. We can see that the loop 1 seems efficient at reducing the effect of noise in the data, with a sharp decrease of σ_{err}^{CO} from loop 0 to loop 1. This can be expected since a lot more pixels are used to determine the CO RDCs starting from loop 1, reducing the impact of the noise. However, the average error $\overline{\delta L^{err, CO} B_{CO}}$ degrades with the first 3 loops, which could indicate that the noise from the data significantly participates to the observed intensity drift. Finally, after loop 5, the CO map does converge towards a stable map, **but** this stable map is *biased* since both $\overline{\delta L^{err, CO} B_{CO}}$ and σ_{err}^{CO} converge towards a non-zero value. This can also be seen in the CO residual map, which, despite being stable, is worse at loop 9 than at loop 0.

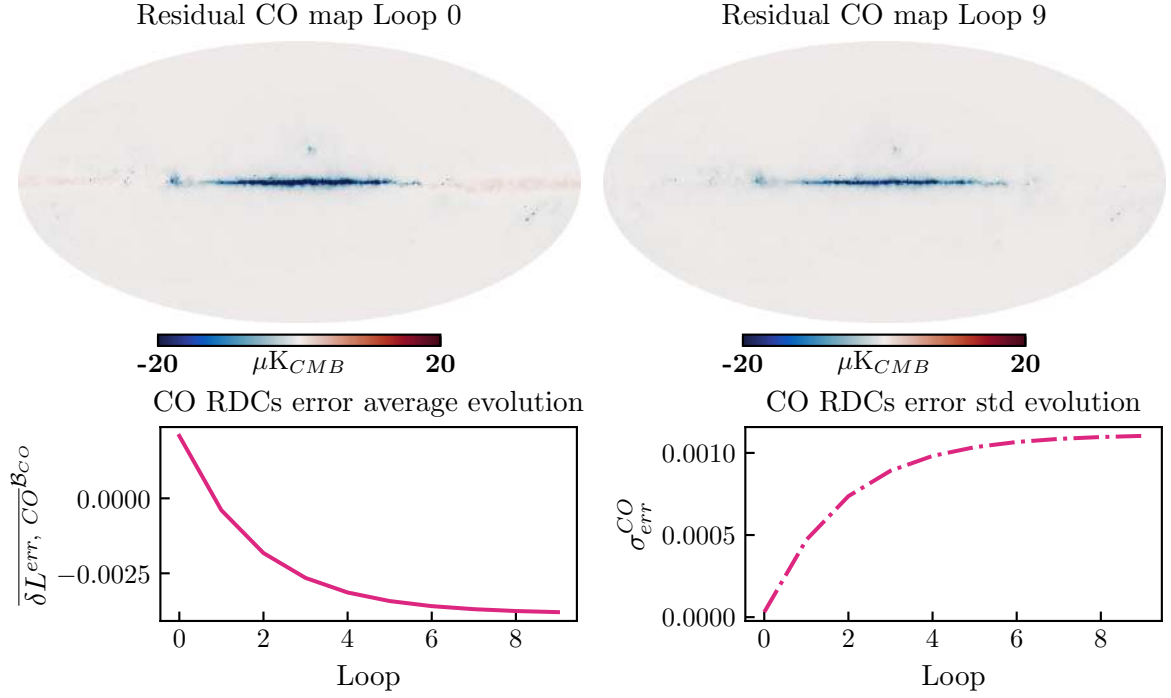


Fig. IV.45 Results of a toy model test to estimate the effect of an erroneous dust input template on the iterative scheme. The degraded dust input template is defined in Eq.IV.19. **Top left:** CO residual map of loop 0; **Top right:** CO residual map of loop 9; **Bottom left:** evolution of the average error on the CO RDCs (see Eq.IV.17); **Bottom right:** evolution of the standard deviation of the error on the CO RDCs (see Eq.IV.18).

c) Dust template error

The second effect I estimated is the impact of an error in the input template of the dust component. Therefore, the sky emission maps are **not** noised, i.e. said maps are computed as defined by Eq.IV.16, and the DustVar and CO template are also the exact sky component maps, i.e. $C_p^{TM, DustVar} = M_p^{TM, DustVar}$ and $C_p^{TM, CO} = M_p^{TM, CO}$. I simulate an error in the dust input template by defining it as follows:

$$C_p^{TM, dust} = 0.9 \times M_p^{TM, dust} + 0.1 \times C_p^{Sroll2.1, dust_I}, \quad (IV.19)$$

where $C_p^{Sroll2.1, dust_I}$ is the intensity map of the SRoll2.1 dust input template, as defined in subsection IV.1.3.a). The values of 90% and 10% for the template construction were arbitrarily taken to introduce a small but noticeable error.

I plotted the results from this test in Fig.IV.45. Here, both the average error $\overline{\delta L_{err, CO}^{B_{CO}}}$ and the standard deviation of the error σ_{err}^{CO} degrade overall from loop 0 to loop 9. This effect is then also a candidate that can participate to the observed intensity drift. In this case too, the CO map does converge towards a stable map, **but** this stable map is *biased* since both $\overline{\delta L_{err, CO}^{B_{CO}}}$ and σ_{err}^{CO} converge towards a non-zero value. Notably, the bias of the stable CO map is different since the residual CO maps seen here are different from the ones from Fig.IV.44.

d) Templates' low resolution

The third effect I tested on the toy model is the impact of the pixelisation of the templates. To this end, I computed the sky emission maps still using the definition of Eq.IV.16, but by

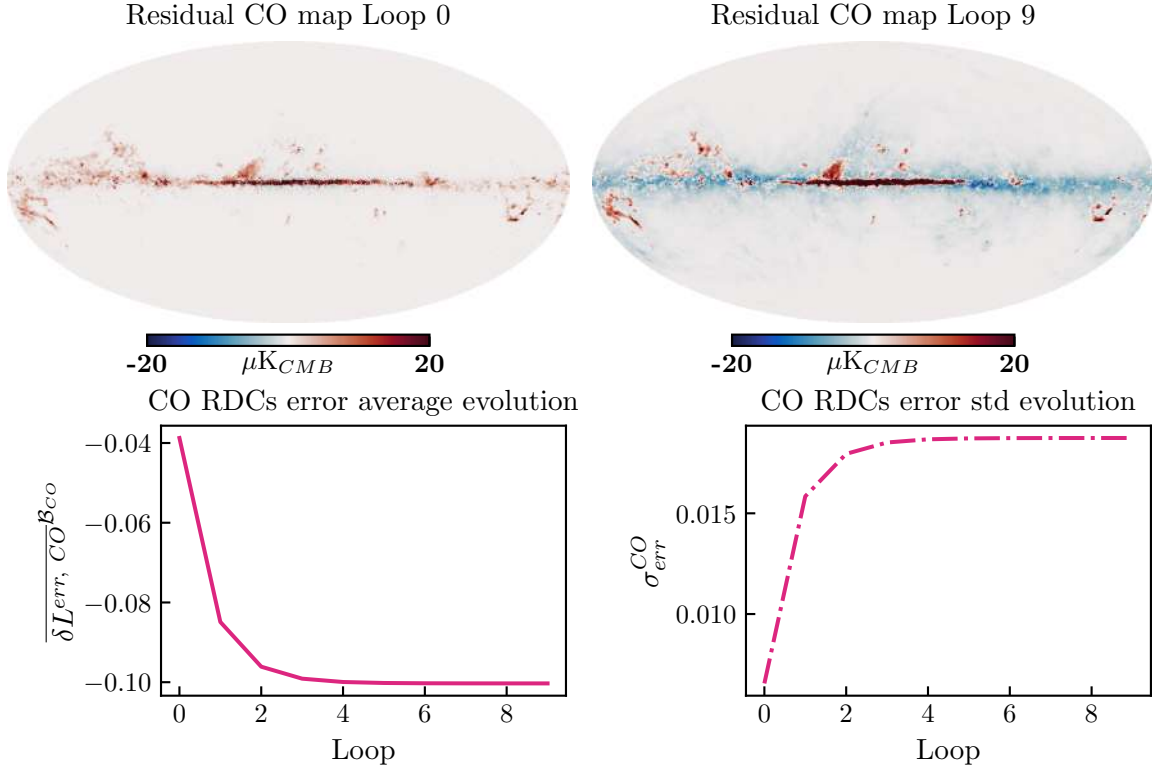


Fig. IV.46 Results of a toy model test to estimate the effect of the low resolution of the input template maps on the iterative scheme. The sky component maps are built at a resolution of $N_{side} = 2048$ while the input template maps are kept at the low resolution of $N_{side} = 128$. **Top left:** CO residual map of loop 0; **Top right:** CO residual map of loop 9; **Bottom left:** evolution of the average error on the CO RDCs (see Eq.IV.17); **Bottom right:** evolution of the standard deviation of the error on the CO RDCs (see Eq.IV.18).

taking the maps $M^{TM, dust}$, $M^{TM, DustVar}$, and $M^{TM, CO}$ at a resolution of $N_{side} = 2048$. More precisely, $M^{TM, dust}$ is taken at its native resolution of $N_{side} = 2048$, and the maps $M^{TM, DustVar}$ and $M^{TM, CO}$, which are originally taken from maps defined at a resolution of $N_{side} = 128$, are upgraded to the resolution $N_{side} = 2048$ by computing a bi-linear interpolation for each pixel using the 4 nearest neighbours. This interpolation cannot create small scale structure, but allows a resolution increase resulting in a smooth map. However, the input template maps are the same maps but kept at the low resolution of $N_{side} = 128$.

I plotted the results from this test in Fig.IV.46. Again, both the average error $\overline{\delta L^{err, CO} B_{CO}}$ and the standard deviation of the error σ_{err}^{CO} degrade overall from loop 0 to loop 9. This effect is a significant additional candidate that can participate to the observed intensity drift, especially since it displays a quick degradation of the RDCs error on the first 3 loops, and leads to the highest average error by a factor of the order of ~ 10 compared to the 2 previous tests. In this case too, the CO map does converge towards a stable map, **but** this stable map is *biased* since both $\overline{\delta L^{err, CO} B_{CO}}$ and σ_{err}^{CO} converge towards a non-zero value, and once more, the bias of the stable CO map is different compared to the ones from the 2 previous tests, as can be seen from the residual CO maps from Figs.IV.44 and IV.45. This test results in a much worse residual error map than in the other tests, which supports the importance of increasing the resolution of the SRoll2.1 input template maps, as was first presented in subsection IV.3.4, and will be further discussed in Section V.2.

e) Toy model conclusion

From these tests, I believe possible to achieve a convergence of the iterative scheme on *SRoll2.1* to improve the templates of the ^{12}CO and ^{13}CO maps. However, this would require first to increase the resolution of the foreground input templates, which raises the others requirements discussed in subsection [IV.3.4](#). Then, one would need to evaluate the bias introduced by the level of noise in the data, and would probably also need to estimate the bias introduced by potential errors on the dust input template, to finally correct for those biases.

This investigative work proved too ambitious for the time I had available, as well as too computationally costly to be carried out during my PhD. For these reasons, we chose to keep *SRoll2.1* in the non-iterative state presented through Sections [IV.1](#) and [IV.2](#).

Chapter V

New CO maps' qualification & exploitation

Now that the new SRoll2.1 ^{12}CO and ^{13}CO maps have been presented, that their computation method has been detailed, characterized, evaluated, and now that I provided estimation of their noise and error level, we are set to explore the content of said maps.

This chapter is precisely aimed at such exploration and exploitation. First, I qualify my maps quality, and characterize the calibration and signal spatial distribution in my new ^{12}CO and ^{13}CO maps by comparing them to state-of-the-art external CO surveys. These various comparisons are presented in Section V.1. We will see a good overall quality, but at the same time uncover a limitation of my method. This limitation will secondly be detailed, discussed, and suggestions for potential corrections will be described in Section V.2. The results from one of the suggested solutions will also be presented in the same section. Finally, I present my results for one of the main goals of my PhD subject: the use of my newly constructed ^{12}CO and ^{13}CO maps to search for CO signal at high latitude and low column density, which may originate from turbulent dissipation region where the production and excitation of the CO is driven by warm chemistry stemming from said dissipation of turbulence, occurring even throughout the diffuse ISM. The method of exploration, definition of a threshold for a confirmed high latitude CO detection, and final results are described in Section V.3.

V.1 New CO maps validation from state-of-the-art distinct CO observations

I will now compare my resulting maps to the state of the art in terms of large scale observation of the CO emission from our galaxy. I will first check how well my SRoll2.1 $J = 1 \rightarrow 0$ CO maps managed to recover the observation used as the SRoll2.1 ^{12}CO and ^{13}CO templates, the Taurus region surveyed by Goldsmith et al. (2008). Secondly, I will compare my $J = 1 \rightarrow 0$ maps with surveys of the Perseus and the Ophiuchus clouds from the COMPLETE survey (see Ridge et al. (2006)). I will then qualify my ^{12}CO map by another external observation, which is the current CO survey with the largest sky coverage, with almost all the northern sky and about a third of the southern sky scanned for the ^{12}CO $J = 1 \rightarrow 0$ line, i.e. the survey of Dame & Thaddeus (2022). At each step of validation of my maps from these external data, I will show how my maps compare to the previous *Planck* legacy CO maps.

In the absence of comparison to $J = 2 \rightarrow 1$ CO survey of large scale sky-coverage, I nonetheless compare my SRoll2.1 $J = 2 \rightarrow 1$ CO maps to the *Planck* legacy $J = 2 \rightarrow 1$ CO

maps to qualify the signal spatial distribution in my maps.

^{12}CO and ^{13}CO separation from Hurier (2019) A mention that should be made is the recent, and first public separation of the ^{12}CO and ^{13}CO isotopologues from the *Planck* data performed in the work of Hurier (2019). I did not have access to the ^{12}CO and ^{13}CO maps resulting from this work in order to compare them to my own results. It should be mentioned that this separation was performed from the *Planck* legacy Type 1 $J = 1 \rightarrow 0$ CO map, and therefore, as previously mentioned in subsection IV.2.1.a), suffers from the same setbacks, maps quality limitations, and residual systematic errors that impacted the *Planck* legacy CO maps and which were partly corrected by the use of the SRoll mapmaking algorithm.

Though I cannot compare my maps to the ones from the work of Hurier (2019), what I can compare is the level of noise in our respective maps: the work of Hurier (2019) report, at $10'FWHM$ angular resolution, at a map resolution brought to $N_{side} = 1024$, an average white noise with a mean standard deviation of $20,6 \text{ K.km.s}^{-1}$ for the ^{12}CO map, and 35.1 K.km.s^{-1} for the ^{13}CO map. As a comparison, the average values of my $J = 1 \rightarrow 0$ statistical error maps (see Fig.IV.28) are much smaller, with 5.2 K.km.s^{-1} for the ^{12}CO map and 3.5 K.km.s^{-1} for the ^{13}CO map. Even if we add the uncertainty on the ^{12}CO and ^{13}CO RDC determination, it only adds a relative uncertainty of $\frac{\Delta_{\delta L^{12CO}}}{\delta L^{12CO}} = 1.96\%$ for the ^{12}CO map, and of $\frac{\Delta_{\delta L^{13CO}}}{\delta L^{13CO}} = 4.17\%$ for the ^{13}CO map.

V.1.1 Comparison to the survey of Goldsmith et al. (2008)

I start my maps qualification by comparing them to the ^{12}CO and ^{13}CO maps from the survey of Goldsmith et al. (2008), which are the maps used as input SRoll2.1 templates of the ^{12}CO and ^{13}CO emissions. However, the SRoll2.1 input templates are at the low resolution of $N_{side} = 128$, and only serve the purpose of determining the RDCs of the corresponding emissions. Therefore, it is a good starting point to check how well my map construction, presented in Section IV.2, managed to recover these SRoll2.1 template input maps, but this time at the resolution of my CO maps, i.e. $N_{side} = 1024$.

I plotted in Fig.V.1 the Taurus region, in the cases of my ^{12}CO and ^{13}CO $J = 1 \rightarrow 0$ maps, the ^{12}CO and ^{13}CO maps from Goldsmith et al. (2008), as well as the *Planck* Type 1 and Type 2 $J = 1 \rightarrow 0$ CO maps, as described in Section II.4. It allows to assess a first visual comparison of the respective maps, as well as their level of noise. The plotted region is centred on $(l, b) = (173^\circ, -15^\circ)$, and is a square of size $15^\circ \times 15^\circ$. A first observation, which will be more visible in later subsection V.1.2, is that although my maps are good at recovering the CO signal distribution at medium and low intensity, they tend to fail in recovering the sharpest and most intense regions, as can be seen in the SRoll2.1 ^{12}CO map of Fig.V.1. I will discuss further this effect, and present a first attempt to correct it, in Section V.2.

For ease of comparison and qualification, and for the rest of **this subsection (V.1.1)**, the maps (my SRoll2.1 maps, as well as those from Goldsmith et al. (2008) and the *Planck* maps) will be smoothed to a Gaussian beam with $FWHM = 30'$.

I first compare my $J = 1 \rightarrow 0$ CO maps to those from the survey of Goldsmith et al. (2008), and then check how this qualification compares to the previous *Planck* $J = 1 \rightarrow 0$ CO maps (in the case of the Type 1 and Type 2 maps).

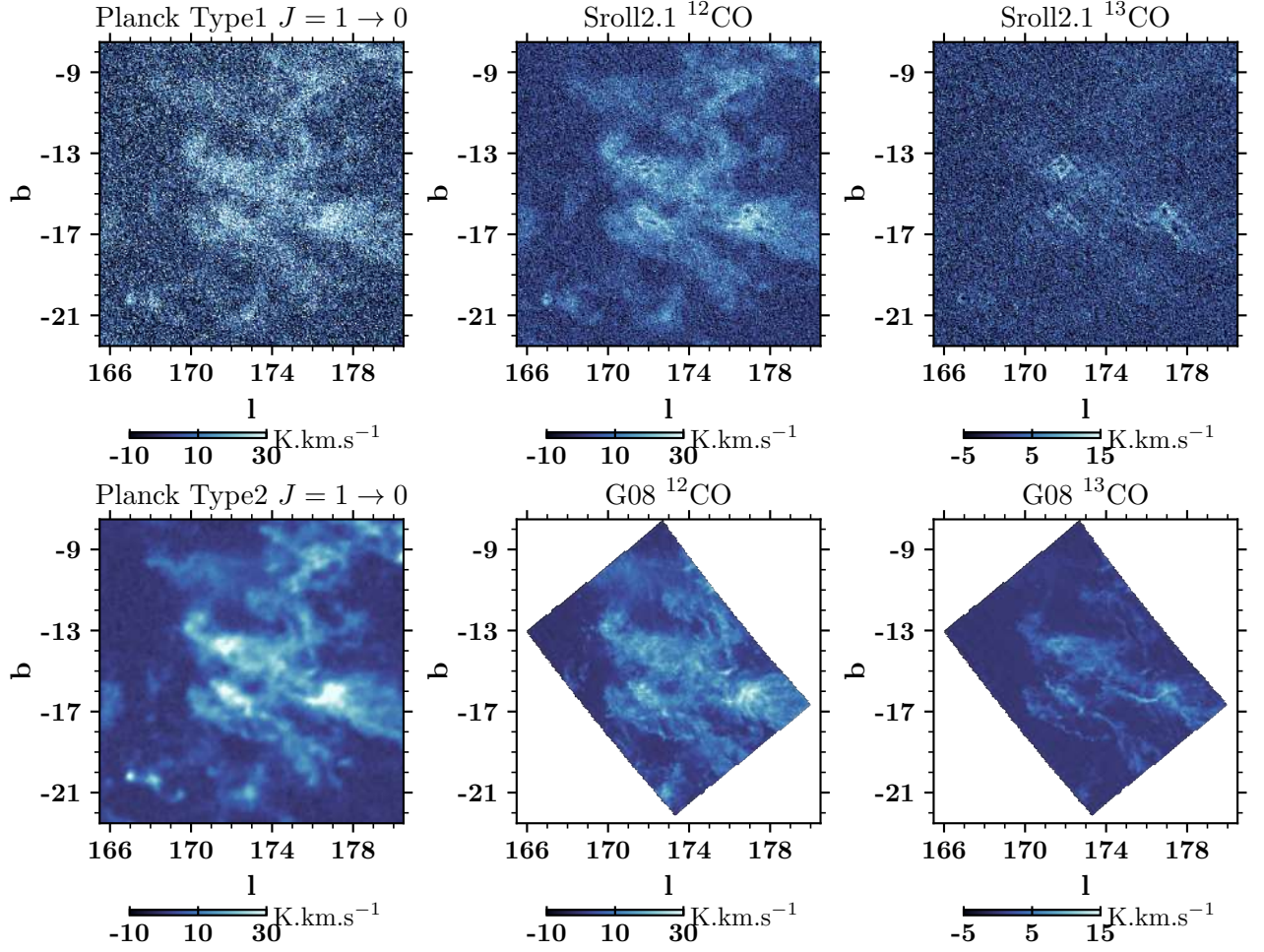


Fig. V.1 Taurus region from various maps. All maps are at the resolution of $N_{side} = 1024$. **Top row:** Planck Type 1 $J = 1 \rightarrow 0$ CO (left), SRoll2.1 ^{12}CO $J = 1 \rightarrow 0$ (middle), SRoll2.1 ^{13}CO $J = 1 \rightarrow 0$ (right); **Bottom row:** Planck Type 2 $J = 1 \rightarrow 0$ CO (left), ^{12}CO from Goldsmith et al. (2008) (middle), ^{13}CO from Goldsmith et al. (2008) (right).

a) Maps from SRoll2.1

A first visual appreciation of my new ^{12}CO and ^{13}CO $J = 1 \rightarrow 0$ maps quality is presented in Fig.V.2, where I displayed the Taurus region from both my ^{12}CO and ^{13}CO $J = 1 \rightarrow 0$ maps as well as from the ^{12}CO and ^{13}CO maps from Goldsmith et al. (2008), all smoothed to a Gaussian beam with $\text{FWHM} = 30'$. The ^{12}CO map quality is excellent, and that of the ^{13}CO is good, while displaying the aforementioned intensity hollows at the locations of the sharpest intensity peaks visible in the ^{13}CO map from Goldsmith et al. (2008). To better visualize this spatial comparison, I also plotted the corresponding difference maps computed as $\text{SRoll2.1} - \text{G08}$. Note that, to mitigate the effect of the calibration mismatch between the maps, the SRoll2.1 maps were recalibrated on the corresponding G08 map before computing the difference. Thanks to this recalibration, the difference maps only account for the discrepancies of signal spatial distribution between the maps. The recalibration was carried out using the affine fit computed in the correlation plot of Fig.V.3, as first defined in subsection IV.1.4.e).

The correlation plot for the SRoll2.1 and Goldsmith et al. (2008) ^{12}CO and ^{13}CO $J =$

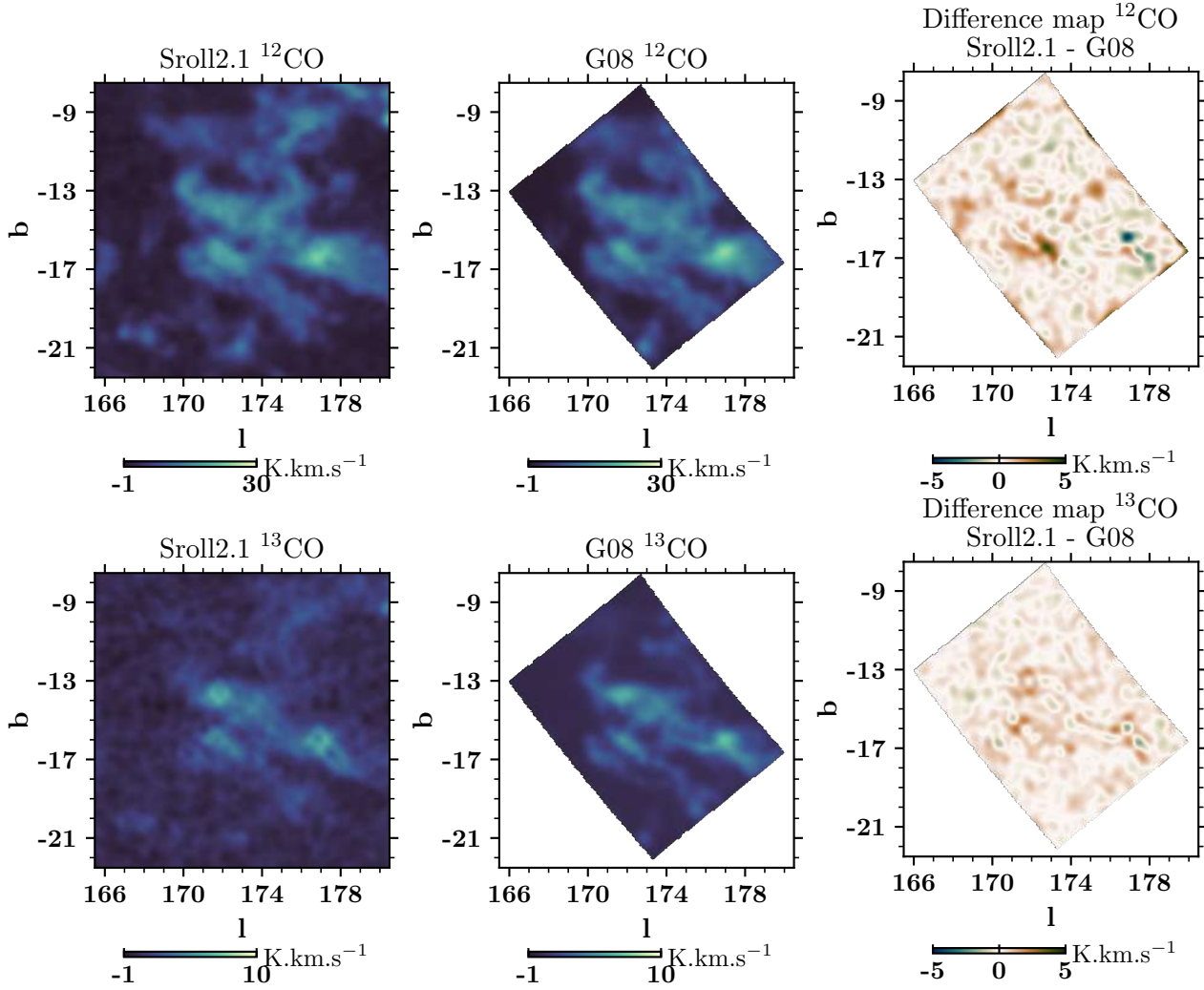


Fig. V.2 CO $J = 1 \rightarrow 0$ maps of the Taurus region smoothed up to a beam of FWHM = $30''$. **Top row:** ^{12}CO , **Bottom row:** ^{13}CO . **Left:** SRoll2.1 map, **Centre:** Goldsmith et al. (2008) survey, **Right:** Difference maps SRoll2.1-G08, after recalibration of SRoll2.1 on G08 to subtract maps with the same calibration.

$1 \rightarrow 0$ maps is computed in Fig.V.3. The correlation plots, as well as the affine fits, are computed only considering pixels with intensity greater than the arbitrary value 1 K.km.s^{-1} in the corresponding map from Goldsmith et al. (2008), to ensure that the comparison focuses only on regions containing actual CO signal. We observe an excellent affine fit for both the ^{12}CO and ^{13}CO maps. The calibration is also well recovered for the ^{12}CO with a factor of 0.987 for the affine fit. The calibration of the ^{13}CO is slightly underestimated, with a factor of 0.865 for the affine fit. This is a relative calibration underestimation of a few percents, even when accounting for the calibration uncertainty from the survey of Goldsmith et al. (2008), which is fully detailed in Narayanan et al. (2008), and amounts to a relative calibration uncertainty of 7% in ^{12}CO and 9.3% in ^{13}CO . This could be due to the increased uncertainty on the ^{13}CO RDCs determination from SRoll2.1, as seen in subsection IV.1.4, and listed in Appendix.D. Another likely reason for this underestimation of the ^{13}CO calibration is the aforementioned failing of my $J = 1 \rightarrow 0$ maps to recover the sharpest and most intense regions. Since these regions are underestimated, they could explain the underestimation detected by the affine fit. This interpretation is explored in later Section V.2 where I attempt the construction of *composite* maps from SRoll2.1 to

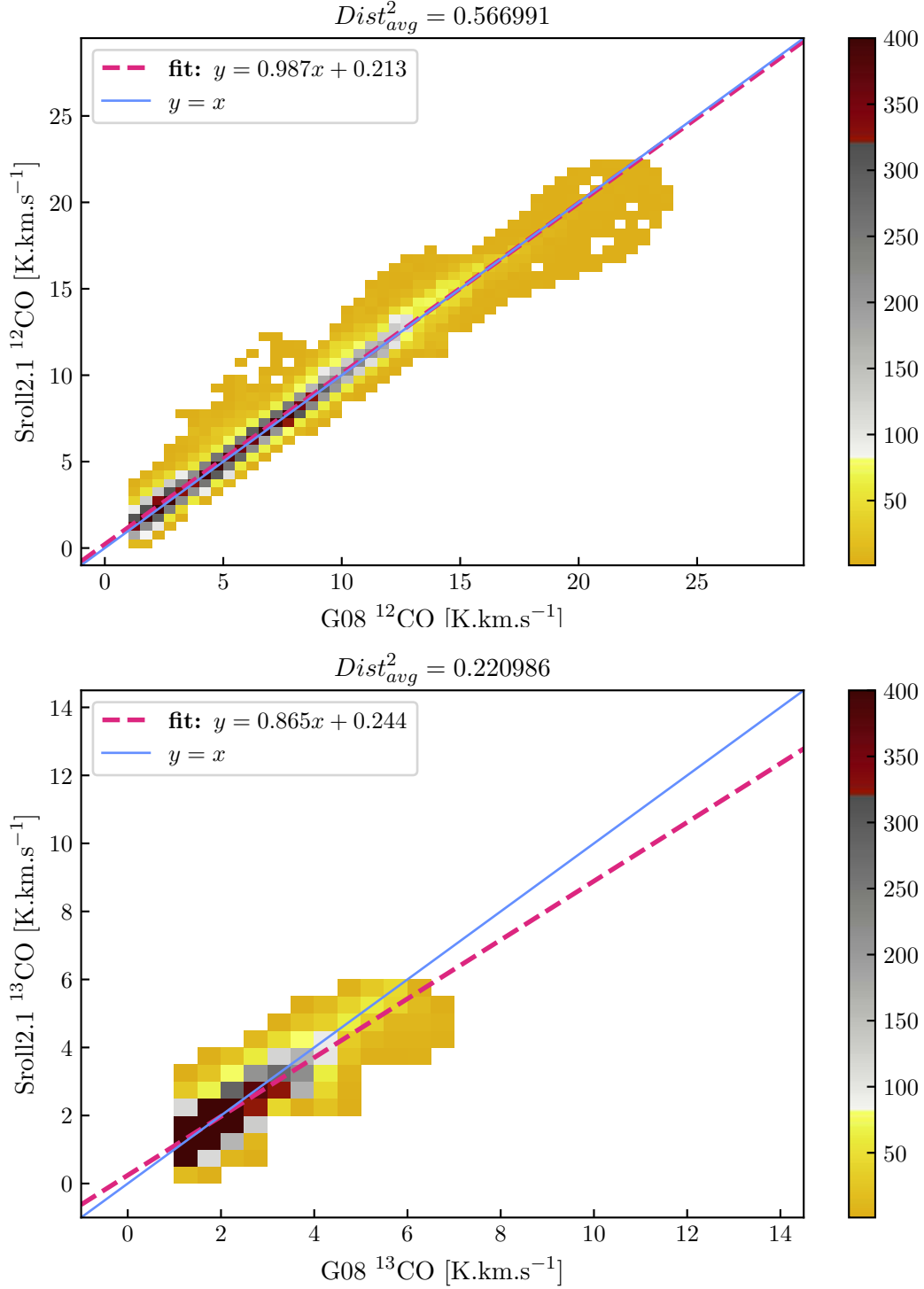


Fig. V.3 Correlation plots between the SRoll2.1 $J = 1 \rightarrow 0$ CO maps and the Taurus survey from Goldsmith et al. (2008). Both maps have been set to a Gaussian beam with FWHM = 30'. Plot computed using only pixels with an intensity superior to 1 K.km.s $^{-1}$ in the G08 map considered. $Dist_{avg}^2$ corresponds to the average of squared distances of points with respect to the fit. **Top:** ^{12}CO , **Bottom:** ^{13}CO . **Red dashed line** is the best affine fit of the cloud of points; **Blue line** is the identity function.

correct this effect.

b) *Planck* CO maps from the legacy release

Here I check how my CO maps compare to the Type 1 and Type 2 maps from the *Planck* legacy release, when using the maps from Goldsmith et al. (2008) as a reference. A visual comparison is plotted in Fig.V.4, where I display the Taurus region from the *Planck* Type 1 and Type 2 $J = 1 \rightarrow 0$ CO maps, the SRoll2.1 ^{12}CO $J = 1 \rightarrow 0$ map, and the ^{12}CO map from Goldsmith et al. (2008). I also plotted the corresponding difference maps with the one from Goldsmith et al. (2008), where the subtraction is computed after the *other* map has been *recalibrated* on the G08 map. This recalibration is again computed using the linear factor from the affine fit of the corresponding correlation plot (see Fig.V.3 for the SRoll2.1 map, and Fig.V.5 for the *Planck* maps). We can already see from the difference maps a better agreement with the map from Goldsmith et al. (2008) in the case of my SRoll2.1 map than with the Type 1 and Type 2 *Planck* $J = 1 \rightarrow 0$ CO maps.

This better agreement is further quantified by the correlation plots between the ^{12}CO map from Goldsmith et al. (2008) and the Type 1 and Type 2 *Planck* $J = 1 \rightarrow 0$ CO maps, displayed in Fig.V.5. The *Planck* CO maps calibration agrees less with that from Goldsmith et al. (2008) with affine fit factor of 1.180 for the Type 1 and 1.458 for the Type 2 map. This is also visible from a better goodness of fit with the SRoll2.1 map than with the *Planck* CO maps, with values of $\text{Dist}_{\text{avg}}^2 = 0.567$ for the SRoll2.1 ^{12}CO map, against values for the Type 1 and Type 2 *Planck* maps of respectively $\text{Dist}_{\text{avg}}^2 = 1.170$ and $\text{Dist}_{\text{avg}}^2 = 1.383$.

^{13}CO contamination in the *Planck* CO maps As previously discussed in subsection II.4.3, it should be noted that the CO maps produced for the *Planck* legacy release are expected to contain some ^{13}CO signal, which may explain both the lesser spatial correlation of the *Planck* CO maps with those from Goldsmith et al. (2008) as well as their calibration mismatch, with intensity observed to be higher in the *Planck* CO maps. I remind the multiplicative factor of the ^{13}CO signal contamination evaluated in Planck Collaboration XIII (2014), and as was defined in Eq.II.6: $X_{J=1 \rightarrow 0}^{\text{Type1}} = 0.53$, $X_{J=1 \rightarrow 0}^{\text{Type2}} = 1.14$, $X_{J=2 \rightarrow 1}^{\text{Type1}} = 0.01$, and $X_{J=2 \rightarrow 1}^{\text{Type2}} = 0.82$.

I then recomputed the correlation plots of Fig.V.5, but this time comparing to the adequate combination of ^{12}CO and ^{13}CO maps from Goldsmith et al. (2008), using the corresponding values of $X_{\text{line}}^{\text{Type } i}$. The results of these new correlation plots are displayed in Fig.V.6. In the case of the Type 1 map, the calibration consistency is improved, with a factor of the affine fit going from 1.180 to 1.035, while the goodness of fit is slightly decreased when including the ^{13}CO signal, going from $\text{Dist}_{\text{avg}}^2 = 1.170$ to $\text{Dist}_{\text{avg}}^2 = 1.220$. As for the Type 2 map, both the calibration consistency and goodness of fit are significantly improved, with an affine factor going from 1.458 to 1.129, and a squared-distance to fit average going from $\text{Dist}_{\text{avg}}^2 = 1.383$ to $\text{Dist}_{\text{avg}}^2 = 0.802$. For comparison, I also plot in Fig.V.6 the correlation plot between the sum maps $^{12}\text{CO} + ^{13}\text{CO}$ from SRoll2.1 compared to Goldsmith et al. (2008). The goodness of fit is slightly better for the SRoll2.1 $^{12}\text{CO} + ^{13}\text{CO}$ sum map compared to the *Planck* Type 2 map, with $\text{Dist}_{\text{avg}}^2 = 0.780$ for the SRoll2.1 sum map. Additionally, the calibration of the SRoll2.1 sum map remains better compared to the *Planck* CO maps, with an affine factor of 0.971 for the SRoll2.1 sum map. Let us also remind that the strong point of the ^{12}CO and ^{13}CO maps from SRoll2.1 is that it manages to *separate* the 2 isotopologues, while reaching very good maps quality, reasonable noise level, and by only loosely constraining the sky model (e.g. no use of foreground emission law).

Finally, the new difference maps, corresponding to the correlation plots of Fig.V.6, are

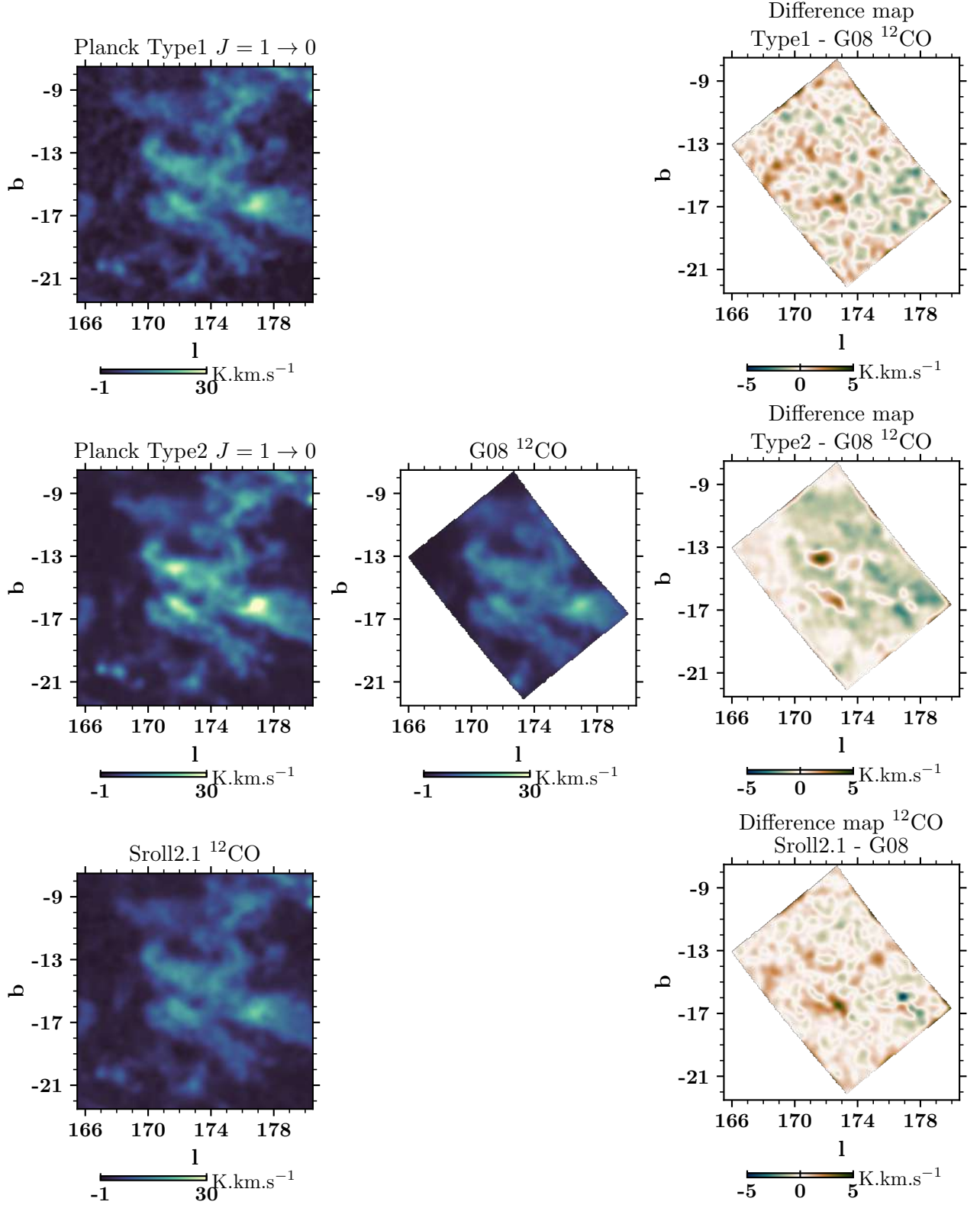


Fig. V.4 CO $J = 1 \rightarrow 0$ maps of the Taurus region smoothed up to a beam of $\text{FWHM} = 30'$. **Left:** CO maps to compare to Goldsmith et al. (2008); **Middle column:** ^{12}CO map from Goldsmith et al. (2008); **Right:** corresponding difference maps between the map on the left column and the map from Goldsmith et al. (2008), after recalibrating the left map on G08 to subtract maps with the same calibration. **Top:** Planck Type 1; **Middle row:** Planck Type 2; **Bottom:** Sroll2.1 ^{12}CO map.

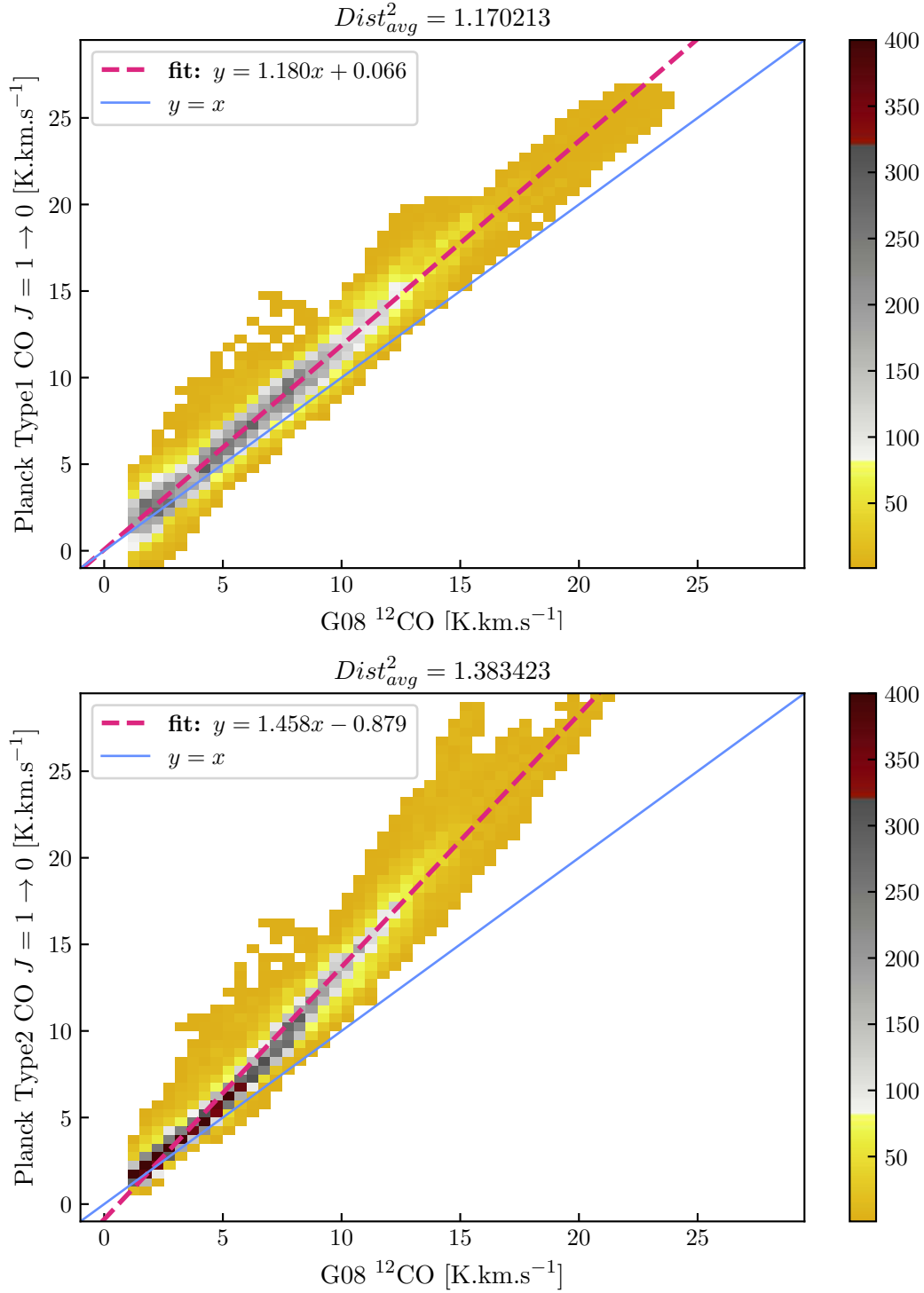


Fig. V.5 Correlation plots between the Planck $J = 1 \rightarrow 0$ CO maps and the Taurus survey from Goldsmith et al. (2008). Both maps have been set to a Gaussian beam with FWHM = 30'. Plot computed using only pixels with an intensity superior to 1 K.km.s $^{-1}$ in the G08 map considered. $Dist_{avg}^2$ corresponds to the average of squared distances of points with respect to the fit. **Top:** Planck Type 1, **Bottom:** Planck Type 2. **Red dashed line** is the best affine fit of the cloud of points; **Blue line** is the identity function.

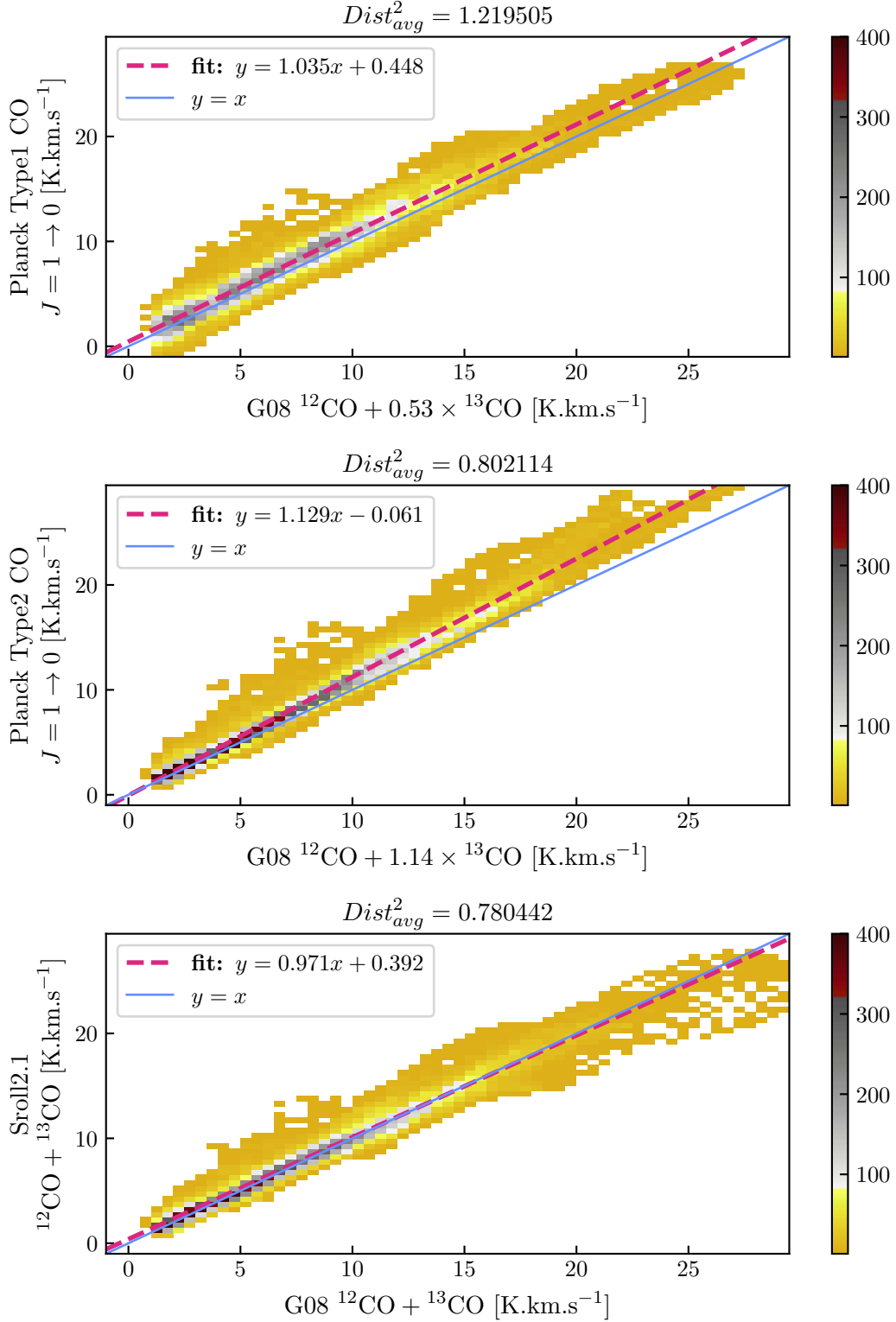


Fig. V.6 Correlation plots between the Planck $J = 1 \rightarrow 0$ CO maps and the adequate combination of ^{12}CO and ^{13}CO maps from the Taurus survey of [Goldsmith et al. \(2008\)](#). All maps have been set to a Gaussian beam with FWHM = 30'. Plot computed using only pixels with an intensity superior to 1 K.km.s $^{-1}$ in the ^{12}CO G08 map. $Dist_{avg}^2$ corresponds to the average of squared distances of points with respect to the fit. **Top:** Planck Type 1, **Middle:** Planck Type 2, **Bottom:** Sroll2.1 $^{12}\text{CO} + ^{13}\text{CO}$. **Red dashed line** is the best affine fit of the cloud of points; **Blue line** is the identity function.

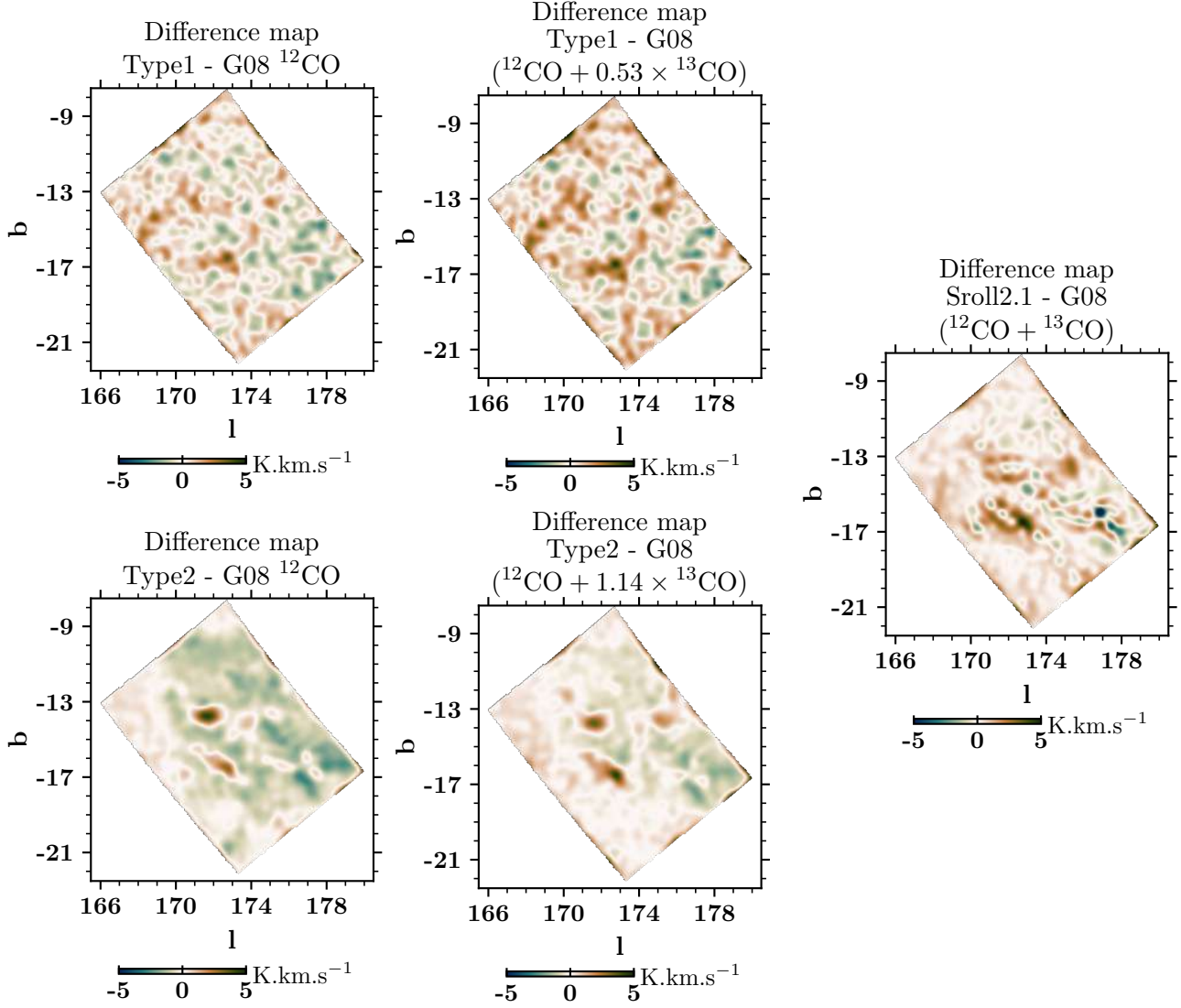


Fig. V.7 $J = 1 \rightarrow 0$ CO difference maps. All maps are smoothed up to a Gaussian beam with FWHM = $30'$. Before subtraction, the non-G08 map (either *Planck* or *SRoll2.1* map) is recalibrated on the corresponding subtracted G08 map, in order to subtract maps with the same calibration. **Top left:** *Planck* Type 1 - ^{12}CO from [Goldsmith et al. \(2008\)](#) (same as top right map from Fig. V.4); **Bottom left:** *Planck* Type 2 - ^{12}CO from [Goldsmith et al. \(2008\)](#) (same as middle right map from Fig. V.4); **Top middle:** *Planck* Type 1 - $^{12}\text{CO} + 0.53 \times ^{13}\text{CO}$ from [Goldsmith et al. \(2008\)](#); **Bottom middle:** *Planck* Type 2 - $^{12}\text{CO} + 1.14 \times ^{13}\text{CO}$ from [Goldsmith et al. \(2008\)](#); **Right:** *SRoll2.1* $^{12}\text{CO} + ^{13}\text{CO} - ^{12}\text{CO} + ^{13}\text{CO}$ from [Goldsmith et al. \(2008\)](#).

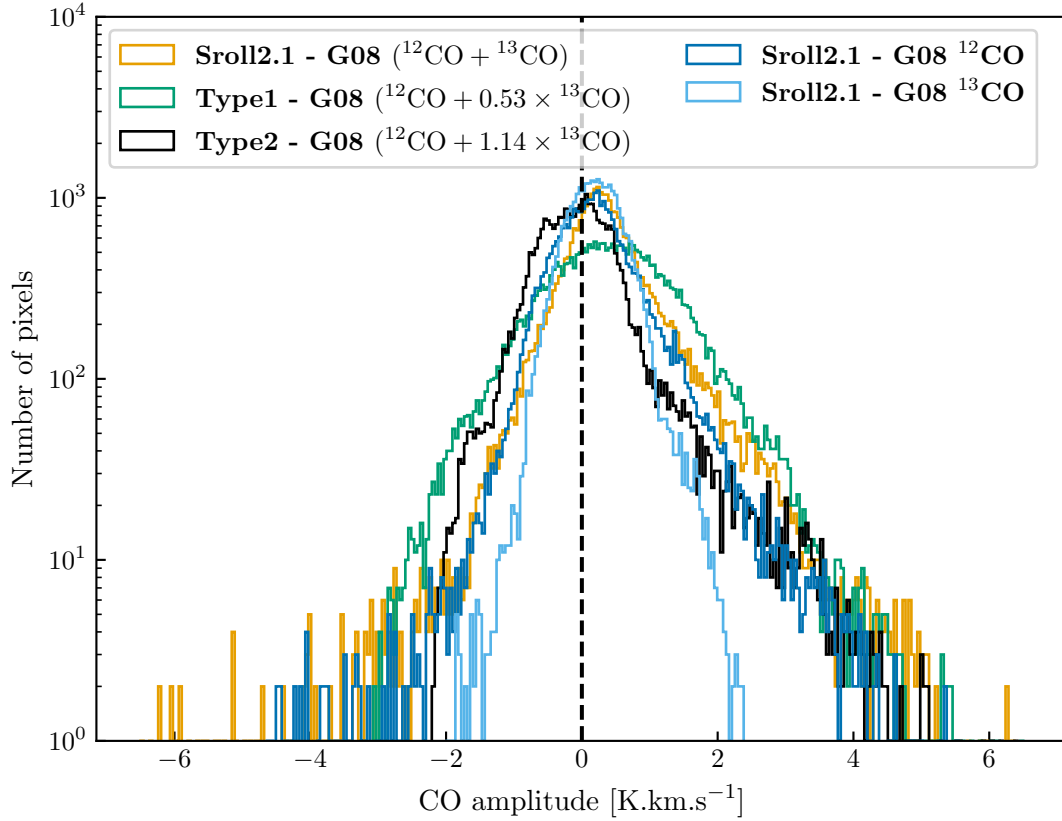


Fig. V.8 Histograms of the difference maps between G08 and the Type 1 and Type 2 *Planck* maps – in the case of the adequate combinations, i.e. corresponding to the maps from the middle column of Fig.V.7 – as well as between G08 and my SRoll2.1 maps – 3 cases: ^{12}CO only (i.e. the map in top right of Fig.V.2), ^{13}CO only (i.e. the map in bottom right of Fig.V.2), and the sum map $^{12}\text{CO} + ^{13}\text{CO}$ (i.e. the map on the right of Fig.V.7).

plotted in Fig.V.7. For comparison, the first difference maps of Fig.V.4, with only the ^{12}CO from Goldsmith et al. (2008), are displayed again in this figure. It then displays the difference maps between the *Planck* Type 1 and Type 2 $J = 1 \rightarrow 0$ CO maps and the adequate corresponding combination of the ^{12}CO and ^{13}CO maps from Goldsmith et al. (2008), as well as the difference map between the SRoll2.1 $^{12}\text{CO} + ^{13}\text{CO}$ sum map and the corresponding $^{12}\text{CO} + ^{13}\text{CO}$ sum map from Goldsmith et al. (2008). In each case, the maps are first recalibrated on the corresponding combination of G08 maps, by dividing them with the affine factor fitted in the correlation plots of Fig.V.6.

Additionally, I display in Fig.V.8 the histograms of the difference maps between G08 and the Type 1 and Type 2 *Planck* maps in the case of the adequate combinations, i.e. corresponding to the maps from the middle column of Fig.V.7, as well as the difference maps between G08 and my SRoll2.1 maps. This figure confirms and extends the observations made on the difference maps: my *individual* SRoll2.1 maps result in a smaller difference map, and the comparison between the *Planck* maps and the SRoll2.1 sum map is that the Type 1 is clearly less consistent with G08, while Type 2 is as consistent as my sum map but more centred on 0 while my sum map is more shifted towards positive values. This shift of my maps towards positive values is a second visualization of the failed peaked signal recovery

which will be discussed in Section V.2.

V.1.2 Comparison to the FCRAO 2011 survey

The second external observations I used to qualify my maps are extracts from the COMPLETE survey (Ridge et al. (2006)), which consisted in an in depth survey of the Perseus and Ophiuchus clouds, providing extinction maps, thermal dust emission maps, as well as emission maps of the H I 21-cm, $^{12}\text{CO } J = 1 \rightarrow 0$, and $^{13}\text{CO } J = 1 \rightarrow 0$ lines. The ^{12}CO and ^{13}CO lines were mapped using the Five College Radio Astronomy Observatory (FCRAO) telescope, the same used for the survey of Goldsmith et al. (2008). As I did previously, I compare my maps, as well as the *Planck* Type 1 and Type 2 CO maps, to the velocity integrated maps from these surveys (cf. COMPLETE team (2011c), COMPLETE team (2011d), COMPLETE team (2011a), COMPLETE team (2011b)) of the Perseus and Ophiuchus regions, which I projected in HEALPix. All maps are compared at the same resolution of $N_{\text{side}} = 1024$.

a) Perseus

The FCRAO observation of the Perseus cloud is centred on $(l, b) = (159^\circ.22, -19^\circ.25)$, and covers a surface of approximately $6^\circ.25 \times 3^\circ$, with a resolution of about $46''$ (see COMPLETE team (2011c), COMPLETE team (2011d), Ridge et al. (2006)). The corresponding region is plotted in Fig.V.9 for the FCRAO survey, the SRoll2.1 ^{12}CO and $^{13}\text{CO } J = 1 \rightarrow 0$ maps, as well as the *Planck* Type 1 and Type 2 $J = 1 \rightarrow 0$ CO maps. The maps of Fig.V.9 are smoothed to a Gaussian beam with $\text{FWHM} = 30'$, centred on $(l, b) = (159^\circ.22, -19^\circ.25)$, and correspond to a square of size $7^\circ.67 \times 7^\circ.67$. Also, I plotted the maps of ^{12}CO and ^{13}CO combinations corresponding to the *Planck* Type 1 and Type 2 $J = 1 \rightarrow 0$ CO maps, as described in the previous subsection V.1.1.b), and introduced in subsection II.4.3. We can see a very good spatial agreement for the ^{12}CO between the SRoll2.1 map and the FCRAO survey, while for the ^{13}CO the agreement seems good with the exception of the brightest intensity peak in the FCRAO ^{13}CO maps which appears as an intensity dip in the SRoll2.1 map. Again, it seems that the SRoll2.1 $J = 1 \rightarrow 0$ maps fails to accurately recover the regions of brightest intensity peaks.

This consistency is further explored by the computation of the correlation plots between my SRoll2.1 ^{12}CO and $^{13}\text{CO } J = 1 \rightarrow 0$ maps and the FCRAO survey, as plotted in Fig.V.10. The affine fit is computed from pixels where the signal intensity is greater than 1 K.km.s^{-1} in the considered FCRAO survey map. I will not linger too much on the issue of the calibration mismatch between my maps and the FCRAO surveys, with an affine fit factor of 0.768 for the ^{12}CO and of 0.614 ^{13}CO , for 3 main reasons:

- I do not know the level of uncertainty of the calibration from these surveys,
- I already checked that my calibration is consistent with the *Planck* legacy CO maps, with the Taurus survey of Goldsmith et al. (2008), and we will see in the next subsection that it is also consistent with the high coverage survey from Dame & Thaddeus (2022),
- This calibration mismatch is aggravated by the failing to recover the high intensity peaked signal in my maps, as will be seen in Section V.2, where my attempt at a solution for this effect will improve the calibration consistency with the FCRAO surveys.

We can still see a good spatial consistency between my ^{12}CO and ^{13}CO maps and the FCRAO Perseus surveys.

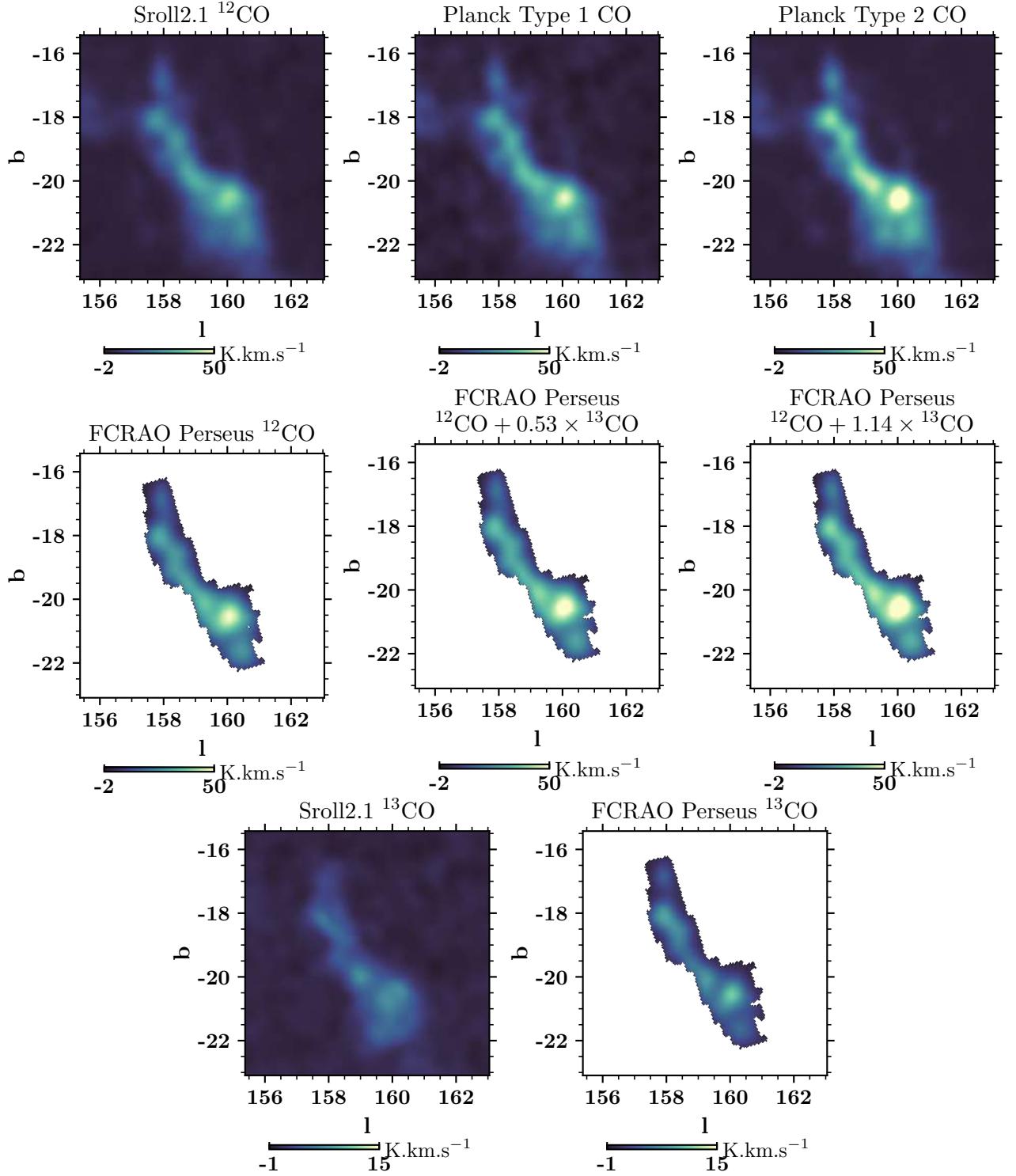


Fig. V.9 CO $J = 1 \rightarrow 0$ maps of the Perseus region smoothed up to a beam of FWHM = 30'. **Top left:** ^{12}CO Sroll2.1; **Top Middle:** Planck Type 1; **Top Right:** Planck Type 2; **Center left:** ^{12}CO FCRAO survey ([COMPLETE team \(2011c\)](#)); **Center middle:** $^{12}\text{CO} + 0.53 \times ^{13}\text{CO}$ FCRAO survey ([COMPLETE team \(2011c\)](#) & [COMPLETE team \(2011d\)](#)); **Center right:** $^{12}\text{CO} + 1.14 \times ^{13}\text{CO}$ FCRAO survey ([COMPLETE team \(2011c\)](#) & [COMPLETE team \(2011d\)](#)); **Bottom left:** ^{13}CO Sroll2.1; **Bottom right:** ^{12}CO FCRAO survey ([COMPLETE team \(2011d\)](#)).

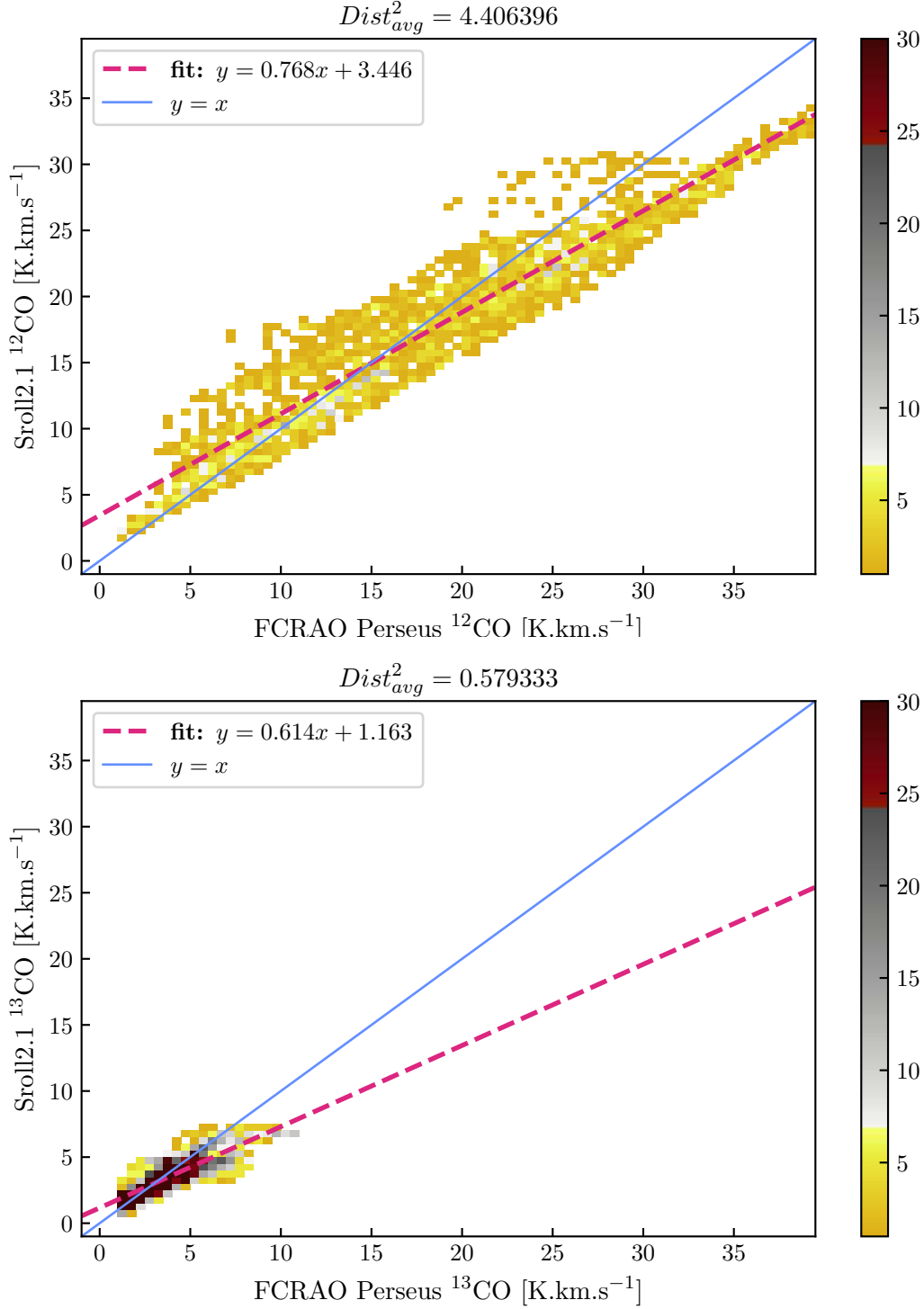


Fig. V.10 Correlation plots between the Sroll2.1 $J = 1 \rightarrow 0$ CO maps and the FCRAO survey of Perseus ([COMPLETE team \(2011c\)](#) & [COMPLETE team \(2011d\)](#)). Both maps have been set to a Gaussian beam with FWHM = 30'. Plot computed using only pixels with an intensity superior to 1 K.km.s⁻¹ in the FCRAO map considered. $Dist_{avg}^2$ corresponds to the average of squared distances of points with respect to the fit. **Top:** ¹²CO, **Bottom:** ¹³CO. **Red dashed line** is the best affine fit of the cloud of points; **Blue line** is the identity function.

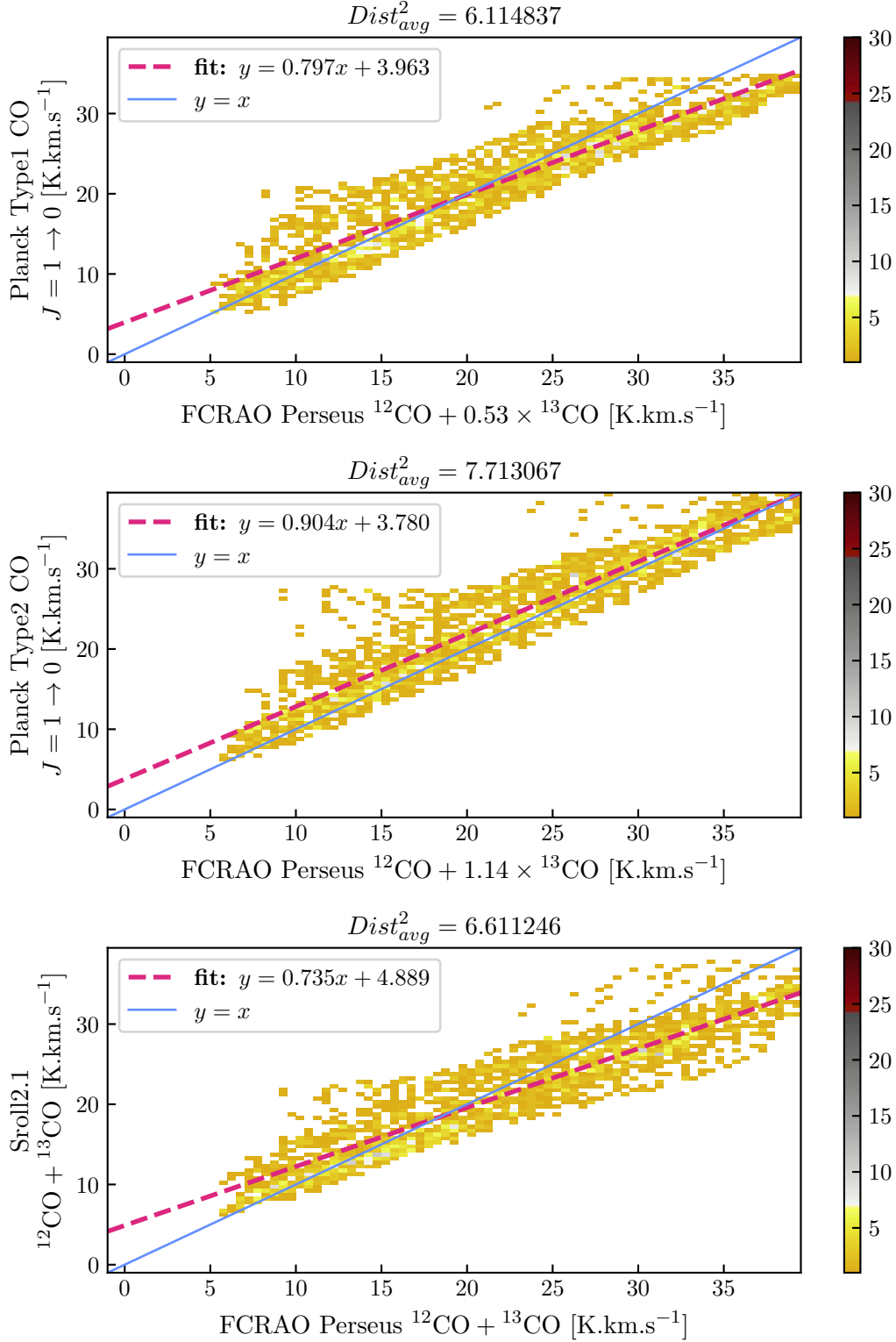


Fig. V.11 Correlation plots between the Planck $J = 1 \rightarrow 0$ CO maps and the adequate combination of ^{12}CO and ^{13}CO maps from the FCRAO survey of Perseus (COMPLETE team (2011c) & COMPLETE team (2011d)). All maps have been set to a Gaussian beam with FWHM = 30'. Plot computed using only pixels with an intensity superior to 1 K.km.s $^{-1}$ in both the ^{12}CO and ^{13}CO FCRAO maps. $Dist_{avg}^2$ corresponds to the average of squared distances of points with respect to the fit. **Top:** Planck Type 1, **Middle:** Planck Type 2, **Bottom:** Sroll2.1 $^{12}\text{CO} + ^{13}\text{CO}$. **Red dashed line** is the best affine fit of the cloud of points; **Blue line** is the identity function.

The consistency between my maps and the FCRAO Perseus survey is also compared to that of the *Planck* legacy CO maps in Fig.V.11. This figure displays the correlation plot between the *Planck* legacy Type 1 and Type 2 $J = 1 \rightarrow 0$ CO maps and the corresponding combination of the ^{12}CO and ^{13}CO components from the FCRAO survey, as described in the previous subsection V.1.1.b). In each case, the affine fit is computed for pixels with intensities greater than 1 K.km.s^{-1} in *both* the ^{12}CO and ^{13}CO maps from the FCRAO survey. We can see that the calibration mismatch is also observed between the FCRAO survey and the *Planck* legacy CO maps, though smaller than with my maps, with affine fit factors of 0.797 for the Type 1 map, 0.904 for the Type 2 map, and 0.735 for the sum of the ^{12}CO and ^{13}CO SRoll2.1 maps. Again, this mismatch is increased by the underestimation in the SRoll2.1 maps of the peak signal, impacting more the ^{13}CO line than the ^{12}CO one as seen in Figs.V.9 and V.10. This effect will be mitigated in Section V.2.

The goodness of fit of my sum map is also slightly lower than that of the *Planck* legacy Type 1 map, with $\text{Dist}_{\text{avg}}^2 = 6.61$ for the SRoll2.1 sum map, compared to $\text{Dist}_{\text{avg}}^2 = 6.11$ for the Type 1 map. It is still better than that of the Type 2 with $\text{Dist}_{\text{avg}}^2 = 7.71$. Additionally, **my maps achieve a separation of the 2 isotopologues**, and still retain a good spatial correlation with the FCRAO survey for each of the ^{12}CO and ^{13}CO lines.

b) Ophiuchus

I then compared my maps to the observation of the Ophiuchus cloud from the COMPLETE survey. The FCRAO observation of the Ophiuchus cloud is centred on $(l, b) = (-4^\circ.6, 16^\circ.15)$, and covers a surface of approximately $7^\circ.75 \times 4^\circ.75$, with a resolution of about $46''$ (see COMPLETE team (2011a), COMPLETE team (2011b), Ridge et al. (2006)). The corresponding region is plotted in Fig.V.12 for the FCRAO survey, the SRoll2.1 ^{12}CO and ^{13}CO $J = 1 \rightarrow 0$ maps, as well as the *Planck* Type 1 and Type 2 $J = 1 \rightarrow 0$ CO maps. The maps of Fig.V.12 are smoothed to a Gaussian beam with $\text{FWHM} = 30'$, centred on $(l, b) = (-4^\circ.6, 16^\circ.15)$, and correspond to a square of size $7^\circ.67 \times 7^\circ.67$. Also, I plotted the maps of ^{12}CO and ^{13}CO combinations from the FCRAO survey of Ophiuchus corresponding to the *Planck* Type 1 and Type 2 $J = 1 \rightarrow 0$ CO maps, as described in the previous subsection V.1.1.b), and first defined in subsection II.4.3.

While we can see an overall good spatial distribution consistency between the SRoll2.1 maps and the FCRAO survey, especially for the ^{12}CO , this region is the most revealing of the effect of peak signal loss in the SRoll2.1 CO maps. Indeed, we can see that the center of brightest intensity in the region appears as a local signal dip in the SRoll2.1 ^{12}CO map, and as an unmistakable pit in the ^{13}CO map, while the rest of the less peaked and lower intensity signal is well recovered.

This effect is also clearly visible in the correlation fits between the SRoll2.1 and the FCRAO survey CO maps, as plotted in Fig.V.13. Indeed, we can see the higher intensity regions of the cloud of points widening in both plots, and displaying the underestimation of said higher intensity regions in the SRoll2.1 maps, especially in the case of the ^{13}CO map. Expectedly, this is also reflected in the correlation plot of the sum map of the 2 SRoll2.1 ^{12}CO and ^{13}CO component, as displayed in Fig.V.14, compared to the corresponding correlation plots with the FCRAO survey in the case of the *Planck* Type 1 and Type 2 CO maps. Again the main observation is the widening of the cloud of points at high intensity, as well as the underestimation of the same region in the SRoll2.1 sum map. This effect is most probably one of the limitations of the use of the bolometers from the 143GHz band to compute my CO maps, as will be further discussed in Section V.2. Note that this effect only affects the most peaked and higher intensity regions of the map, which will not impact my later use of

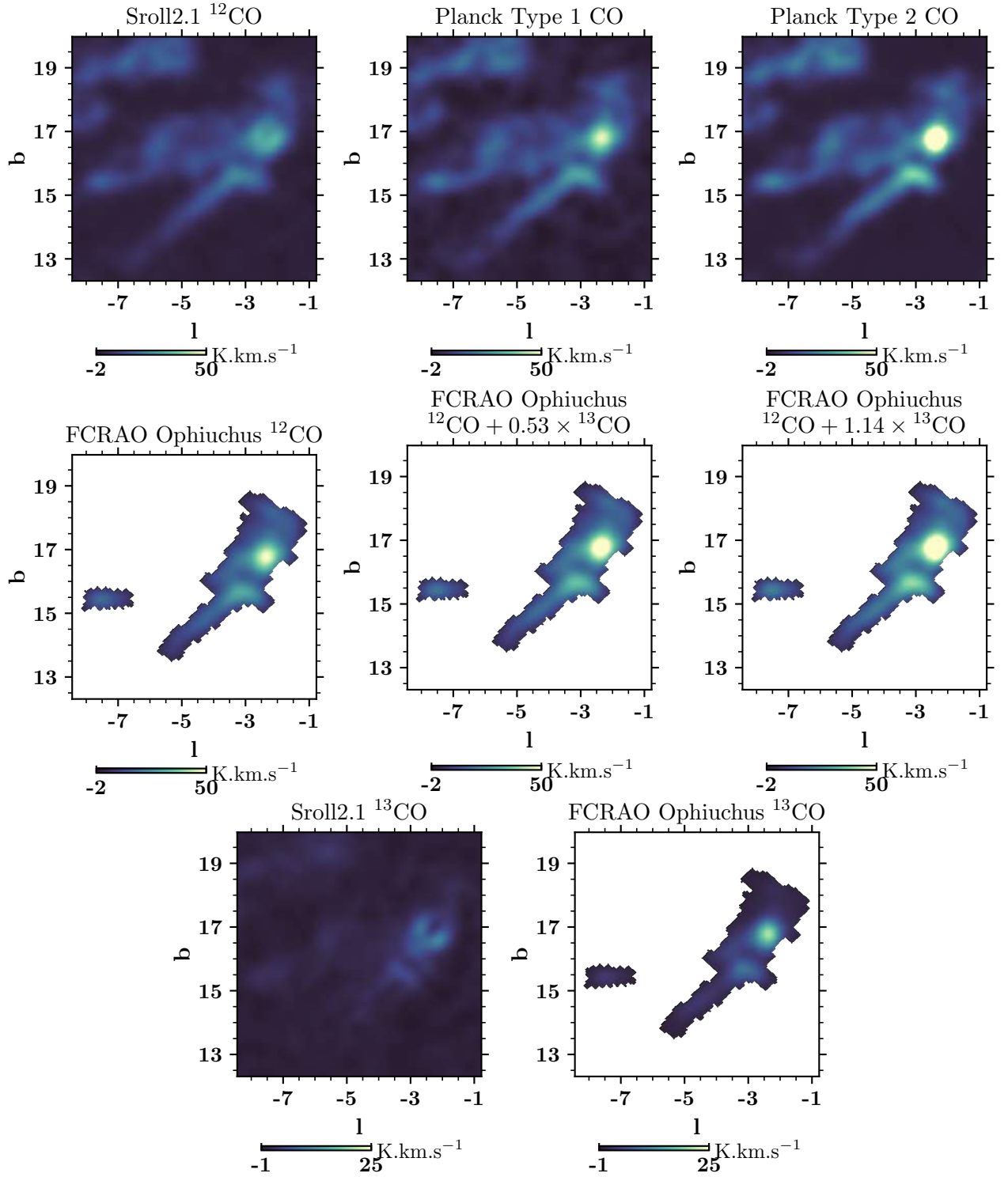


Fig. V.12 CO $J = 1 \rightarrow 0$ maps of the Ophiuchus region smoothed up to a beam of $\text{FWHM} = 30'$. **Top left:** ^{12}CO Sroll2.1; **Top Middle:** Planck Type 1; **Top Right:** Planck Type 2; **Center left:** ^{12}CO FCRAO survey ([COMPLETE team \(2011a\)](#)); **Center middle:** $^{12}\text{CO} + 0.53 \times ^{13}\text{CO}$ FCRAO survey ([COMPLETE team \(2011a\)](#) & [COMPLETE team \(2011b\)](#)); **Center right:** $^{12}\text{CO} + 1.14 \times ^{13}\text{CO}$ FCRAO survey ([COMPLETE team \(2011a\)](#) & [COMPLETE team \(2011b\)](#)); **Bottom left:** ^{13}CO Sroll2.1; **Bottom right:** ^{13}CO FCRAO survey ([COMPLETE team \(2011b\)](#)).

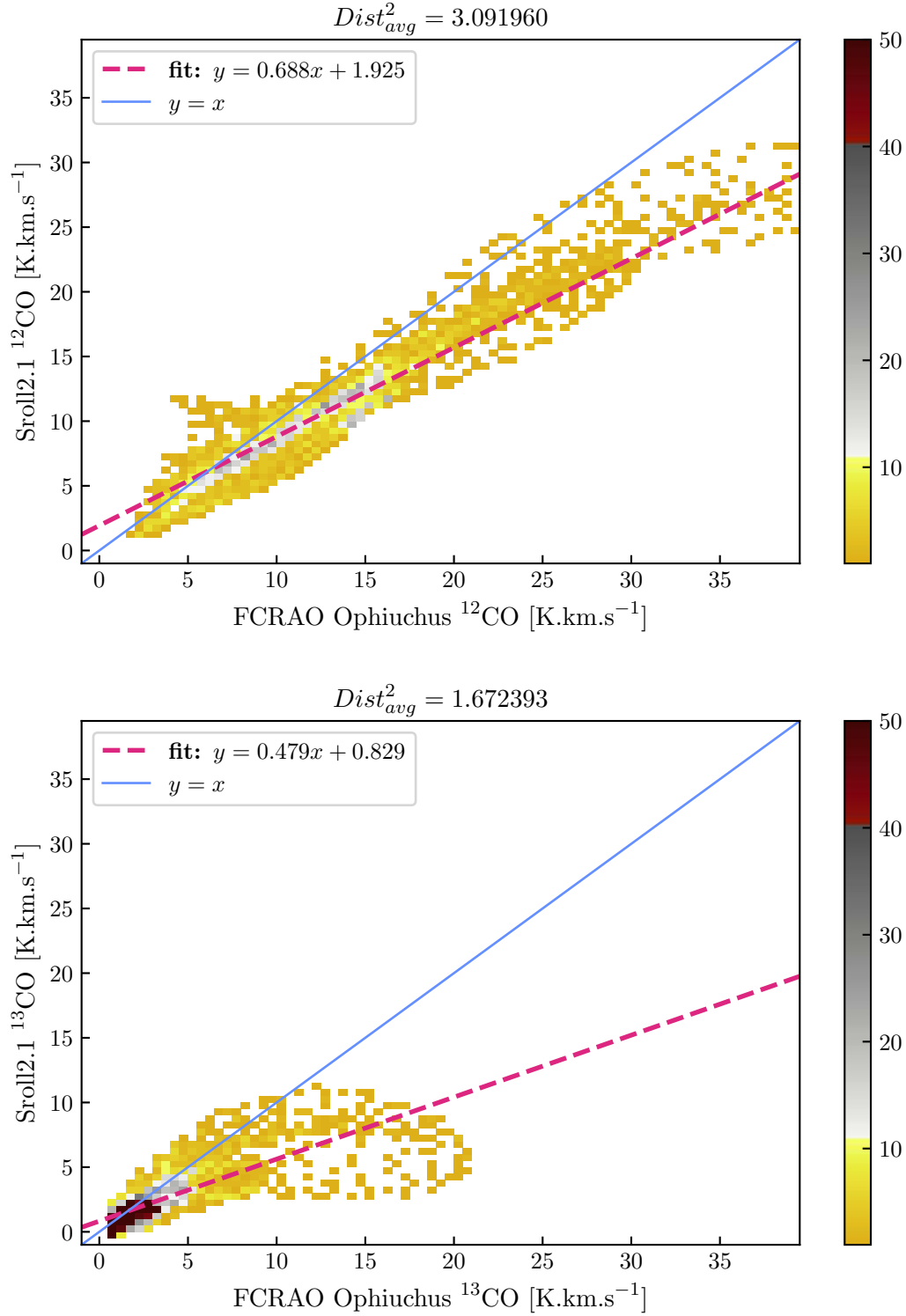


Fig. V.13 Correlation plots between the SRoll2.1 $J = 1 \rightarrow 0$ CO maps and the FCRAO survey of Ophiuchus ([COMPLETE team \(2011a\)](#) & [COMPLETE team \(2011b\)](#)). Both maps have been set to a Gaussian beam with FWHM = 30'. Plot computed using only pixels with an intensity superior to 1 K.km.s⁻¹ in the FCRAO map considered. $Dist_{avg}^2$ corresponds to the average of squared distances of points with respect to the fit. **Top:** ¹²CO, **Bottom:** ¹³CO. **Red dashed line** is the best affine fit of the cloud of points; **Blue line** is the identity function.

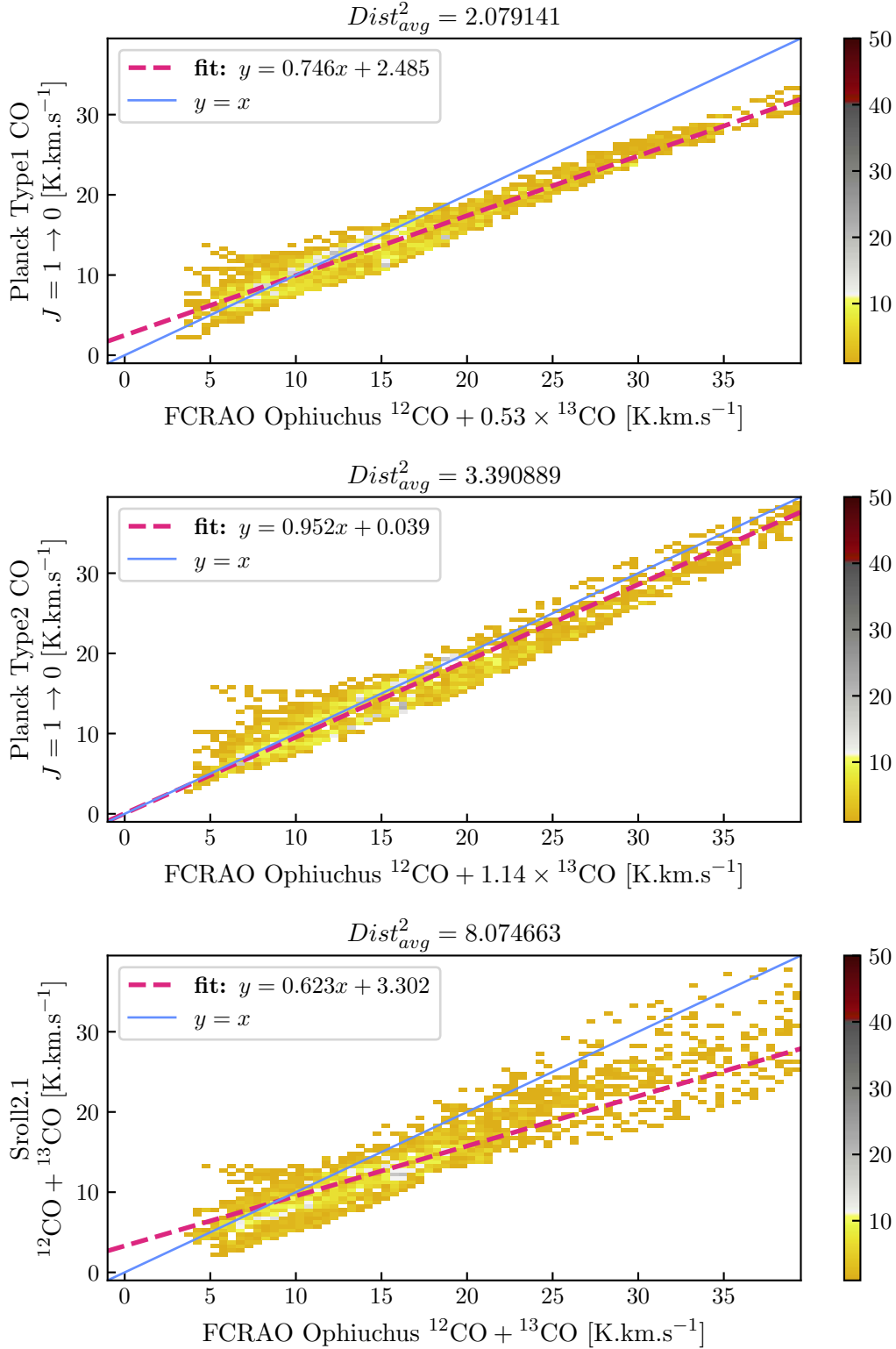


Fig. V.14 Correlation plots between the Planck $J = 1 \rightarrow 0$ CO maps and the adequate combination of ^{12}CO and ^{13}CO maps from the FCRAO survey of Ophiuchus (*COMPLETE team (2011a)* & *COMPLETE team (2011b)*). All maps have been set to a Gaussian beam with FWHM = 30'. Plot computed using only pixels with an intensity superior to 1 K.km.s $^{-1}$ in both the ^{12}CO and ^{13}CO FCRAO maps. $Dist_{avg}^2$ corresponds to the average of squared distances of points with respect to the fit. **Top:** Planck Type 1, **Middle:** Planck Type 2, **Bottom:** Sroll2.1 $^{12}\text{CO} + ^{13}\text{CO}$. **Red dashed line** is the best affine fit of the cloud of points; **Blue line** is the identity function.

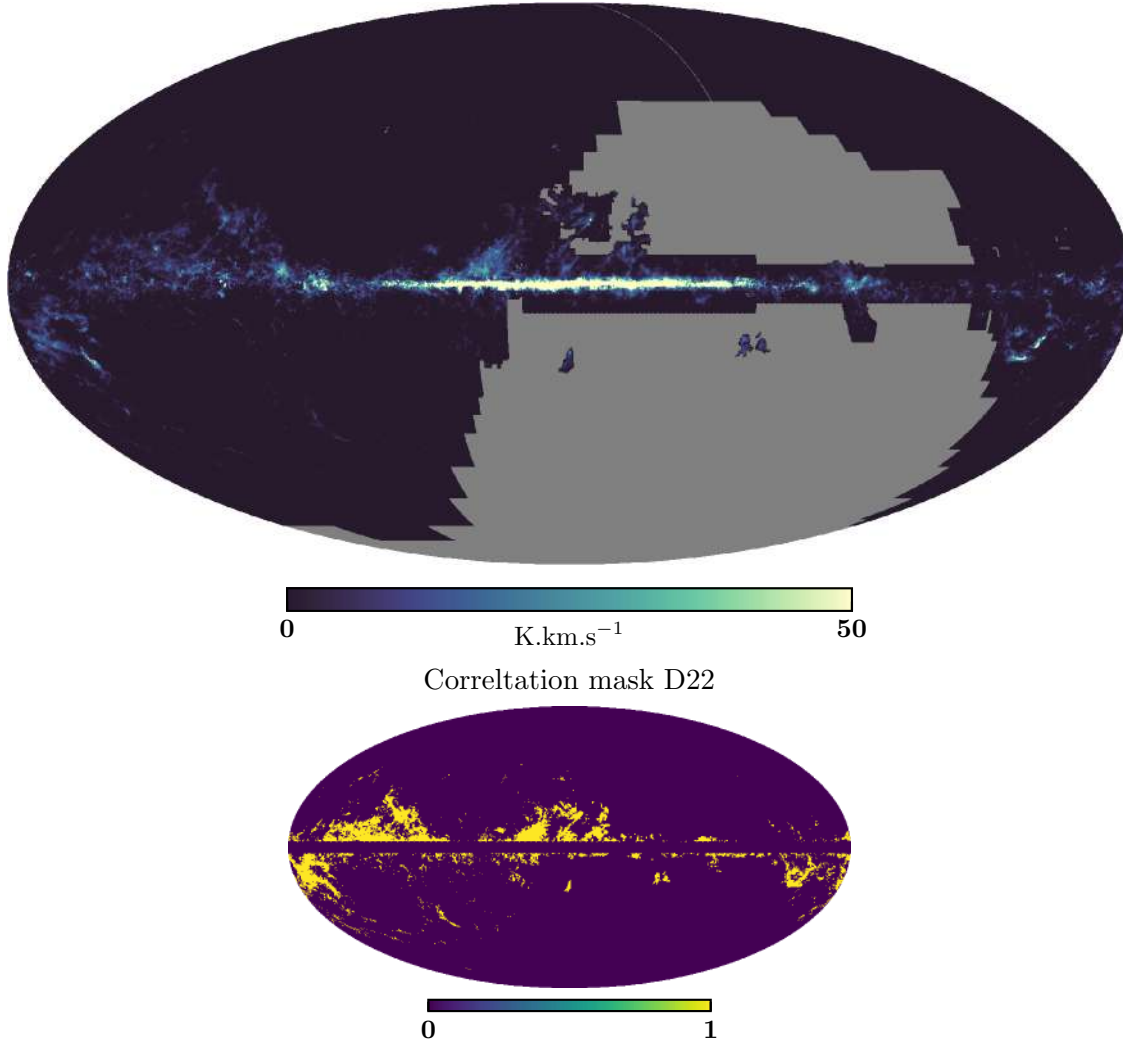


Fig. V.15 **Top:** Velocity integrated map of the $J = 1 \rightarrow 0$ ^{12}CO survey of [Dame & Thaddeus \(2022\)](#). The map was projected in HEALPix with a resolution of $N_{\text{side}} = 2048$. **Bottom:** mask indicating the pixels considered for the correlation plots with D22, i.e. with an intensity superior to 1 K.km.s^{-1} in D22, as well as with a latitude either superior to 3° or inferior to -3° .

these maps to search for CO signal at high latitude in regions of low column density.

V.1.3 Comparison to the ^{12}CO survey of [Dame & Thaddeus \(2022\)](#)

The last survey I used to qualify my SRoll2.1 $J = 1 \rightarrow 0$ maps is the ^{12}CO survey of [Dame & Thaddeus \(2022\)](#), which is the current CO survey with the largest sky coverage, covering almost the entire northern sky, as well as about a third of the southern sky, amounting to a total of $\sim 20000\text{deg}^2$. The survey uniformly sampled the sky with a true-angle spacing of $\sim 0.25^\circ$ or better. The map of the combined moment-masked cube from the survey (cf. No.5 from Table.3 in [Dame & Thaddeus \(2022\)](#)), integrated along the velocity and projected in HEALPix (see Appendix.A) with a resolution of $N_{\text{side}} = 2048$ is plotted in Fig.V.15. Note that most of the high latitude regions are set to zero by the moment-masking performed on the survey data, masking regions with no significant signal detection.

As I did with the previous surveys, I compare my SRoll2.1 $J = 1 \rightarrow 0$ ^{12}CO map to the

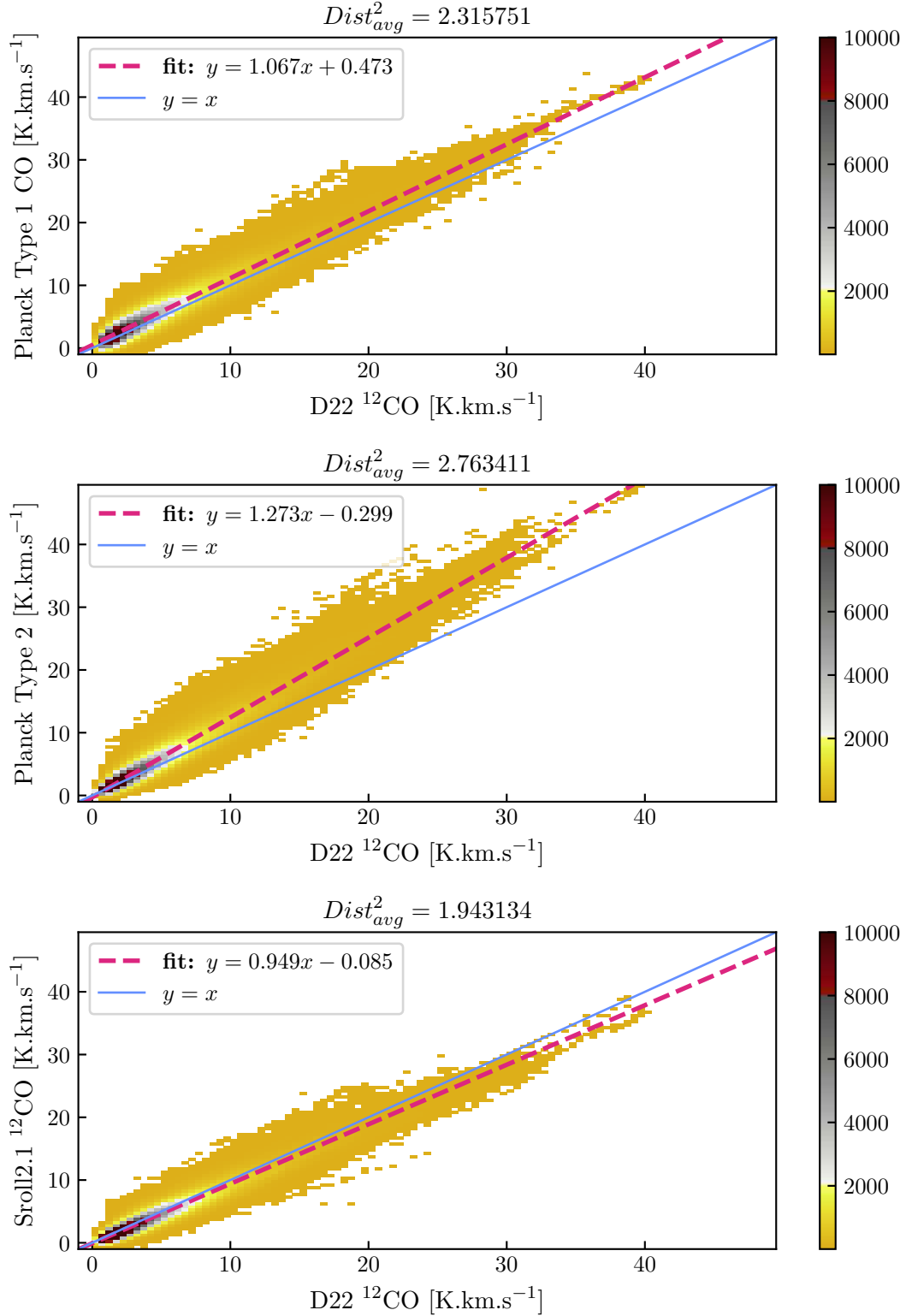


Fig. V.16 Correlation plots with the $J = 1 \rightarrow 0$ ^{12}CO survey of Dame & Thaddeus (2022). All maps have been set to a Gaussian beam with FWHM = 30'. Plot computed using only pixels with an intensity superior to 1 K.km.s $^{-1}$ in the map from Dame & Thaddeus (2022), as well as with a latitude either superior to 3° or inferior to -3° in order to exclude the innermost part of the galactic disk. $Dist_{avg}^2$ corresponds to the average of squared distances of points with respect to the fit. **Top:** Planck Type 1, **Middle:** Planck Type 2, **Bottom:** Sroll2.1 ^{12}CO . **Red dashed line** is the best affine fit of the cloud of points; **Blue line** is the identity function.

survey from Dame & Thaddeus (2022) by computing the corresponding correlation plot. For comparison, I also plotted the same correlation plots with the *Planck* $J = 1 \rightarrow 0$ Type 1 and Type 2 CO maps. The 3 resulting correlation plots are displayed in Fig.V.16. Note that the affine fit is, in the 3 cases, computed only on pixels with a signal intensity superior to 1 K.km.s^{-1} in the integrated map from Dame & Thaddeus (2022), as well as located at a latitude either superior to 3° or inferior to -3° in order to exclude the innermost part of the galactic disk. This innermost part of the galactic disk is a strong signal gradient region, where the subpixel variation of signal is expected to be high, leading to poor CO map quality in both the SRoll2.1 and the *Planck* legacy maps. This pixel selection for the affine fit computation keeps a region on the sky equal to 5.66% of the total sky surface, and is plotted in Fig.V.16. The maps are also all put to the same resolution of $N_{\text{side}} = 1024$ and smoothed to a Gaussian beam of $\text{FWHM} = 30'$. We can see a very good calibration consistency with the survey from Dame & Thaddeus (2022) for the SRoll2.1 ^{12}CO map and the Type 1 *Planck* map, displaying affine fit factors of respectively 0.949 and 1.067. The Type 2 *Planck* CO map is less consistent, with an affine fit of 1.273, but this is most probably due to the presence of ^{13}CO in the Type 2 CO map, as was previously seen in subsection V.1.1.b). The goodness of fit is also better for the SRoll2.1 ^{12}CO map, with $\text{Dist}_{\text{avg}}^2 = 1.94$, compared to $\text{Dist}_{\text{avg}}^2 = 2.31$ for the Type 1 map, and $\text{Dist}_{\text{avg}}^2 = 2.76$ for the Type 2 map. However, this difference should be taken with a grain of salt, as the Type 1 and Type 2 maps contain ^{13}CO signal not accounted for in these correlation plots.

V.1.4 $J = 2 \rightarrow 1$ comparison to the *Planck* CO maps

I did not have at my disposal maps of CO surveys of the $J = 2 \rightarrow 1$ lines with a *large* sky coverage to qualify my SRoll2.1 ^{12}CO and ^{13}CO maps. However, I can still use the *Planck* legacy $J = 2 \rightarrow 1$ CO maps as a reference to evaluate the spatial distribution of the signal in my CO maps. It should be noted that these 2 maps come from the same data, and are therefore not independent. Additionally, I remind that, as was detailed in subsection IV.2.2, my $J = 2 \rightarrow 1$ ^{12}CO and ^{13}CO maps are *recalibrated* using the *Planck* Type 1 $J = 2 \rightarrow 1$ CO map.

I first provide a visual appreciation of the aforementioned comparison in Fig.V.17, where I plotted the Taurus region from the Type 1 and Type 2 *Planck* $J = 2 \rightarrow 1$ CO maps, as well as my SRoll2.1 $J = 2 \rightarrow 1$ ^{12}CO and ^{13}CO maps. All 4 maps were brought to a Gaussian beam of $\text{FWHM} = 30'$ to ease the comparison. The 2 main observations we can draw from these maps are that my maps seem to efficiently recover the CO emission from the sky signal, **but** the recovered emission's spatial distribution appears a lot *sharper* than in the *Planck* legacy CO maps. Also, the denser regions of higher intensity (similar to the ones previously observed in the $J = 1 \rightarrow 0$ maps) appear to display *greater intensity* in my SRoll2.1 maps than in the *Planck* legacy ones. It is the exact *inverse* observation to the one that was seen in my $J = 1 \rightarrow 0$ CO maps. Nevertheless, as will be further discussed in Section V.2 below, I reckon that these 2 *inverse* observations in my $J = 1 \rightarrow 0$ and $J = 2 \rightarrow 1$ CO maps are due to the same effect, stemming from the limitations of using the 143GHz frequency band in the computation of my ^{12}CO and ^{13}CO maps.

I further quantified this comparison of my maps with the *Planck* legacy ones by computing the correlation plot between my maps and the *Planck* Type 2 $J = 2 \rightarrow 1$ CO map, displayed in Fig.V.18. This figure displays both the correlation plots of my $J = 2 \rightarrow 1$ SRoll2.1 ^{12}CO map with the *Planck* Type 2 map, as well as that of the combination of SRoll2.1 ^{12}CO and ^{13}CO maps corresponding to the $J = 2 \rightarrow 1$ Type 2 ^{12}CO and ^{13}CO contributions, i.e. $^{12}\text{CO} + 0.82 \times ^{13}\text{CO}$ (cf. subsection II.4.3), with the *Planck* Type 2 map.

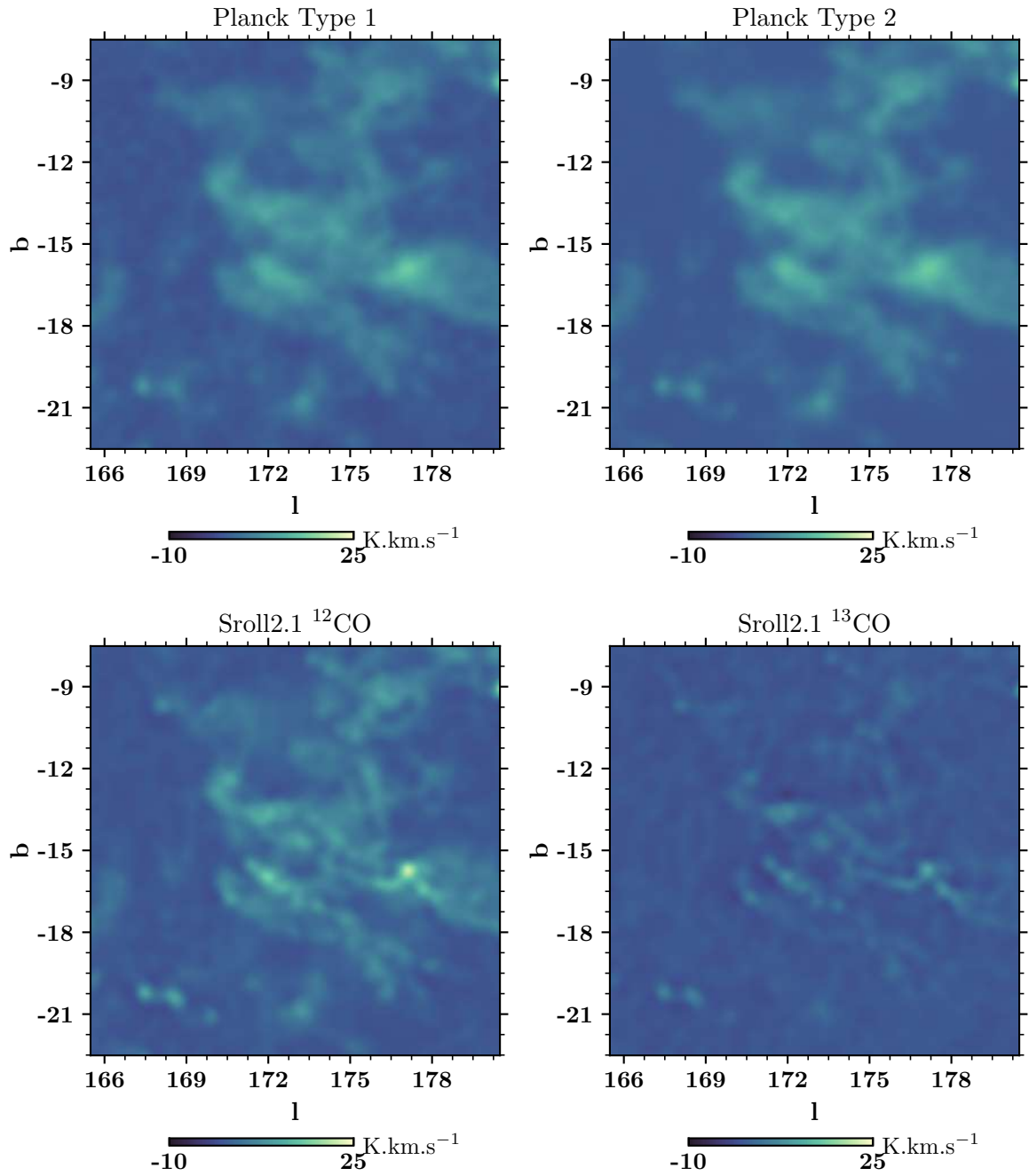


Fig. V.17 CO $J = 2 \rightarrow 1$ maps of the Taurus region smoothed up to a Gaussian beam of FWHM = 30'. **Top left:** Planck Type 1; **Top right:** Planck Type 2; **Bottom left:** Sroll2.1 ^{12}CO ; **Bottom right:** Sroll2.1 ^{13}CO .

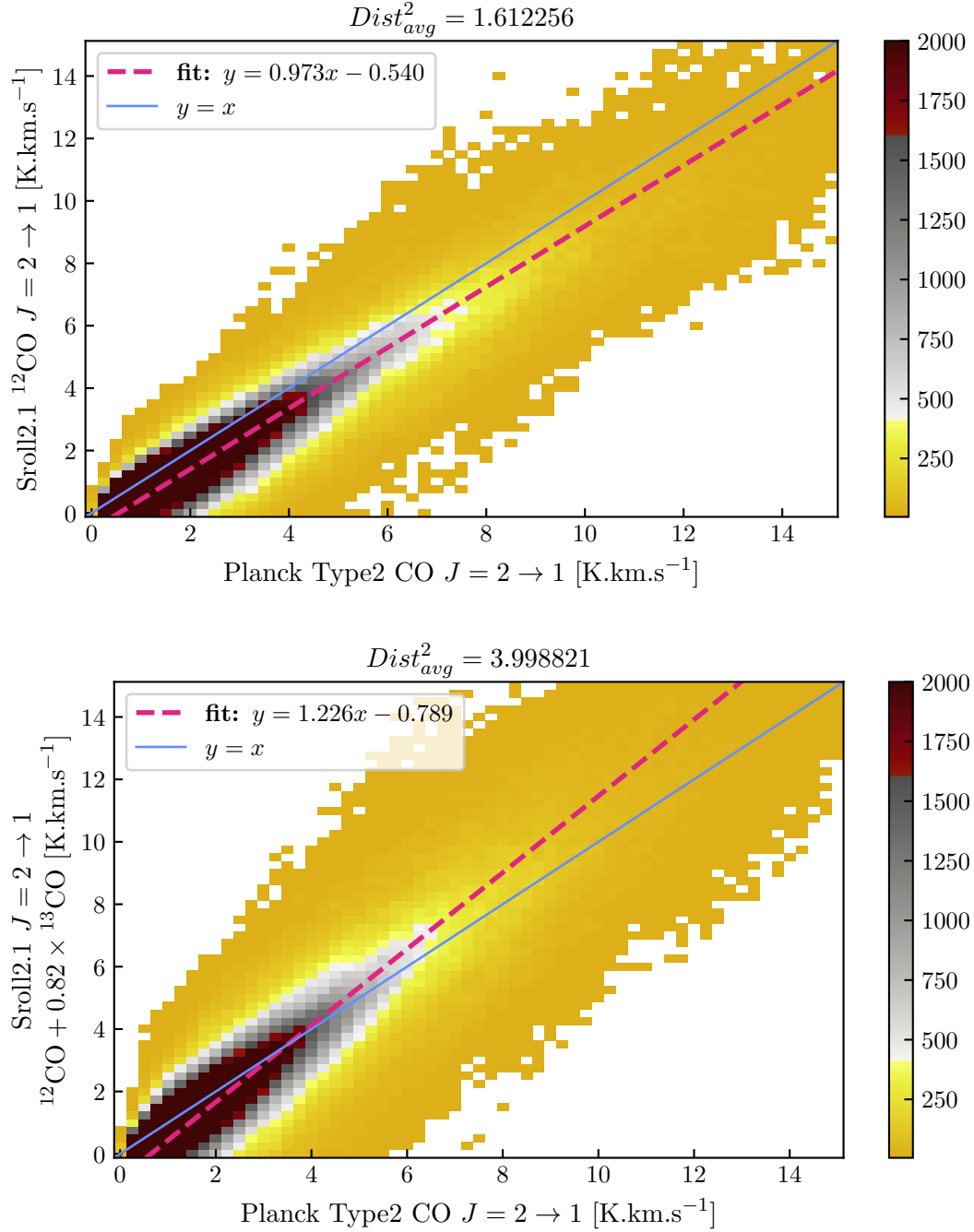


Fig. V.18 Correlation plots between the SRoll2.1 $J = 2 \rightarrow 1$ maps and the Planck Type 2 $J = 2 \rightarrow 1$ CO maps. Both maps have been set to a Gaussian beam with FWHM = 30', and performed using only pixels of intensity greater than 0.5 K.km.s⁻¹ in the unsmoothed Type 2 map, as well as located at a latitude either superior to 3° or inferior to -3° in order to exclude the innermost part of the galactic disk. **Top:** correlation plot using the SRoll2.1 ^{12}CO ; **Bottom:** correlation plot using the combination of SRoll2.1 ^{12}CO and ^{13}CO maps corresponding to the Type 2 map, i.e. $^{12}\text{CO} + 0.82 \times ^{13}\text{CO}$ (cf. subsection II.4.3).

I remind that the correlation plot between my SRoll2.1 CO maps and the *Planck* Type 1 map was plotted in Fig. IV.24 to determine the SRoll2.1 maps *recalibration*. The overall correlation between the SRoll2.1 CO maps and the *Planck* legacy CO maps is good. However, an interesting observation is that both the calibration consistency and the goodness-of-fit seen in Fig. V.18 are better with the SRoll2.1 ^{12}CO map *alone* than with the ^{12}CO and ^{13}CO combination. This observation may indicate a signal leakage from the ^{13}CO to the ^{12}CO SRoll2.1 map. Indeed, if my map supposed to contain *only* ^{12}CO gives the best correlation with a map *known* to contain ^{13}CO , then my ^{12}CO map most probably also contains some ^{13}CO signal. This leakage in turn may be explained by the combination of 2 effects: the small difference between the average RDCs of the ^{12}CO and ^{13}CO foregrounds (see Fig. IV.13), **and** the low quality of the ^{13}CO RDC distribution distinctness, as indicated by the low value of the ratio $\left. \frac{\sigma_{\delta L^{13}\text{CO}}}{\Delta_{\delta L^{13}\text{CO}}} \right|_{217} = 1.36$. These 2 combined effects, paired with the high spatial correlation between the ^{12}CO and ^{13}CO foregrounds, probably explain this ^{13}CO to ^{12}CO leakage in the $J = 2 \rightarrow 1$ SRoll2.1 maps. Also, it should be noted that this suspected leakage is supported by the fact that an attempt at a ^{12}CO and ^{13}CO maps construction using **only the bolometers from the 217GHz band** leads to the production of a ^{13}CO map mostly negative. Additionally, the aforementioned effect of *overestimation* of the intensity in the peaked high density region of the ^{12}CO and ^{13}CO $J = 2 \rightarrow 1$ SRoll2.1 maps, further discussed in Section V.2, is expected to increase these observed discrepancies of Fig. V.18.

V.2 $J = 1 \rightarrow 0$ **composite CO maps to correct the peaked signal loss**

We have seen in the previous section that my SRoll2.1 maps tend to fail to accurately recover the most sharply peaked regions of signal in the ^{12}CO and ^{13}CO maps. This section is dedicated to discussing my interpretation of this issue, followed by suggestions of ways in which this might be corrected.

V.2.1 Limit of the 143GHz band to compute the CO maps

To sum up the issue at hand: the aforementioned SRoll2.1 maps failing at accurately recovering the most sharply peaked regions of signal is present in both ^{12}CO and ^{13}CO maps, though most evident in the ^{13}CO map, and also appears for both the $J = 1 \rightarrow 0$ and the $J = 2 \rightarrow 1$ lines. However, the nature of this effect is reversed between said lines: in the $J = 1 \rightarrow 0$ maps the peaked regions of higher signal intensity tend to be *underestimated* and appear as signal dips in the maps, whereas in the $J = 2 \rightarrow 1$ maps the *same regions* appear *overestimated*, and the overall spatial distribution of signal displays significantly sharper features than expected.

My understanding of this phenomenon is the following: this issue is the symptom of the limitations of my method of maps construction, and especially is due to the use of the bolometers from the 143GHz frequency band to compute said maps. Indeed, the addition of the information from the bolometers of the 143GHz band adds complexity to the overall structure and content of the sky signal, i.e. it complexifies my sky model. This is mainly the reason why it brings the improvements that were listed in subsection IV.1.1.a), but it also requires to take this increased complexity into account in my sky model. This is typically why the additional DustVar template was introduced in the SRoll2.1 version of the algorithm. However, it is probable that the regions of highest column density, and of

sharpest signal spatial features, also correspond to the regions of highest emission *colour* variation, i.e. to the regions of greatest spatial variation of SED for the various foregrounds. If so, it is possible that my simple model of this effect fails in these regions. In turn, the failing in accurately modelling this effect would let the separation algorithm interpret this spatial variation of SED between the 100GHz and 143GHz band, or equivalently the 217GHz and 143GHz bands, as CO signal.

Additionally, the separation model described in Section IV.2 is based on the idea that the signal of *all bolometers used to build the maps* (i.e. along the 2 frequency bands) have been brought to the same average sky signal along the distribution of said bolometers, thanks to the *realignment* procedure of SRoll2.1 (see Eq.IV.1). From this hypothesis, the 3 maps of the remaining sky signal from the separation, i.e. I^{other} , Q^{other} , and U^{other} , are built to be *the same* along the 2 frequency bands used. However, the templates used to compute the realignment have a low resolution of only $N_{side} = 128$, which fails to capture the small-scale variations of peaked signal. Therefore, in regions of peaked high intensity signal, the *non-CO* foreground emissions may retain some discrepancies between the 100GHz band (or 217GHz band) and the 143GHz one, as opposed to the hypothesis made in the maps computation. If so, the aforementioned discrepancies, most probably dominated by the dust emission and its SED spatial variation, are expected to display higher intensity in the 217GHz band than in the 143GHz band, which in turn is expected to be more intense than the 100GHz band in these regions. Since I force the I^{other} , Q^{other} , and U^{other} maps to be the same for *all bolometers*, the only lever left to the algorithm to account for these discrepancies is through the ^{12}CO and ^{13}CO maps. Finally, given the aforementioned intensity ordering between frequency bands, the imprinting of these discrepancies is expected to be exactly as it is observed in the maps: a signal dip in the 100-143GHz maps and a stronger peak in the 217-143GHz ones. This interpretation is in turn entirely consistent with the distribution of the observed dust $\rightarrow$ CO leakage which was presented in subsection IV.3.1 and which coincides with the regions of signal dip in the 100-143GHz ^{12}CO and ^{13}CO maps.

An easy way to check this interpretation is to compute the ^{12}CO and ^{13}CO maps without using the bolometers from the 143GHz band, and checking if the effect disappears. The result from this test for the $J = 1 \rightarrow 0$ line is displayed in Fig.V.19. I plotted the Taurus region from the ^{12}CO and ^{13}CO maps built from both 100GHz and 143GHz frequencies as well as the same maps computed using *only* the bolometers from the 100GHz frequency band. All maps were smoothed up to a Gaussian beam with $\text{FWHM} = 15'$, i.e. the widest beam size among the maps plotted. We can see that the signal dip in the peak high intensity regions disappears when computing the maps from *only* the bolometers from the 100GHz frequency band, confirming my interpretation.

Another way to visualize this effect, as well as to confirm my interpretation of its underlying mechanism, is to plot the full CO maps histogram. This plot, which corresponds to the plot of Fig.34 from [Planck Collaboration X \(2016\)](#), is displayed in Fig.V.20. This figure displays the unmasked histogram of the SRoll2.1 ^{12}CO maps of the $J = 1 \rightarrow 0$ and $J = 2 \rightarrow 1$ lines, as well as those of the *Planck* legacy Type 1 and Type 2 maps for the same two lines. The interesting part is the shape of the negative part of this histogram. Since these CO maps are supposed to represent an incoming power intensity of the CO emission, the *negative* pixels should only be caused by the noise, and are therefore expected to display a Gaussian distribution. This is mostly verified for all maps except the SRoll2.1 $J = 2 \rightarrow 1$ map, which displays a non-Gaussian negative tail in the histogram. I understand this tail as signal from the same effect discussed above. Again, to verify this interpretation, I also plotted in Fig.V.20 the histogram from the SRoll2.1 ^{12}CO $J = 2 \rightarrow 1$ map built from *only* the bolometers from the 217GHz frequency band. We can see that the tail disappears when

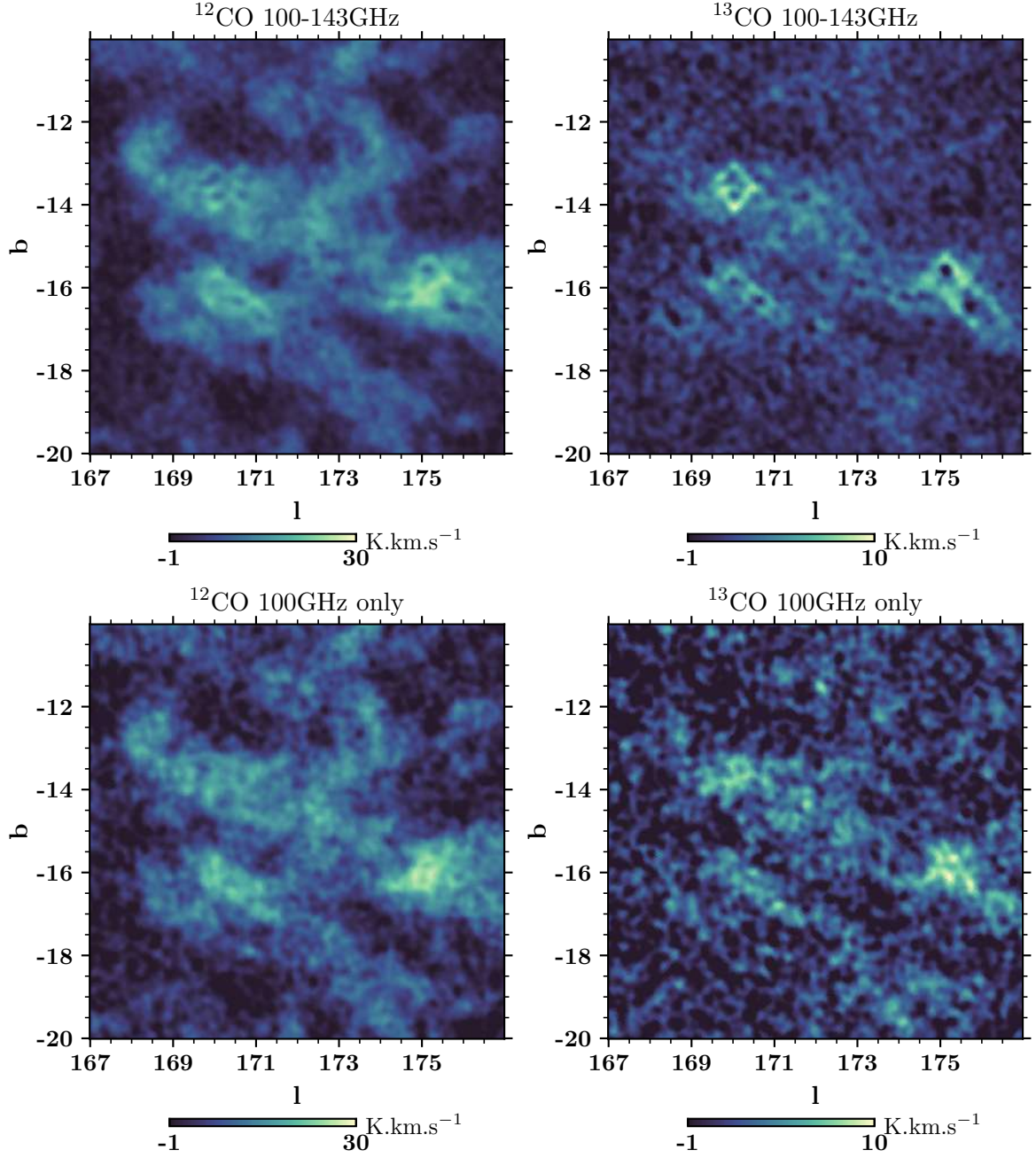


Fig. V.19 $J = 1 \rightarrow 0$ ^{12}CO (*left*) and ^{13}CO (*right*) maps built from the output of a 100-143GHz SRoll2.1 run. **Top:** computed using all bolometers from both the 100GHz and 143GHz bands; **Bottom:** computed using only the bolometers from the 100GHz band. All maps were smoothed up to a Gaussian beam with $\text{FWHM} = 15'$.

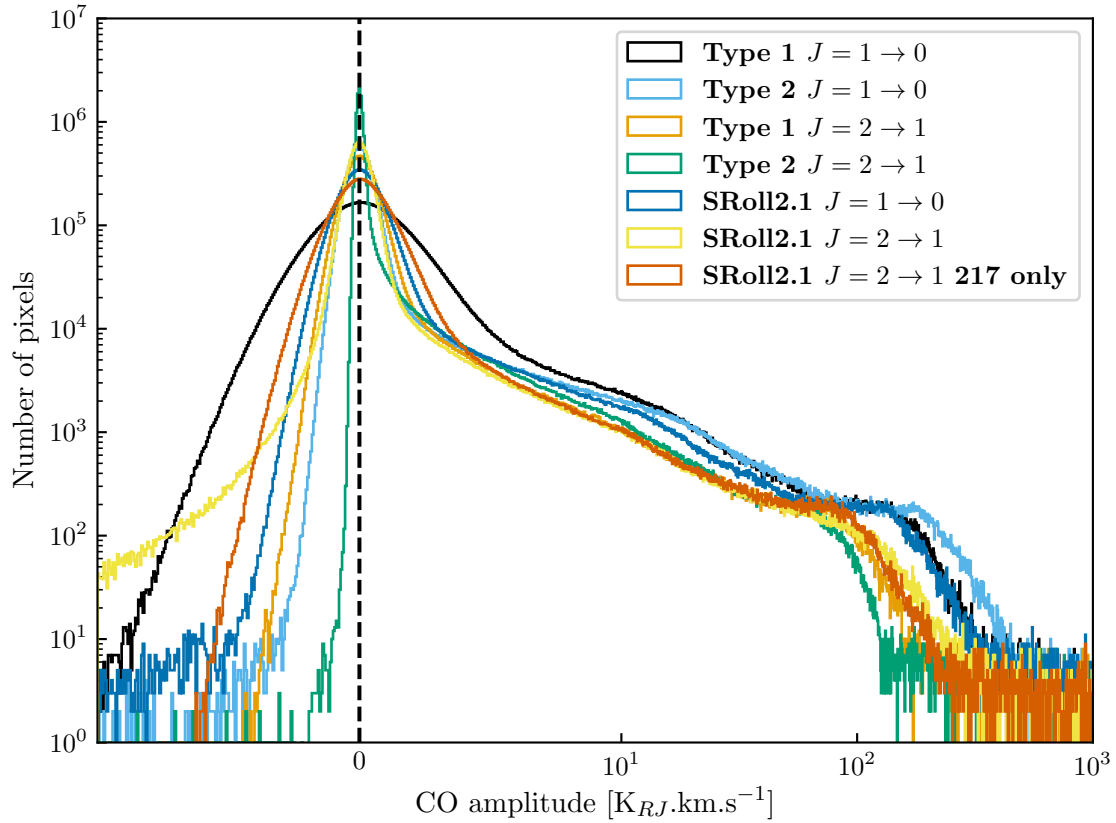


Fig. V.20 Histogram of SRoll2.1 ^{12}CO maps as well as the Planck legacy Type 1 and Type 2 CO maps. In each case, both $J = 1 \rightarrow 0$ and $J = 2 \rightarrow 1$ CO lines are plotted. The SRoll2.1 ^{12}CO $J = 2 \rightarrow 1$ map computed using **only** the bolometers from the 217GHz band is also plotted. All maps are brought to a Gaussian beam with FWHM = $15'$, i.e. the widest among the original maps. X-axis is in semi-logarithmic scale: the $[-10;10]$ interval is linear and the $[10;1000]$ interval is logarithmic.

Corresponds to the plot of Fig.34 from *Planck Collaboration X (2016)*.

not using the bolometers from the 143GHz band, again confirming my interpretation of the previously described mechanisms at play.

NB: Nevertheless, I should note that, as was observed in the maps, this effect is seen only in the most peaked *and* higher signal intensity regions. Therefore, this effect is not expected to impact my results in the following Section V.3 where I will explore the CO signal at high latitude *and* low column density, i.e. low intensity, regions. Additionally, the intermittent structures from which any potential CO signal at high latitude may originate are too small to be resolved by my maps at their resolution of $N_{\text{side}} = 1024$, therefore minimizing the risk of being impacted by this effect.

V.2.2 A straightforward solution: *composite* CO maps

The best way to correct this effect at its root would probably be to improve directly the SRoll2.1 foreground templates. Namely, the best improvements to reduce this effect would be twofold:

1. Improve the spatial variation of SED of the templates, e.g. using a better DustVar

template map, such as the dust SED variation model determined in Delouis, J.-M. et al. (2021), and adding higher order templates for the dust SED variation,

2. Increasing the resolution of the SRoll2.1 template maps.

The first point only adds complexity to the sky model but can be straightforwardly included in the current algorithm structure. However, the second point would require to correct the signal of all bolometers for the difference in beam shape and size, as previously discussed in subsection IV.3.4.

Another correction, which could be attempted at the level of the maps construction, is the addition in the separation model of Eq.IV.8 of an intensity map to be fitted on the 143GHz band bolometers only. This would add another map to be determined, therefore slightly increase the statistical uncertainty in the separation, but it would provide the algorithm with a "lever" to account for the remaining sky signal discrepancies between the 100GHz (or 217GHz depending of the case) and the 143GHz band, avoiding these discrepancies to be forcibly imprinted in the CO maps.

Given that I lacked the time to explore these 2 potential solutions, I devised a more straightforward correction which does not require any modification of the algorithms presented in this work, and only rests upon the maps already built. Indeed, as can be seen in Fig.V.19, the CO maps built only using the bolometers from the 100GHz band, though significantly more noisy, do not display a signal dip in the regions of peaked signal. Therefore, the correction can be implemented by building *composite* CO maps: the regions of intensity dip in the CO maps built from both 100GHz and 143GHz can be replaced by the same pixels from the maps built using *only* the 100GHz band. Since I had at my disposal only maps from external surveys for the $J = 1 \rightarrow 0$ line, as discussed in the previous section, I tested this solution only on the $J = 1 \rightarrow 0$ CO maps, from a 100-143GHz SRoll2.1 run.

a) Composite maps building method

The correction method I got to try out therefore relies on: firstly identifying the regions from the 100-143GHz CO maps with a signal dip, and secondly replacing those regions by the corresponding ones from the maps built using the 100GHz band only.

I computed the selection of the regions to be replaced by using my ^{12}CO SRoll2.1 $J = 1 \rightarrow 0$ maps. The idea is to identify regions of intensity dip in my maps, which should be of fairly high intensity. To this end, I build a mask based on 2 conditions which **must** be satisfied (these 2 conditions are defined on the maps smoothed up to a Gaussian beam with FWHM = 15'):

1. The ratio between the ^{12}CO map built from both 100-143GHz frequency bands over the one built *only* from the 100GHz band must be inferior to 0.85, i.e. $M_{100-143}^{12\text{CO}}/M_{100}^{12\text{CO}} < 0.85$, where $M_{100-143}^{12\text{CO}}$ is the ^{12}CO map built from both 100-143GHz frequency bands, and $M_{100}^{12\text{CO}}$ is the ^{12}CO map built *only* from the 100GHz band,
2. The intensity in the ^{12}CO map built *only* from the 100GHz band must be greater than 16 K.km.s $^{-1}$, i.e. $M_{100}^{12\text{CO}} > 16 \text{ K.km.s}^{-1}$. The threshold value of 16 K.km.s $^{-1}$ was determined by trial and error, by manually tuning it so that the selected regions were sufficiently representative of the dip regions observed in the G08 and FCRAO surveys.

Then, to avoid the introduction of spurious signal "jumps" in the maps along the edges of the replaced regions, I smoothed the mask defined by the 2 conditions defined above, with a Gaussian beam of FWHM = 2'. This smooths the edges of the mask. This smoothed version of the mask, used for the computation of the composite maps, is displayed in Fig.V.21.

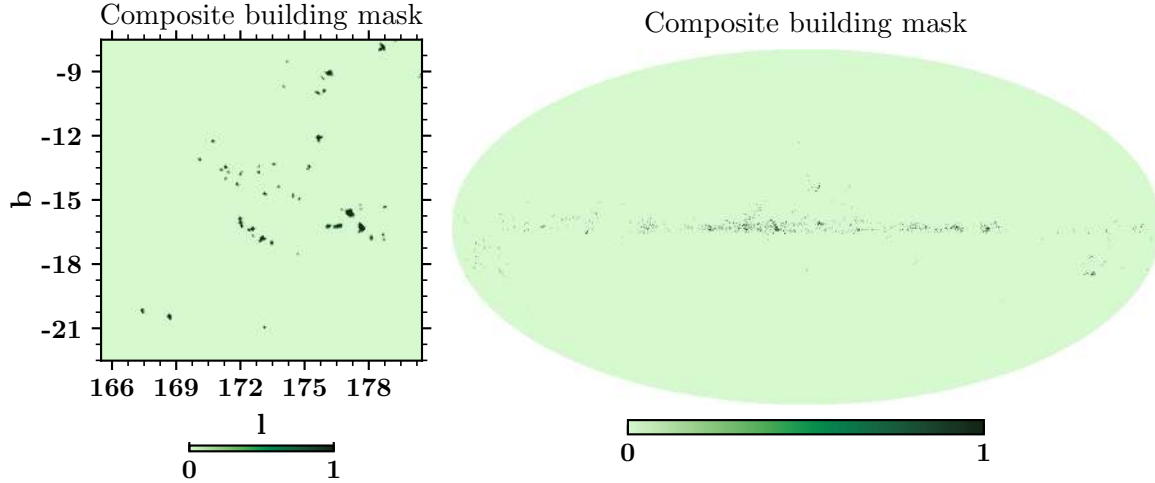


Fig. V.21 Mask built to compute the composite *S*Roll2.1 $J = 1 \rightarrow 0$ CO maps (see Eq.V.1). First, a signal dip selection mask is built, determined by keeping pixels verifying both conditions $M_{100-143}^{12CO}/M_{100}^{12CO} < 0.85$ and $M_{100}^{12CO} > 16 \text{ K.km.s}^{-1}$, which is then smoothed with a Gaussian beam of FWHM = $2'$. **Left:** Taurus region; **Right:** full-sky map.

Finally, the composite ^{12}CO and ^{13}CO $J = 1 \rightarrow 0$ maps can be constructed using the following expression:

$$\begin{aligned} M_{compo}^{12CO} &= M_{100-143}^{12CO}(1 - M_{compo}^{mask}) + M_{100}^{12CO} M_{compo}^{mask}, \\ M_{compo}^{13CO} &= M_{100-143}^{13CO}(1 - M_{compo}^{mask}) + M_{100}^{13CO} M_{compo}^{mask}, \end{aligned} \quad (\text{V.1})$$

where M_{compo}^{mask} is the smoothed mask defined above.

The composite map construction thereby defined can be enhanced by improving the method to build the mask M_{compo}^{mask} . Indeed, my method of selection of the regions to be replaced is crude and can be improved by making use of edge detection algorithms, e.g. Canny-edge (cf. [Canny \(1986\)](#)), to detect the peaked region even at lower intensity (which can be seen in the 100-143GHz CO maps at intensities of $\sim 7 \text{ K.km.s}^{-1}$).

I displayed the final ^{12}CO and ^{13}CO composite maps hereby produced in Fig.V.22.

b) Correction results

I checked the quality of the *composite* CO maps built using the method defined above by comparing them to the same external CO surveys introduced and exploited in Section V.1. Notably, I recomputed the correlation plots with each survey and compared the results between the original *S*Roll2.1 100-143GHz maps and the *composite* maps.

Taurus region ([Goldsmith et al. \(2008\)](#)) First, I compared again with the [Goldsmith et al. \(2008\)](#) survey. I plotted in Fig.V.23 the ^{12}CO and ^{13}CO emissions of the Taurus region from the original *S*Roll2.1 100-143GHz maps, the *composite* maps, and the maps of the [Goldsmith et al. \(2008\)](#) survey, where all maps are smoothed up to a beam of FWHM = $30'$. We can see (better seen on the ^{13}CO map than on the ^{12}CO one) that the signal dips have been filled, reaching features closer to the ones seen in the survey of [Goldsmith et al. \(2008\)](#). Then, in Fig.V.24, I computed the correlation plots between the *composite* CO maps and the ones from the survey of [Goldsmith et al. \(2008\)](#). For comparison, I also displayed the correlation plots of the original *S*Roll2.1 100-143GHz corresponding

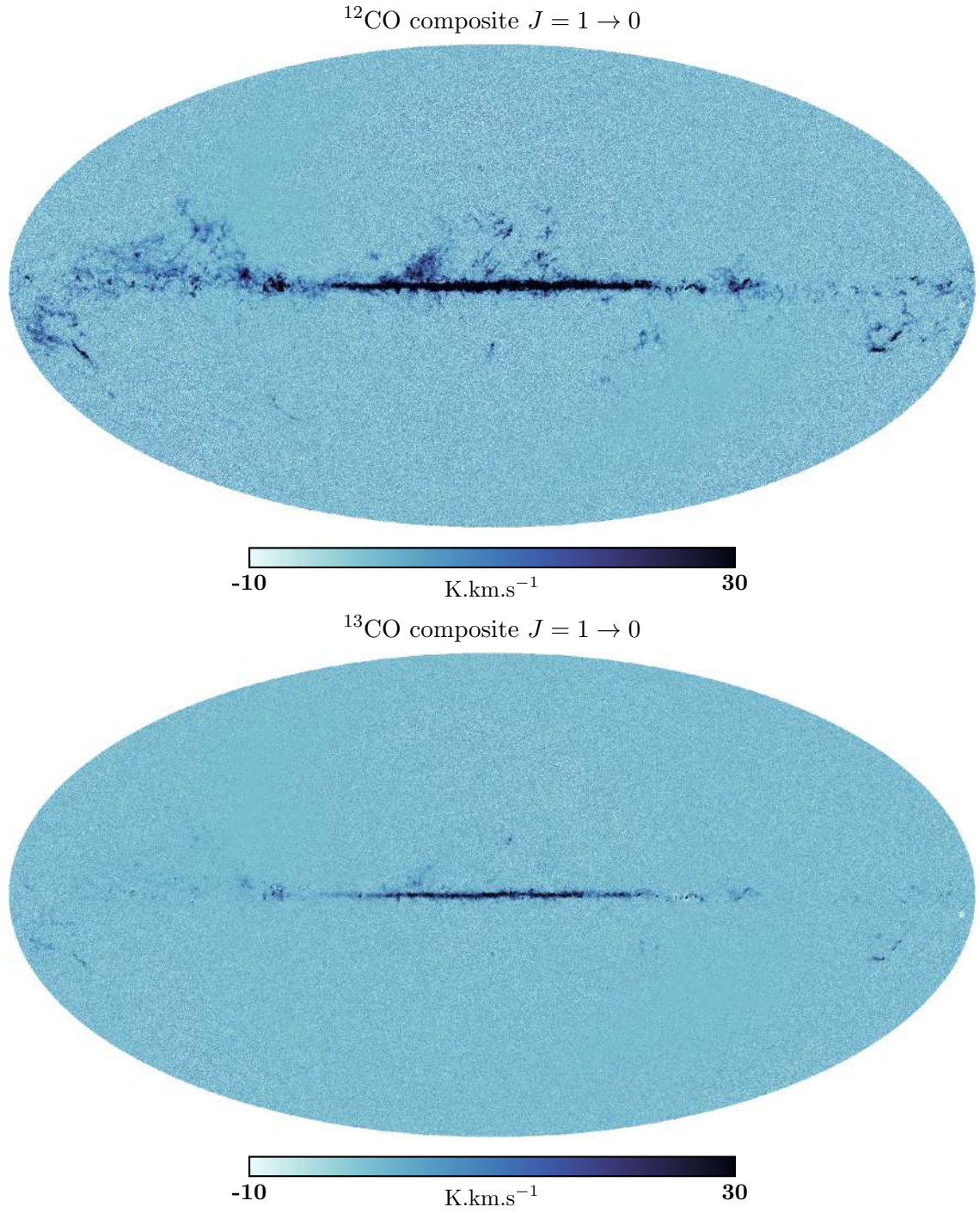


Fig. V.22 *Fullsky* composite CO maps of the $J = 1 \rightarrow 0$ line. **Top:** ^{12}CO , **Bottom:** ^{13}CO .

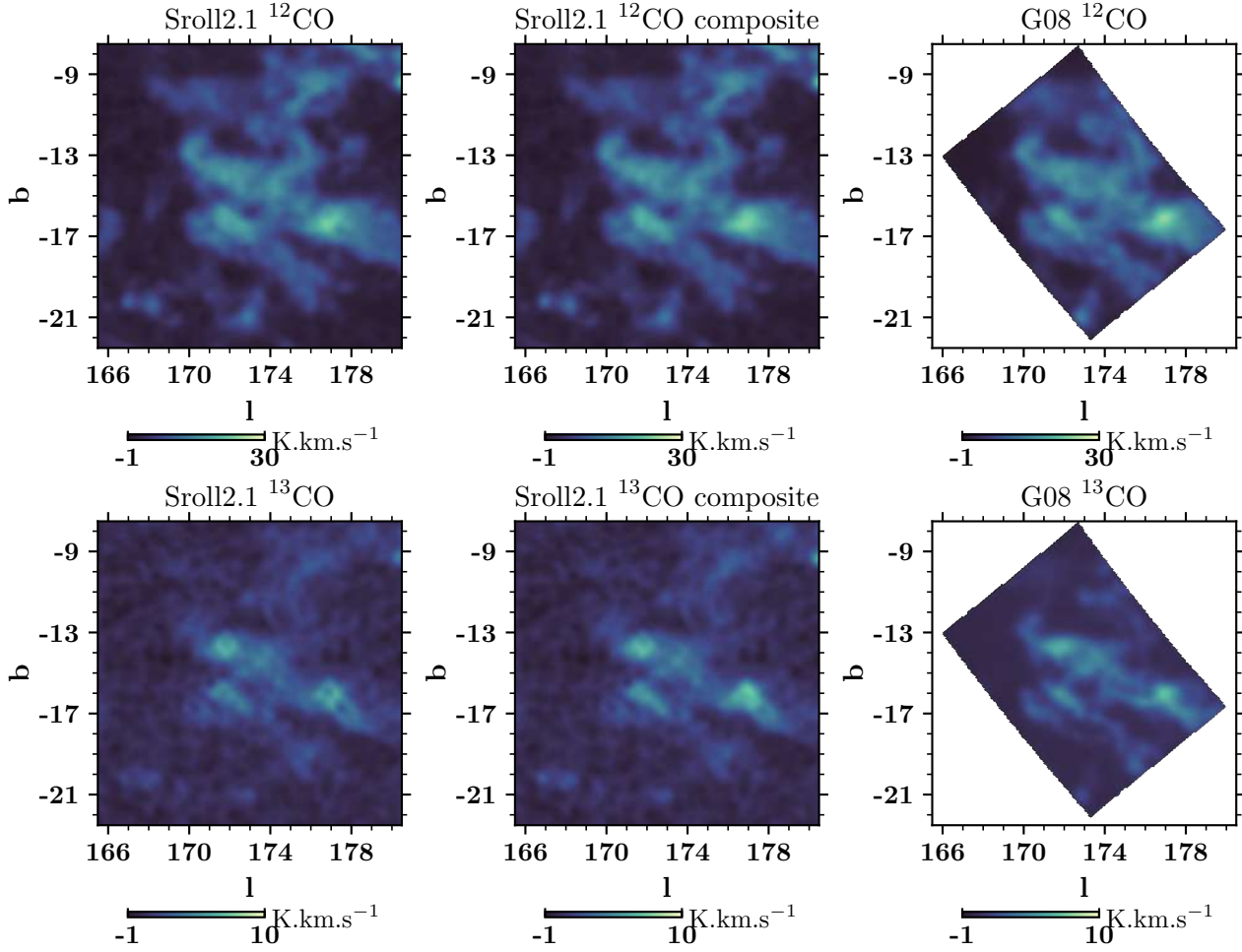


Fig. V.23 CO $J = 1 \rightarrow 0$ maps of the Taurus region smoothed up to a beam of FWHM = $30'$. **Top row:** ^{12}CO , **Bottom row:** ^{13}CO . **Left:** SRoll2.1 typical maps, **Centre:** SRoll2.1 composite maps, **Right:** Goldsmith *et al.* (2008) survey.

to the ones from Fig.V.3. All correlation plots are computed in the same conditions: all maps have been set to a Gaussian beam with FWHM = $30'$, and the plots are computed using only pixels with an intensity higher than 1 K.km.s^{-1} in the G08 maps. We can see an improvement in *almost* all aspects of the plots: for the ^{12}CO the goodness-of-fit is dimly increased overall, and, furthermore, we can see a clear improvement in the region of higher intensity where the cloud of points has collapsed significantly closer to the affine fit; the calibration consistency is as good as before while the value of the offset in the affine fit grew greatly closer to zero, going from 0.213 to 0.027. As for the ^{13}CO , the calibration consistency was greatly improved, with the affine fit factor going from 0.865 to 1.045, and the value of the offset in the affine fit grew greatly closer to zero, going from 0.244 to -0.006. The goodness-of-fit slightly decreased overall, which I suspect is due to the perfectible quality of the selection mask used to build the *composite* maps, especially since said mask was built from the SRoll2.1 ^{12}CO map. I expect that a better selection mask construction, with edge detection algorithm and performed separately for *both* ^{12}CO and ^{13}CO maps, would lead to a better goodness-of-fit. Finally, note that despite the overall slight decrease in the goodness-of-fit for the ^{13}CO *composite* map, the cloud of points has collapsed significantly closer to the affine fit in the region of higher intensity, as is the case for the ^{12}CO *composite*

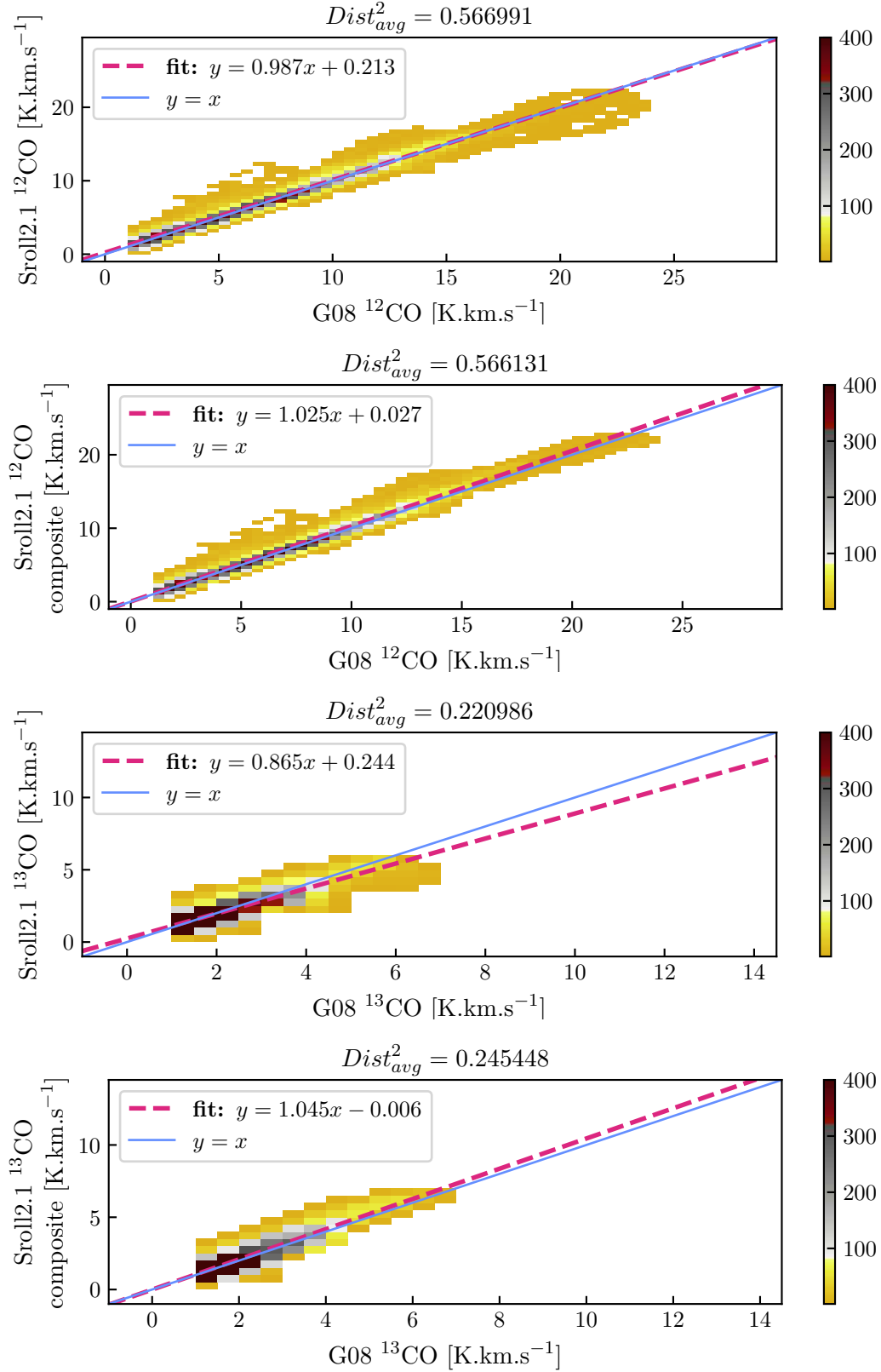


Fig. V.24 Correlation plots between the Sroll2.1 $J = 1 \rightarrow 0$ CO maps and the corresponding Taurus survey CO map from [Goldsmith et al. \(2008\)](#). All maps have been set to a Gaussian beam with FWHM = 30'. Plot computed using only pixels with an intensity superior to 1 K.km.s $^{-1}$ in the G08 maps. **From top to bottom:** ^{12}CO ; composite ^{12}CO ; ^{13}CO ; composite ^{13}CO . **Red dashed line** is the best affine fit of the cloud of points; **Blue line** is the identity function.

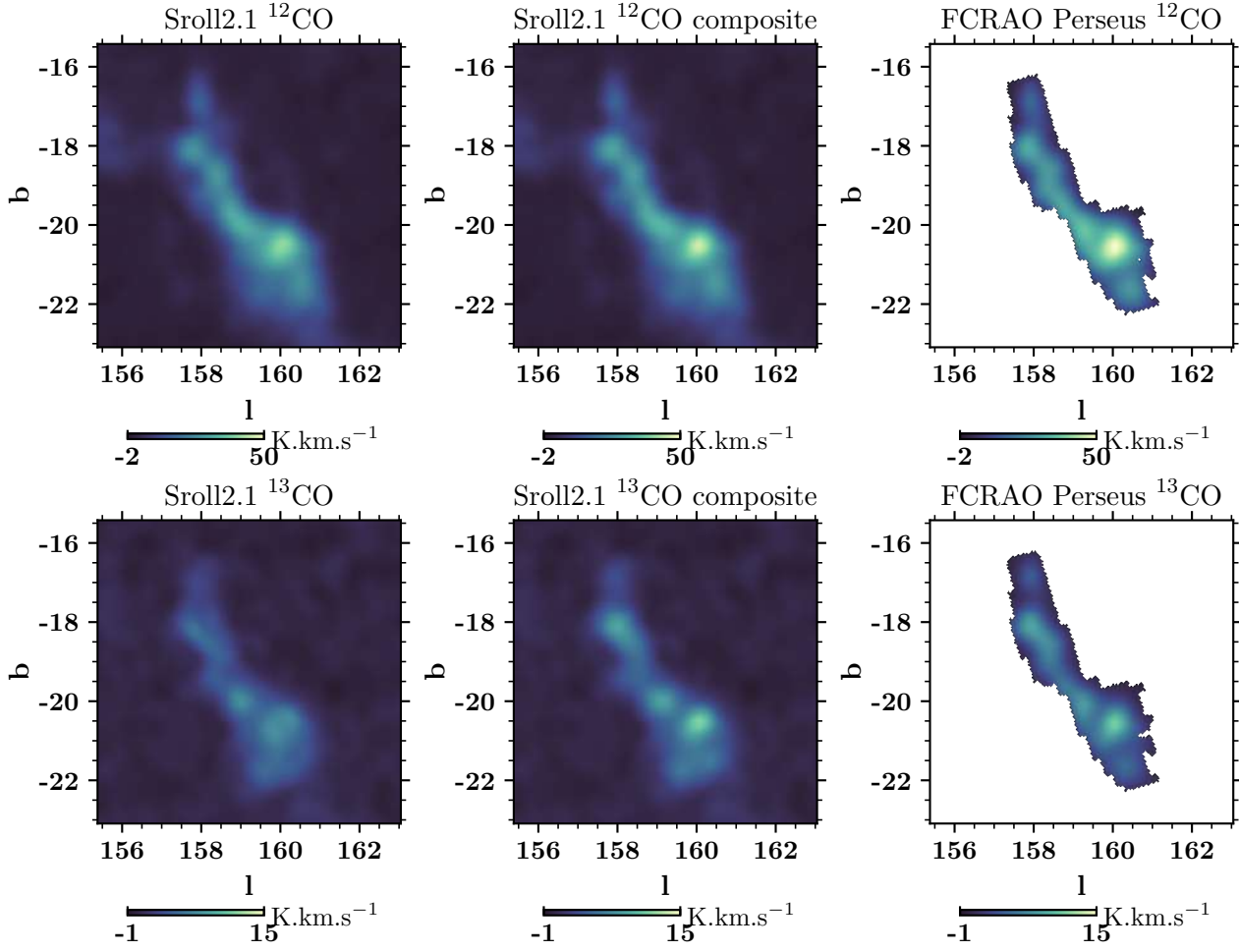


Fig. V.25 CO $J = 1 \rightarrow 0$ maps of the Perseus region smoothed up to a beam of FWHM = 30'. **Top row:** ^{12}CO , **Bottom row:** ^{13}CO . **Left:** SRoll2.1 typical maps, **Centre:** SRoll2.1 composite maps, **Right:** FCRAO survey (*COMPLETE team* (2011c) & *COMPLETE team* (2011d)).

map.

Perseus region (*Ridge et al. (2006)*, *COMPLETE team (2011c)* & *COMPLETE team (2011d)*) In a second time, I recomputed the comparison with the regions mapped by the COMPLETE survey of *Ridge et al. (2006)*. I started with the Perseus cloud from said survey: I plotted in Fig.V.25 the ^{12}CO and ^{13}CO emissions of the Perseus cloud from the original SRoll2.1 100-143GHz maps, the *composite* maps, and the maps of the COMPLETE survey, where all maps are smoothed up to a beam of FWHM = 30'. Again, a clear improvement can be seen in the peaked region of signal of the *composite* maps compared to the original SRoll2.1 100-143GHz ones. Then I recomputed in Fig.V.26 the correlation plots between the *composite* CO maps and the ones from the Perseus region of the COMPLETE survey of *Ridge et al. (2006)*. For comparison, I also displayed the correlation plots of the original SRoll2.1 100-143GHz corresponding to the ones from Fig.V.10. All correlation plots are computed in the same conditions: all maps have been set to a Gaussian beam with FWHM = 30', and the plots are computed using only pixels with an intensity superior to 1 K.km.s $^{-1}$ in the FCRAO maps. In this case, literally **every** aspect

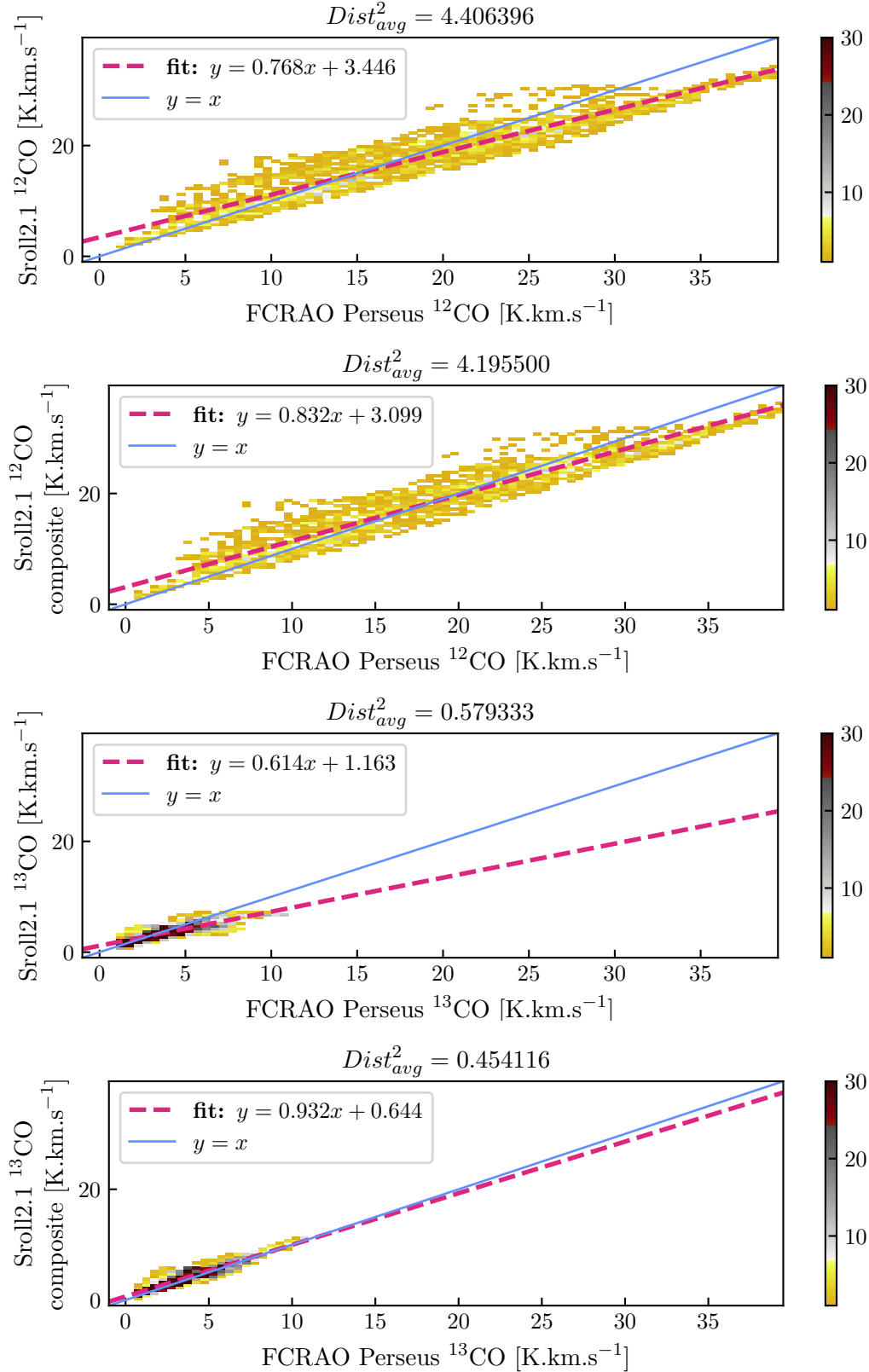


Fig. V.26 Correlation plots between the Sroll2.1 $J = 1 \rightarrow 0$ CO maps and the corresponding Perseus FCRAO survey CO map (COMPLETE team (2011c) & COMPLETE team (2011d)). All maps have been set to a Gaussian beam with FWHM = 30'. Plot computed using only pixels with an intensity superior to 1 K.km.s⁻¹ in the FCRAO maps. **From top to bottom:** ^{12}CO ; composite ^{12}CO ; ^{13}CO ; composite ^{13}CO . **Red dashed line** is the best affine fit of the cloud of points; **Blue line** is the identity function.

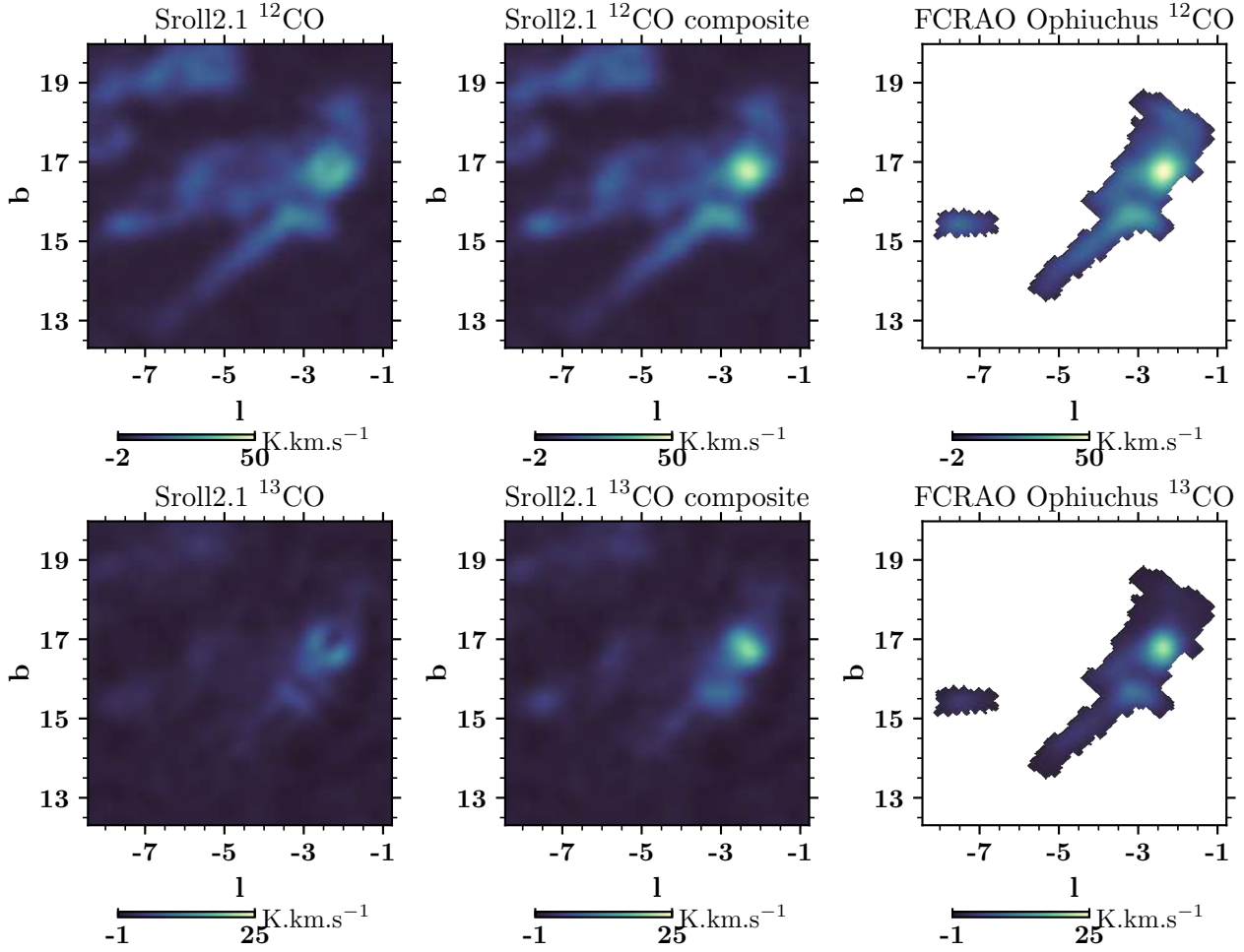


Fig. V.27 CO $J = 1 \rightarrow 0$ maps of the Ophiuchus region smoothed up to a beam of FWHM = 30'. **Top row:** ^{12}CO , **Bottom row:** ^{13}CO . **Left:** SRoll2.1 typical maps, **Centre:** SRoll2.1 composite maps, **Right:** FCRAO survey ([COMPLETE team \(2011a\)](#) & [COMPLETE team \(2011b\)](#)).

of the plots is improved for **both** the ^{12}CO and ^{13}CO maps. The calibration consistencies are improved (affine fit factor going from 0.768 to 0.832 for the ^{12}CO , and from 0.614 to 0.932 for the ^{13}CO), the offset parameter of the affine fit is closer to zero in both cases, and the goodness-of-fit is visibly *and* quantitatively improved in both cases.

Ophiuchus region ([Ridge et al. \(2006\)](#), [COMPLETE team \(2011a\)](#) & [COMPLETE team \(2011b\)](#)) I pursued with the Ophiuchus cloud from the COMPLETE survey: I plotted in Fig.V.27 the ^{12}CO and ^{13}CO emissions of the Ophiuchus cloud from the original SRoll2.1 100-143GHz maps, the *composite* maps, and the maps of the COMPLETE survey, where all maps are smoothed up to a beam of FWHM = 30'. Here, the improvement in the recovery of the peaked region of signal is remarkable, especially for the ^{13}CO emission. Then, once more, I recomputed the corresponding correlation plots for the Ophiuchus region, and displayed the results in Fig.V.28. For comparison, I also displayed the correlation plots of the original SRoll2.1 100-143GHz corresponding to the ones from Fig.V.13. All correlation plots are computed in the same conditions: all maps have been set to a Gaussian beam with FWHM = 30', and the plots are computed using only pixels with

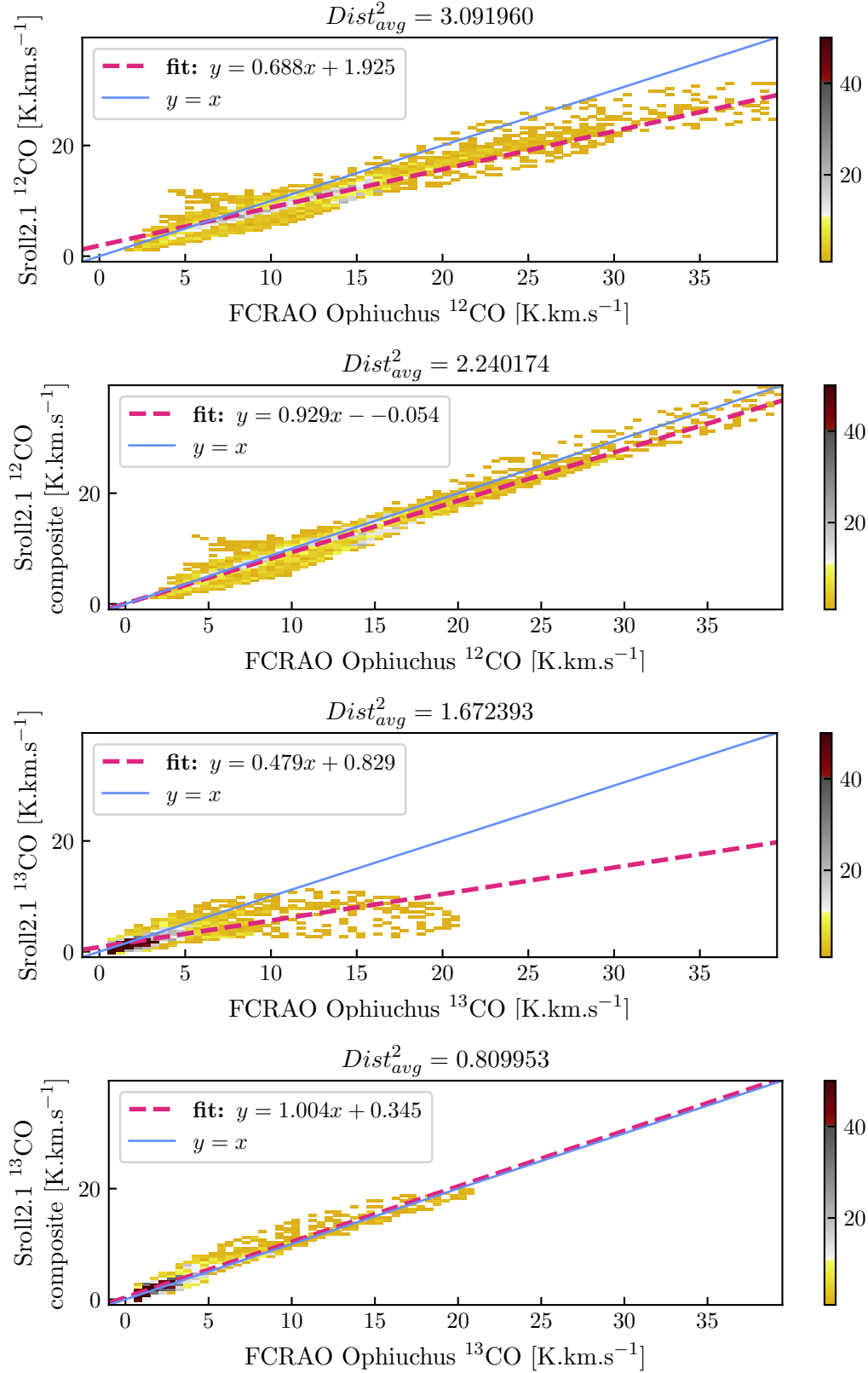


Fig. V.28 Correlation plots between the Sroll2.1 $J = 1 \rightarrow 0$ CO maps and the corresponding Ophiuchus FCRAO survey CO map ([COMPLETE team \(2011a\)](#) & [COMPLETE team \(2011b\)](#)). All maps have been set to a Gaussian beam with $\text{FWHM} = 30'$. Plot computed using only pixels with an intensity superior to 1 K.km.s^{-1} in the FCRAO maps. **From top to bottom:** ^{12}CO ; composite ^{12}CO ; ^{13}CO ; composite ^{13}CO . **Red dashed line** is the best affine fit of the cloud of points; **Blue line** is the identity function.

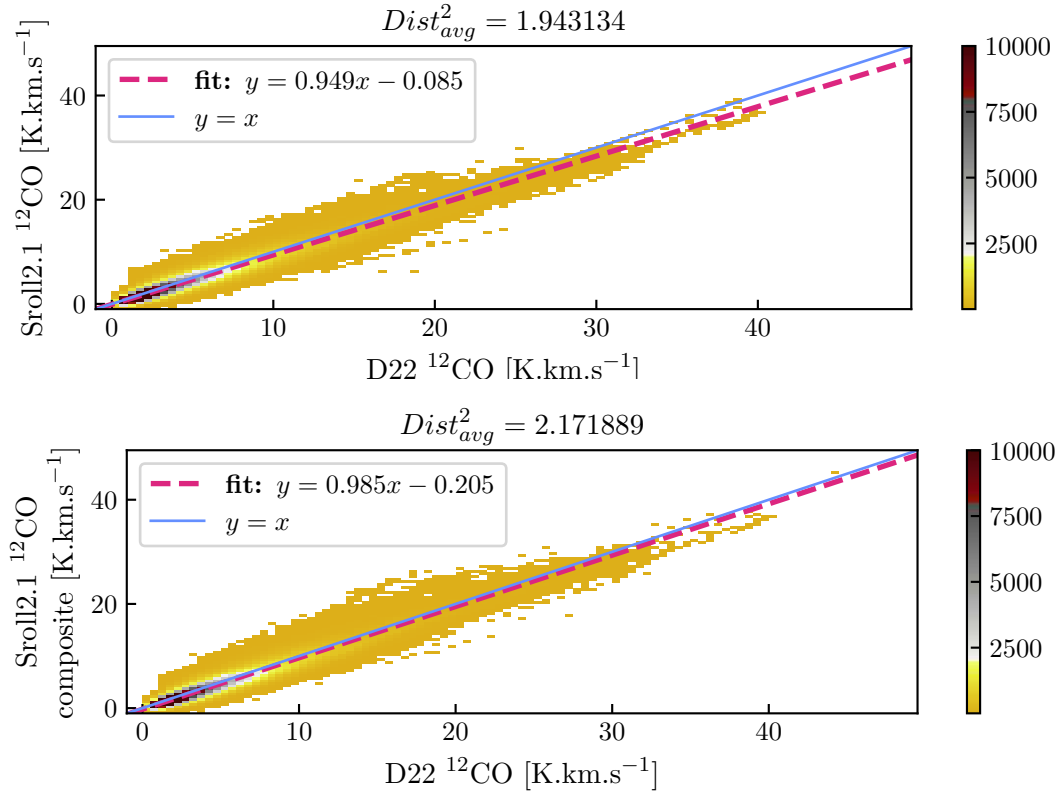


Fig. V.29 Correlation plots between the SRoll2.1 $J = 1 \rightarrow 0$ ^{12}CO map and the $J = 1 \rightarrow 0$ ^{12}CO survey of Dame & Thaddeus (2022). All maps have been set to a Gaussian beam with FWHM = $30'$. Plot computed using only pixels with an intensity higher than 1 K.km.s^{-1} in the map from Dame & Thaddeus (2022), as well as with a latitude either higher than 3° or lower than -3° in order to exclude the innermost part of the galactic disk. **Top:** ^{12}CO ; **Bottom:** composite ^{12}CO . **Red dashed line** is the best affine fit of the cloud of points; **Blue line** is the identity function.

an intensity superior to 1 K.km.s^{-1} in the FCRAO maps. Once more, **every** aspect of the plots is quite dramatically improved for **both** the ^{12}CO and ^{13}CO maps. The calibration consistencies are improved (affine fit factor going from 0.688 to 0.929 for the ^{12}CO , and from 0.479 to 1.004 for the ^{13}CO), the offset parameter of the affine fit is closer to zero in both cases (from 1.925 to -0.054 for the ^{12}CO , and from 0.829 to 0.345 for the ^{13}CO), and the goodness-of-fit is vastly improved both visibly *and* quantitatively in both cases.

Largest ^{12}CO survey (Dame & Thaddeus (2022)) Finally, I recomputed the correlation plot with the $J = 1 \rightarrow 0$ ^{12}CO survey of Dame & Thaddeus (2022) using the *composite* ^{12}CO map. I plotted the result in Fig.V.29, and added for comparison the correlation plot with the original SRoll2.1 100-143GHz corresponding to the one displayed at the bottom of Fig.V.16. In this case, the calibration consistency is improved, with an affine fit factor going from 0.949 to 0.985, but the goodness-of-fit and the offset of the affine fit are slightly degraded. I suspect again that this decrease in goodness-of-fit can be dissipated by improving the selection mask construction for the *composite* maps' construction.

I believe it worthwhile to emphasize again that the effect discussed here, and attempted to be corrected with the *composite* SRoll2.1 CO maps, does not impede the main purpose

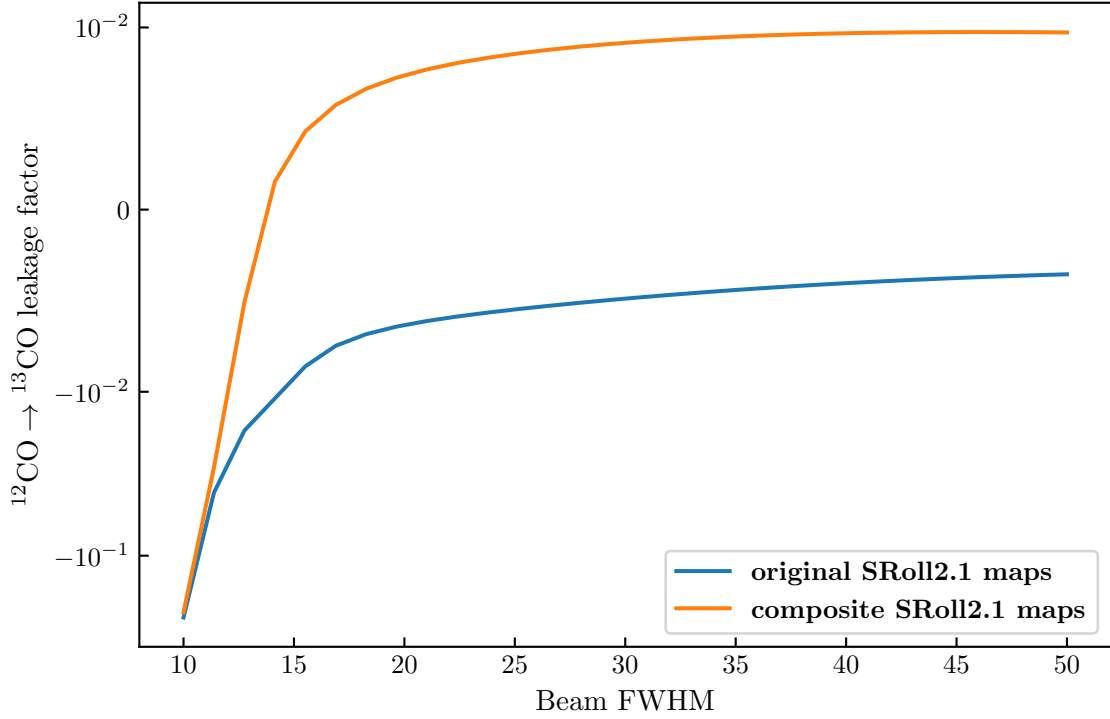


Fig. V.30 $\chi_{leak}^{12CO \rightarrow 13CO}$ leakage factor computed using the expression of Eq. IV.12, as a function of the smoothing applied on maps, determined on either the original SRoll2.1 100-143GHz maps (in blue), or the composite maps (in orange). Each determination of $\chi_{leak}^{12CO \rightarrow 13CO}$ is computed on the Taurus region as surveyed by Goldsmith et al. (2008), where all maps were smoothed up to a Gaussian beam with FWHM given in abscissa. The first value (at FWHM = 10') corresponds to unsmoothed SRoll2.1 maps. The y-axis is in symlog scale, with a linear scale for values inside the range $[-10^{-2}, 10^{-2}]$, and a log scale for values outside said range.

of the construction of my new SRoll2.1 ^{12}CO and ^{13}CO maps, i.e. the exploration of the high latitude and low column density regions of the sky to search for CO emission which would therefore not be explained by the typical self-shielding mechanism (see subsection I.4.2) observed in dense molecular clouds. This search is the main object of the next and last section of this manuscript.

V.2.3 Leakage $^{12}\text{CO} \rightarrow ^{13}\text{CO}$ from composite maps

As was discussed in subsection IV.3.2, the evaluation of the $^{12}\text{CO} \rightarrow ^{13}\text{CO}$ leakage, and especially the direction of said leakage, may have been impacted by the effect discussed throughout the present section. I therefore chose to check this effect once more, but now using the composite ^{12}CO and ^{13}CO maps presented in the subsections above.

The method is exactly the same as the one presented in subsection IV.3.2, but now using the composite maps instead of the original SRoll2.1 100-143GHz maps. I plotted the result of this new evaluation in Fig. V.30, where I reported the result from subsection IV.3.2 on the SRoll2.1 100-143GHz maps for comparison. Note that the y-axis is in symlog scale, with a linear scale for values inside the range $[-10^{-2}, 10^{-2}]$, and a log scale for values outside said range. As was suggested in subsection IV.3.2, the direction of the leakage $^{13}\text{CO} \rightarrow ^{12}\text{CO}$ was caused by the signal dip in regions of sharp signal at higher intensity, and the

leakage computed on the composite maps displays a $^{12}\text{CO} \rightarrow ^{13}\text{CO}$ leakage (positive values of $\mathcal{X}_{leak}^{^{12}\text{CO} \rightarrow ^{13}\text{CO}}$). However, the main conclusion remains unchanged: this effect corresponds to less than $< 1\%$ of the ^{12}CO map.

V.2.4 ^{13}CO power spectrum: the "bump"

As was observed in Fig. IV.27, an unexplained feature was visible on the power spectrum of the ^{13}CO map, as well as in the HM1 * HM2 cross-spectrum, more visible for the $J = 1 \rightarrow 0$ line: a "bump" between multipoles $130 \lesssim \ell \lesssim 1000$. At first, my (erroneous) interpretation was that this apparent excess of correlation was due to the features of signal dip resulting from the failing of the peaked signal recovery (i.e. due to the effect described above), and that it was the correlation of the contamination from the 143GHz band causing the aforementioned failing.

Thankfully, the construction of the composite maps later provided me with a way to check this interpretation: indeed, if the hypothesis above is correct, then the power spectrum of the composite ^{13}CO map should not display the "bump", or it should at least be significantly modified and reduced. I therefore plotted the power spectrum of the composite CO maps, as seen in Fig. V.31. We can see that the ^{13}CO composite map displays the same "bump" as the standard SRoll2.1 map when using a mask $f_{sky} = 90\%$, which invalidates my first interpretation. Additionally, I plotted the power spectrum from the maps built using **only** the 100GHz frequency band. We can see that in those maps the "bump" *vanishes*. This means that the "bump" is due to some *unidentified* effect caused by adding the 143GHz band to the CO maps construction. I reckon that this "bump" could therefore be caused by the beam differences, as discussed in subsection IV.3.4. Correcting this effect as suggested in the aforementioned subsection might then also remove this "bump".

However, this effect is limited to regions of low latitude and high column density, since the "bump" vanishes at all other more restrictive masks (i.e. for $f_{sky} = 80\%$ -10%) even for the standard and composite maps (the effect falls under the noise level). This can be seen in Fig. V.31 where I also plotted all the aforementioned spectra with a mask $f_{sky} = 80\%$. Nevertheless, we will see in the next section that a similar shape appears in the cross-spectra computed from simulations.

V.3 High latitude & low column density exploration

As previously introduced and discussed in subsection I.4.3, models predict that CO can be produced (and later excited to emit rotational lines) through turbulence driven chemistry. This warm chemistry, appearing in structures produced by bursts of turbulent dissipation, characteristic of the intermittent nature of said turbulent dissipation in both time and space, has no reason to be confined to the dense molecular clouds. On the contrary, it is expected to be present throughout the whole diffuse ISM, even in regions not dense enough to be dominated and maintained by the typical self-shielding mechanism observed in molecular clouds. This means that some CO signal, coming from such intermittent structures, may be present in the entire sky, even at high latitudes.

However, the scale of the turbulent dissipation regions, in which the non-equilibrium warm chemistry occurs, is very small, typically too small to be resolved in any of the current comprehensive MHD turbulence numerical simulations ($1'$ width for more than $> 20'$ length, see subsection I.4.3). Additionally, in the diffuse ISM, characterised by a low density of gas and dust, the CO intensity coming from such regions is expected to be faint. In these

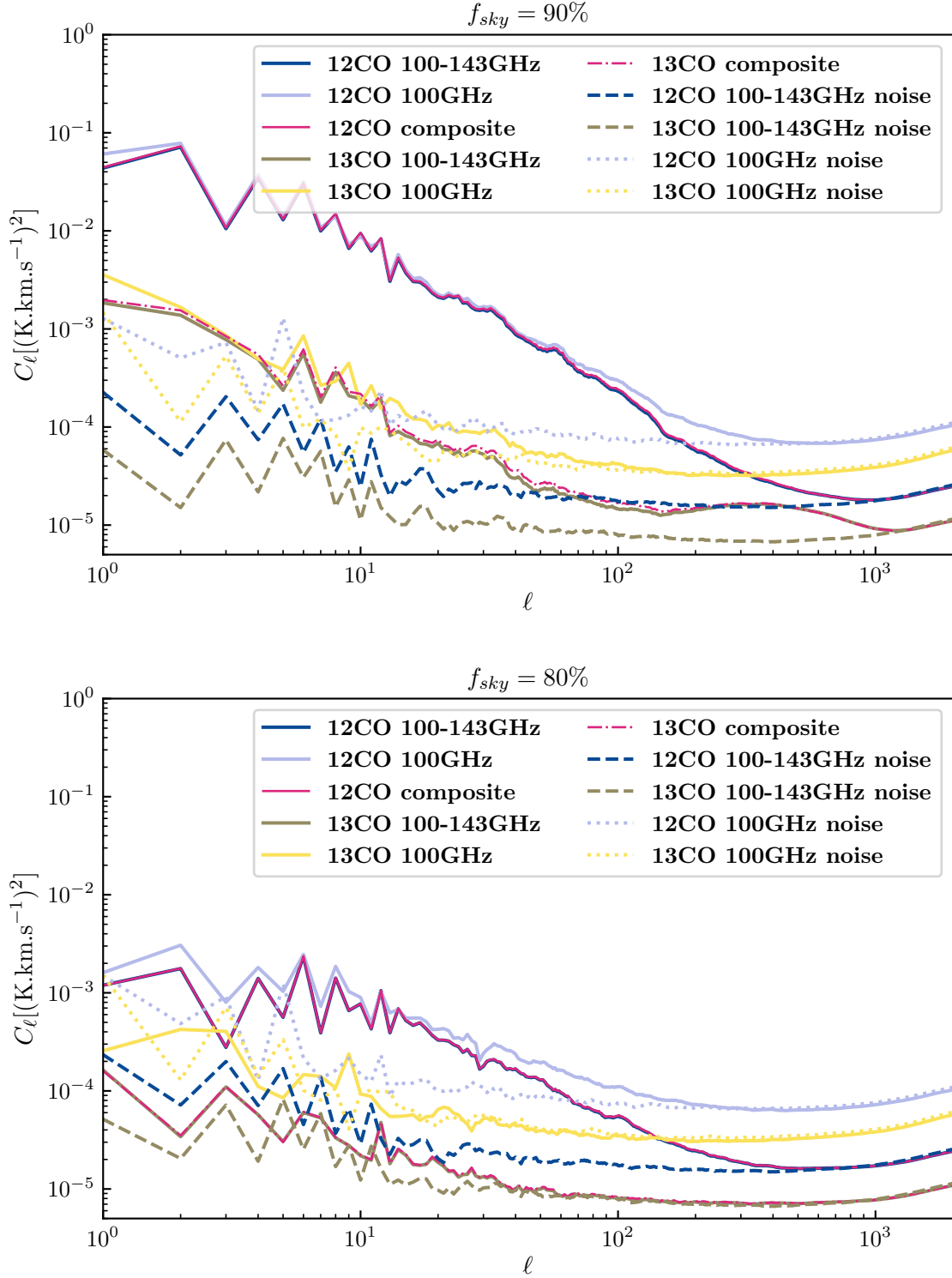


Fig. V.31 ^{12}CO and ^{13}CO maps spectra of the $J = 1 \rightarrow 0$ line, log-smoothed with a multipole fraction of $\ell_{frac} = 15$ (cf. Appendix.C). **Top:** computed using a mask keeping a sky fraction $f_{sky} = 90\%$; **Bottom:** using a mask keeping a sky fraction $f_{sky} = 80\%$. The power spectra are those of the standard SRoll2.1 maps, the composite maps, and the 100GHz only maps, as well as the power spectrum of the difference maps built as HM1-HM2. The difference maps power spectra have been divided by 4 to be brought back to the original maps noise level and therefore correspond to the power spectrum of the noise in the corresponding maps.

conditions, the only straightforward ways to observe such faint CO emission in the diffuse ISM, typically seen higher galactic latitudes, are the following:

- Either use a kind of radio-telescope with both extremely high resolution and sensitivity, leading to very small filling factor and therefore a small chance of detection with such small field-of-view,
- Or, accumulate a sufficient sky coverage of CO observation to search for this emission using *statistical* tools.

Thus, the main asset of my SRoll2.1 ^{12}CO and ^{13}CO maps for this search is the combination of a low level of noise, a low risk of significant residual effects, and a full-sky coverage of the CO emission. Using these maps, paired with the useful qualification brought by the SRoll2.1 simulations, I explored the high latitude and low column density regions, i.e. the regions filled by the diffuse ISM, searching for a potential CO detection. Any such detection however cannot in itself inform on the physical properties and chemical dynamics at the origins of the formation of the CO in these region. We cannot exclude the fact that it is simply "classically" formed CO which was never seen before because not looked for.

I will hereby present the method I used to carry out this search, based off maps cross-correlations (see Appendix.C for the various spectra definitions), and display the corresponding results for the ^{12}CO and ^{13}CO emissions.

V.3.1 Exploration method

Since the intensity of the CO signal we are looking for is expected to be well below the noise, I need a statistical way of looking for signal inside the maps' noise. This is typically possible by computing the cross-correlation between 2 observations of the same sought signal, but impacted by 2 distinct and uncorrelated noise realizations.

Therefore, I search for the aforementioned faint CO signal by computing the cross-correlation between my HM1 and HM2 SRoll2.1 ^{12}CO and ^{13}CO maps. This is done on the standard SRoll2.1 maps. The CO signal observed should be the same in both maps, but they were built from distinct subsets of the *Planck* data, and therefore should be impacted by 2 uncorrelated realizations of noise. The computation of the cross-correlation HM1*HM2 is thereby expected to eliminate the noise and bring out only the signal. Nevertheless, as small as they are expected to be, random and *spurious* correlations are always present, and I will need a way to estimate their level to ensure whether or not I actually detect some CO signal. This is done using the results from the SRoll2.1 simulations, as detailed in the subsections below.

Such cross-correlations are computed using all the sky masks defined in Appendix.C, masking the region of the sky of highest dust intensity, and therefore of highest column density, by increments of 10%. At the sky fraction $f_{sky} = 40\%$, all known molecular clouds are masked and only the low regions of the diffuse ISM remain, corresponding typically to the regions I aim to explore.

Note that, since I use the results from the SRoll2.1 simulation runs to identify a *genuine* high latitude CO detection, as described in the next 2 subsections, I only performed this high latitude exploration on the $\mathbf{J} = \mathbf{1} \rightarrow \mathbf{0}$ SRoll2.1 ^{12}CO and ^{13}CO maps.

a) Evaluate random and *spurious* correlation level using simulations

The SRoll2.1 simulations are perfect to evaluate the level of any random, or even any *spurious*, correlations appearing at high latitude. Indeed, the sky emission model of the simulations **does not contain any** high latitude CO signal. Therefore, any cross-correlation

at high latitude between the HM1 and HM2 SRoll2.1 ^{12}CO and ^{13}CO maps from the simulations are either due to random correlations, or to *spurious* correlations stemming from systematic residuals (instrumental error, correlated non-Gaussian noise, leakage, etc). These maps are then perfect for the identification of a *genuine* high latitude CO detection. Indeed, I should emphasize that the simulations allow to account for **any** residual error, i.e. even if the various effect discussed up to now, such as the failing of peaked signal or the ^{13}CO "bump", as well as any effect from Section IV.3, were to have more effect at high latitude than anticipated, these effects will also appear *in the simulations*. Therefore, any detection above the level of the simulations is *not only* above the random chance correlation but *also* above the potential error residuals.

To estimate my level of random and spurious correlations, I compute the cross-correlation of the HM1 and HM2 simulation CO **residual** maps (after removing the signal map), averaged over the set of 10 available simulations. Consequently, the cross-correlation computed to determine the aforementioned evaluation can be expressed, for the ^{12}CO and ^{13}CO , as follows:

$$\begin{aligned} \forall \ell \in \mathbb{N}, C_\ell^{12\text{CO} \text{ simu. avg}} &= \frac{1}{10} \sum_{i=1}^{10} C_\ell(M_{\text{HM1},i}^{12\text{CO}_{res}} * M_{\text{HM2},i}^{12\text{CO}_{res}}), \\ \forall \ell \in \mathbb{N}, C_\ell^{13\text{CO} \text{ simu. avg}} &= \frac{1}{10} \sum_{i=1}^{10} C_\ell(M_{\text{HM1},i}^{13\text{CO}_{res}} * M_{\text{HM2},i}^{13\text{CO}_{res}}), \end{aligned} \quad (\text{V.2})$$

where $M_{\text{HM1},i}^{12\text{CO}_{res}}$ is the HM1 *residual* map of ^{12}CO from the simulation of index i , and with any ^{12}CO *residual* map being computed as $M^{12\text{CO}_{res}} = M^{12\text{CO}} - M_{\text{template}}^{12\text{CO}}$ where $M^{12\text{CO}}$ is the SRoll2.1 ^{12}CO map and $M_{\text{template}}^{12\text{CO}}$ is the input template map of ^{12}CO for the SRoll2.1 simulations, as displayed in Fig. IV.8. The notations follow the same definitions for the HM2 maps and the ^{13}CO emission.

In each case of HM1 * HM2 cross-correlation computed on the maps from the data, I also compute the hereby defined cross-correlation of random and spurious correlations' evaluation, as expressed in Eq. V.2, and display it in the upcoming resulting plots in **red**.

b) Definition of the detection threshold

The last element needed to identify a *genuine* high latitude CO detection is the definition of a threshold for the minimum required distance separating the cross-correlation on the data from the random and spurious correlations level, in order to deem it high enough to be a detection. This detection threshold is defined by using the standard deviation of the HM1 * HM2 cross-correlations of residual maps over the 10 simulations.

More precisely, I define the standard deviation cross-correlation, for the ^{12}CO and ^{13}CO , as follows:

$$\begin{aligned} \forall \ell \in \mathbb{N}, C_\ell^{12\text{CO} \text{ simu. std}} &= \sqrt{\frac{1}{10} \sum_{i=1}^{10} (C_\ell(M_{\text{HM1},i}^{12\text{CO}_{res}} * M_{\text{HM2},i}^{12\text{CO}_{res}}) - C_\ell^{12\text{CO} \text{ simu. avg}})^2}, \\ \forall \ell \in \mathbb{N}, C_\ell^{13\text{CO} \text{ simu. std}} &= \sqrt{\frac{1}{10} \sum_{i=1}^{10} (C_\ell(M_{\text{HM1},i}^{13\text{CO}_{res}} * M_{\text{HM2},i}^{13\text{CO}_{res}}) - C_\ell^{13\text{CO} \text{ simu. avg}})^2}, \end{aligned} \quad (\text{V.3})$$

where C_ℓ is the result of the log-smoothing of spectrum C_ℓ .

From this definition of the standard deviation cross-correlation of Eq. V.3, I define the levels of detection which I call 1σ , 2σ , and 3σ , for any CO map, by the following respective

expressions:

$$\begin{aligned}\forall \ell \in \mathbb{N}, \mathcal{C}_\ell^{CO \text{ simu. } 1\sigma} &= \mathcal{C}_\ell^{CO \text{ simu. avg}} + \mathcal{C}_\ell^{CO \text{ simu. std}}, \\ \forall \ell \in \mathbb{N}, \mathcal{C}_\ell^{CO \text{ simu. } 2\sigma} &= \mathcal{C}_\ell^{CO \text{ simu. avg}} + 2\mathcal{C}_\ell^{CO \text{ simu. std}}, \\ \forall \ell \in \mathbb{N}, \mathcal{C}_\ell^{CO \text{ simu. } 3\sigma} &= \mathcal{C}_\ell^{CO \text{ simu. avg}} + 3\mathcal{C}_\ell^{CO \text{ simu. std}}.\end{aligned}\tag{V.4}$$

In each case of HM1*HM2 cross-correlation computed on the maps from the data, I also compute the hereby defined 1σ , 2σ , and 3σ detection levels, as expressed in Eq.V.4, and respectively display them in the upcoming resulting plots in **light yellow**, **dark yellow**, and **black**. Any scales for which the *data* cross-correlation lands on or above the 3σ detection threshold is considered to be a *detection* of high latitude CO signal.

V.3.2 Results on ^{12}CO

I present here the results of the exploration method defined above, applied on the SRoll2.1 $J = 1 \rightarrow 0$ ^{12}CO map. I plotted the HM1 * HM2 cross-spectra computed with masks of decreasing f_{sky} in Figs.V.32, V.33, and V.34, respectively displaying the spectra computed with masks of sky fraction $f_{sky} = 90\text{-}80\text{-}70\%$, $f_{sky} = 60\text{-}50\text{-}40\%$, and $f_{sky} = 30\text{-}20\text{-}10\%$.

We can see that already at the $f_{sky} = 80\%$ value, the data cross-spectrum falls below the detection threshold of 3σ at small scales of $\ell \gtrsim 1000$, which corresponds to angular scales of $\lesssim 10'.5$ (see Appendix.A). At a sky fraction of $f_{sky} = 60\%$, most molecular clouds are already masked, and only a few high latitude clouds remain (see the plot of the masks in Fig.C.1), and still the data cross-spectrum is either above or grazing the 3σ detection threshold along many of the smaller scales ($150 \lesssim \ell \lesssim 850$).

The most striking result though, is the scales displaying a ^{12}CO **signal detection** at sky fractions of $f_{sky} \leq 40\%$. Notably, the cross-spectrum using the $f_{sky} = 40\%$ mask, corresponding to gas column densities less than $N_{\text{H}} < 5.5 \times 10^{20} \text{cm}^{-2}$, displays ^{12}CO signal detection for multipoles $240 \lesssim \ell \lesssim 560$ ($44'.0 \gtrsim \text{ang. scale} \gtrsim 18'.8$). Also, a ^{12}CO 3σ detection remains even down to $f_{sky} = 10\%$, corresponding to gas column densities less than $N_{\text{H}} < 2.5 \times 10^{20} \text{cm}^{-2}$. At such high latitudes, the detection is very localized in scale, around $\ell \simeq 400$ (ang. scale $\simeq 26'.4$).

Expectedly, all these high latitude ^{12}CO detections are close to the 3σ detection threshold, which should encourage further confirmation from future observations, but these detections are a stimulating observational incentive to further search for direct observation supporting the models of production and excitation of molecules, and especially CO, from turbulence driven warm chemistry in intermittent turbulent dissipation structures (Falgarone & Puget (1995), Godard et al. (2014), Lesaffre et al. (2020)).

Note that in these 3 figures (i.e. results on the ^{12}CO), as well as in the 3 figures of the next subsection (i.e. results on the ^{13}CO), a shape similar to the "bump" discussed in subsection V.2.4 is visible in the cross-spectra of the simulations. It is spread along varying ranges of scales depending on the considered f_{sky} . This "bump" shape may be a residual error stemming from the beam differences between bolometers, as discussed in subsection IV.3.4. Nevertheless, the strength of employing simulations to estimate the detection threshold is that *all such residual errors* are also affecting the simulations. Therefore, the simulations offer an evaluation of the total residual error level in the maps, and all detection significantly deviating from this evaluation can thus be *confidently* considered a detection.

V.3.3 Results on ^{13}CO

I present here the results of the exploration method defined above, applied on the SRoll2.1 $J = 1 \rightarrow 0$ ^{13}CO map. I plotted the HM1 * HM2 cross-spectra computed with masks of

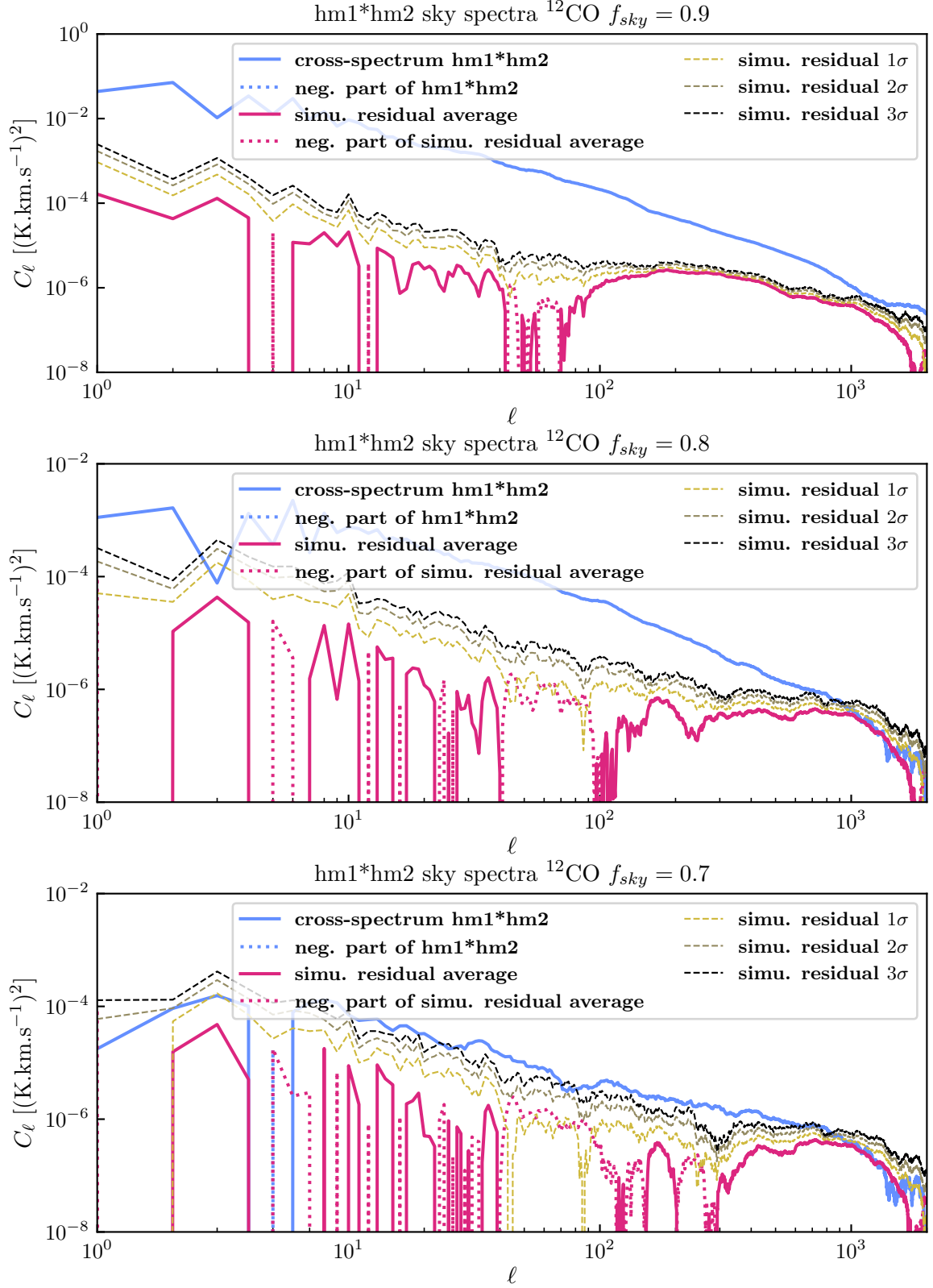


Fig. V.32 ^{12}CO HM1 * HM2 cross-spectra with various sky fraction masks, and log-smoothed with a multipole fraction of $\ell_{\text{frac}} = 10$ (cf. Appendix C). **Top:** $f_{\text{sky}} = 90\%$ mask; **Middle:** $f_{\text{sky}} = 80\%$ mask; **Bottom:** $f_{\text{sky}} = 70\%$ mask. The plotted spectra are from the SRoll2.1 ^{12}CO $J = 1 \rightarrow 0$ map (in blue), the average HM1 * HM2 cross-spectrum of ^{12}CO residual over the 10 SRoll2.1 simulations (in red), and the 1σ , 2σ , and 3σ deviations from said average cross-spectrum of ^{12}CO residual (respectively in light yellow, dark yellow, and black).

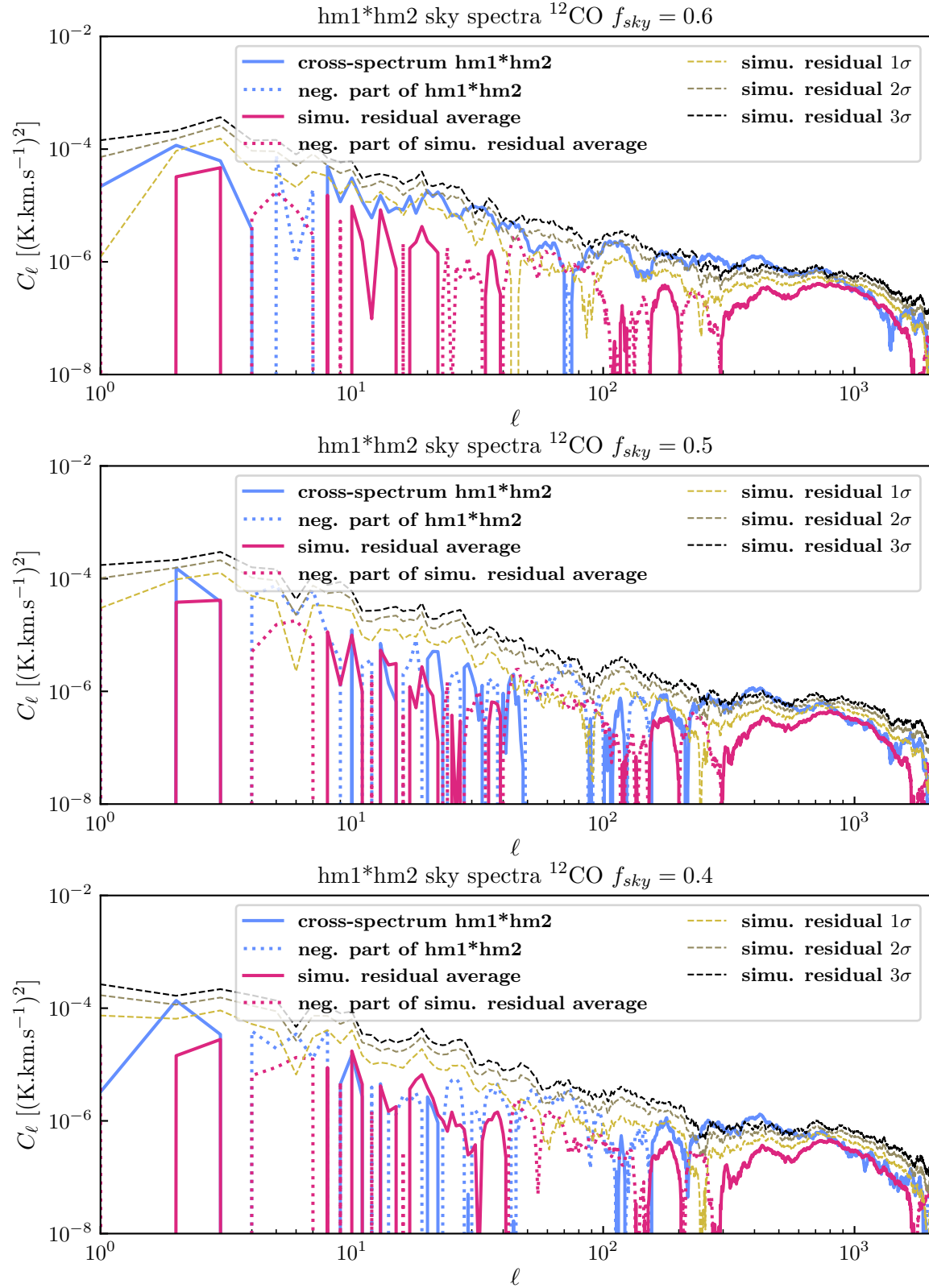


Fig. V.33 ^{12}CO HM1 * HM2 cross-spectra with various sky fraction masks, and log-smoothed with a multipole fraction of $\ell_{\text{frac}} = 10$ (cf. Appendix.C). **Top:** $f_{\text{sky}} = 60\%$ mask; **Middle:** $f_{\text{sky}} = 50\%$ mask; **Bottom:** $f_{\text{sky}} = 40\%$ mask. The plotted spectra are from the SRoll2.1 ^{12}CO $J = 1 \rightarrow 0$ map (in blue), the average HM1 * HM2 cross-spectrum of ^{12}CO residual over the 10 SRoll2.1 simulations (in red), and the 1σ , 2σ , and 3σ deviations from said average cross-spectrum of ^{12}CO residual (respectively in light yellow, dark yellow, and black).

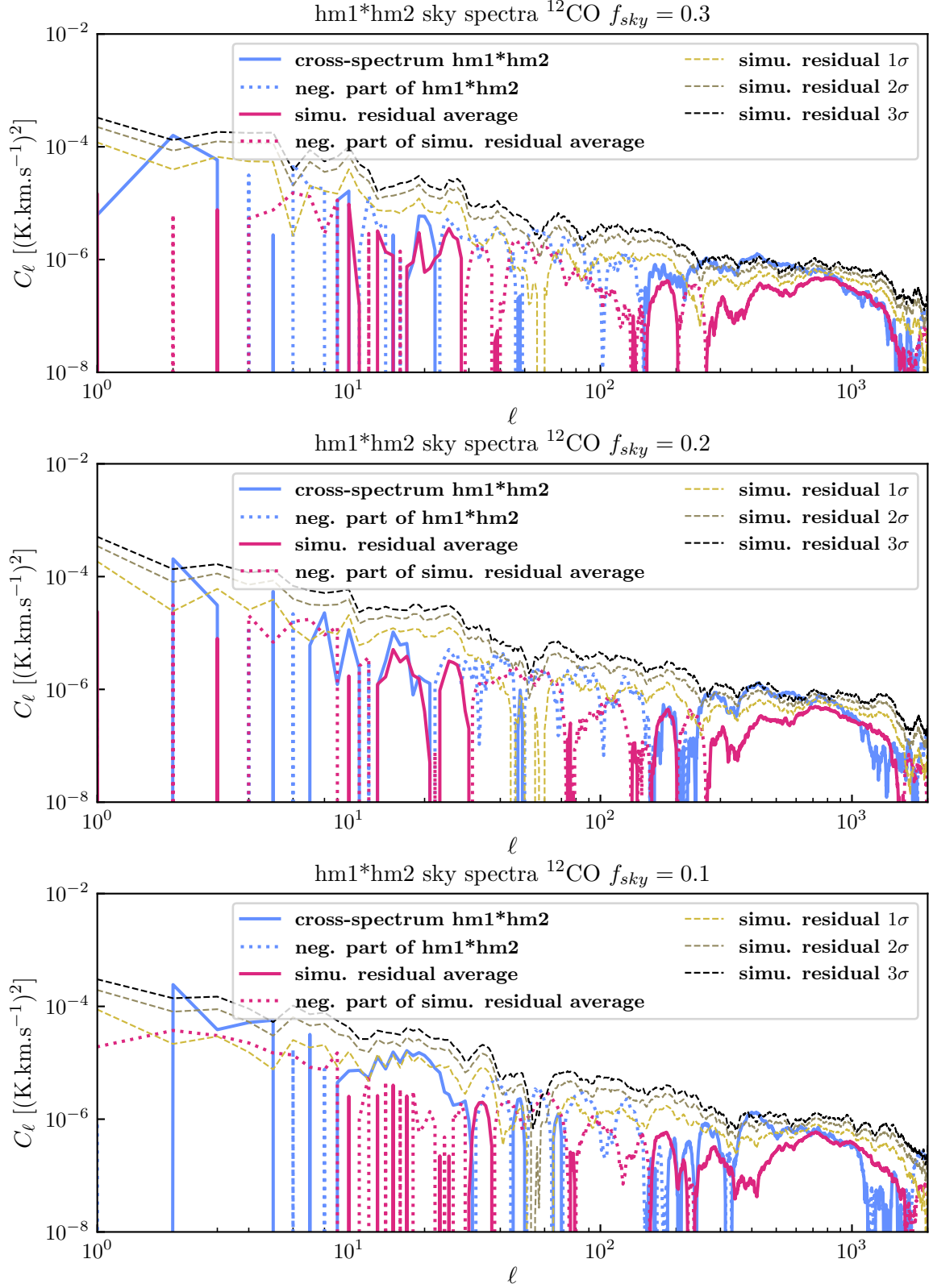


Fig. V.34 ^{12}CO HM1 * HM2 cross-spectra with various sky fraction masks, and log-smoothed with a multipole fraction of $\ell_{frac} = 10$ (cf. Appendix C). **Top:** $f_{sky} = 30\%$ mask; **Middle:** $f_{sky} = 20\%$ mask; **Bottom:** $f_{sky} = 10\%$ mask. The plotted spectra are from the SRoll2.1 ^{12}CO $J = 1 \rightarrow 0$ map (in blue), the average HM1 * HM2 cross-spectrum of ^{12}CO residual over the 10 SRoll2.1 simulations (in red), and the 1σ , 2σ , and 3σ deviations from said average cross-spectrum of ^{12}CO residual (respectively in light yellow, dark yellow, and black).

decreasing f_{sky} in Figs. V.35, V.36, and V.37, respectively displaying the spectra computed with masks of sky fraction $f_{sky} = 90\text{-}80\text{-}70\%$, $f_{sky} = 60\text{-}50\text{-}40\%$, and $f_{sky} = 30\text{-}20\text{-}10\%$.

The first observation is that, as discussed in subsection V.2.4, the "bump" vanishes for all sky fractions less than $f_{sky} \leq 90\%$.

Finally, we observe no conclusive high latitude ^{13}CO detection from these cross-spectra. However, we can see a correlation peak at high latitude ($f_{sky} \leq 40\%$) around scales of $\ell \sim 400$, which correspond to scales which displayed ^{12}CO signal detection even at highest latitudes. This peak culminates between the 2σ and the 3σ thresholds, which is insufficient to conclude to a detection, but should again encourage future endeavours to search for confirmation (or refutation) of any ^{13}CO signal detection, along with potential ^{12}CO signal detection, in high latitude and low column density regions.

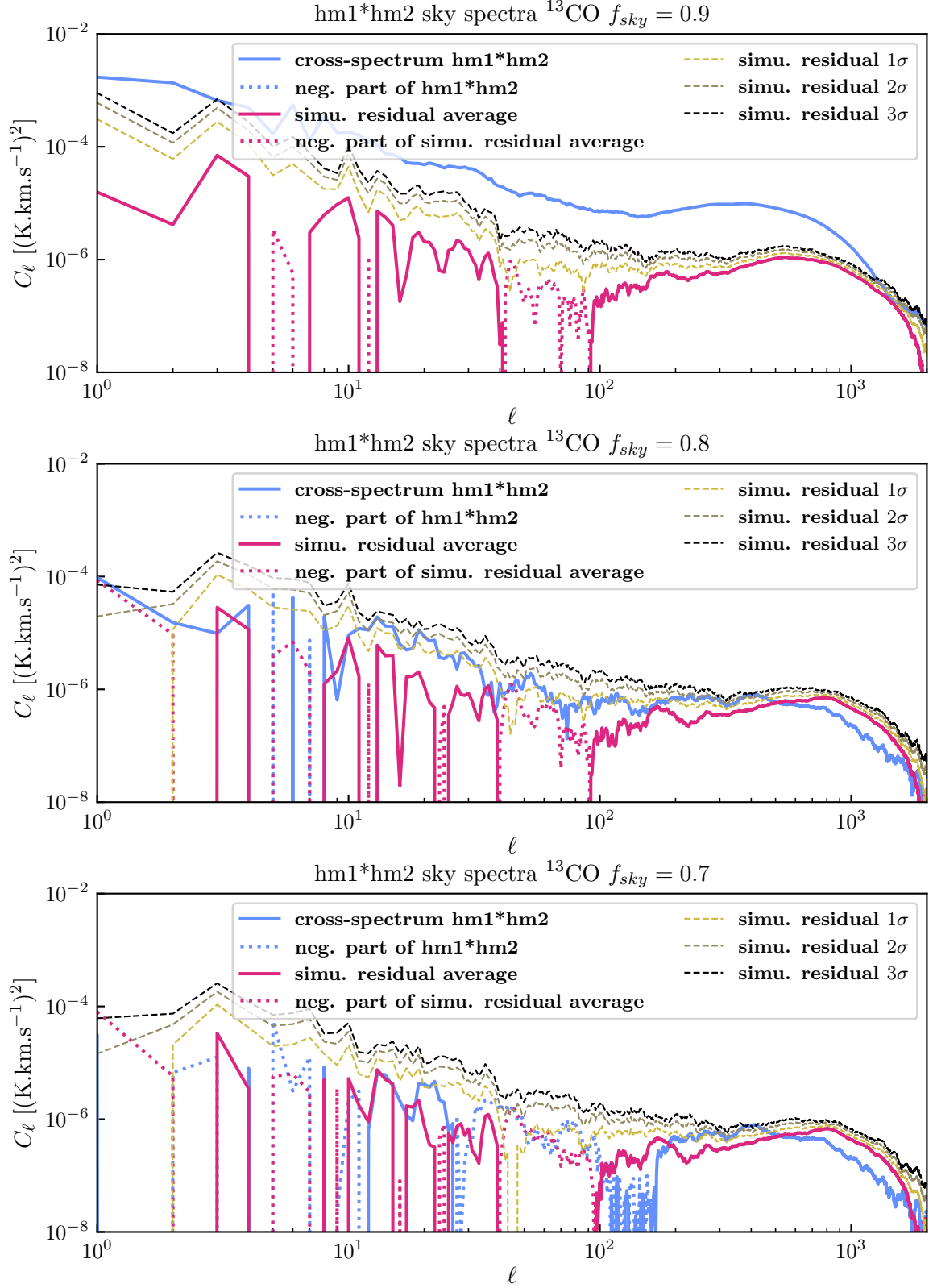


Fig. V.35 ^{13}CO HM1 * HM2 cross-spectra with various sky fraction masks, and log-smoothed with a multipole fraction of $\ell_{frac} = 10$ (cf. Appendix.C). **Top:** $f_{sky} = 90\%$ mask; **Middle:** $f_{sky} = 80\%$ mask; **Bottom:** $f_{sky} = 70\%$ mask. The plotted spectra are from the SRoll2.1 $^{13}\text{CO } J = 1 \rightarrow 0$ map (in blue), the average HM1 * HM2 cross-spectrum of ^{13}CO residual over the 10 SRoll2.1 simulations (in red), and the 1σ , 2σ , and 3σ deviations from said average cross-spectrum of ^{13}CO residual (respectively in light yellow, dark yellow, and black).

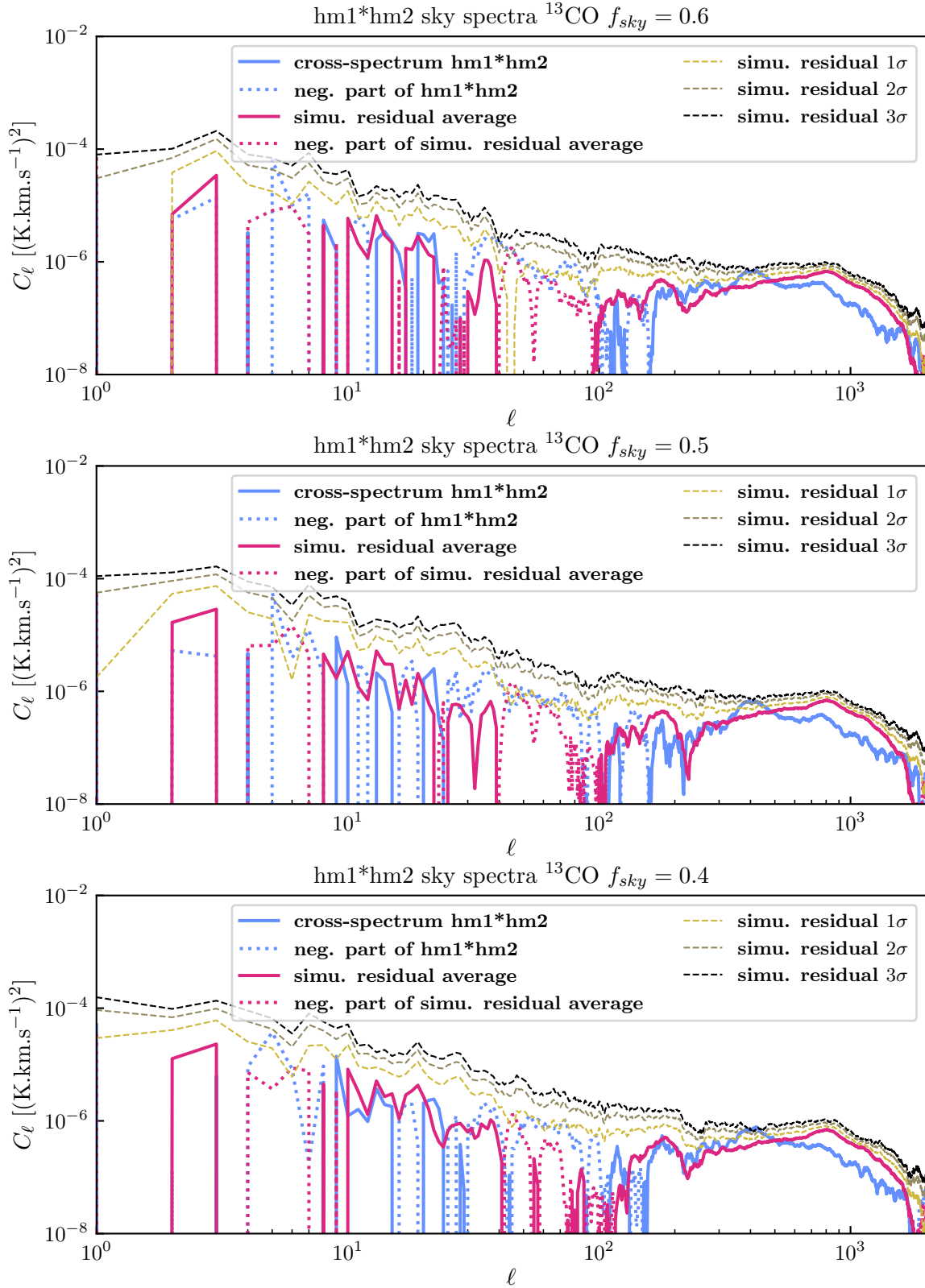


Fig. V.36 ^{13}CO HM1 * HM2 cross-spectra with various sky fraction masks, and log-smoothed with a multipole fraction of $\ell_{\text{frac}} = 10$ (cf. Appendix C). **Top:** $f_{\text{sky}} = 60\%$ mask; **Middle:** $f_{\text{sky}} = 50\%$ mask; **Bottom:** $f_{\text{sky}} = 40\%$ mask. The plotted spectra are from the SRoll2.1 ^{13}CO $J = 1 \rightarrow 0$ map (in blue), the average HM1 * HM2 cross-spectrum of ^{13}CO residual over the 10 SRoll2.1 simulations (in red), and the 1σ , 2σ , and 3σ deviations from said average cross-spectrum of ^{13}CO residual (respectively in light yellow, dark yellow, and black).

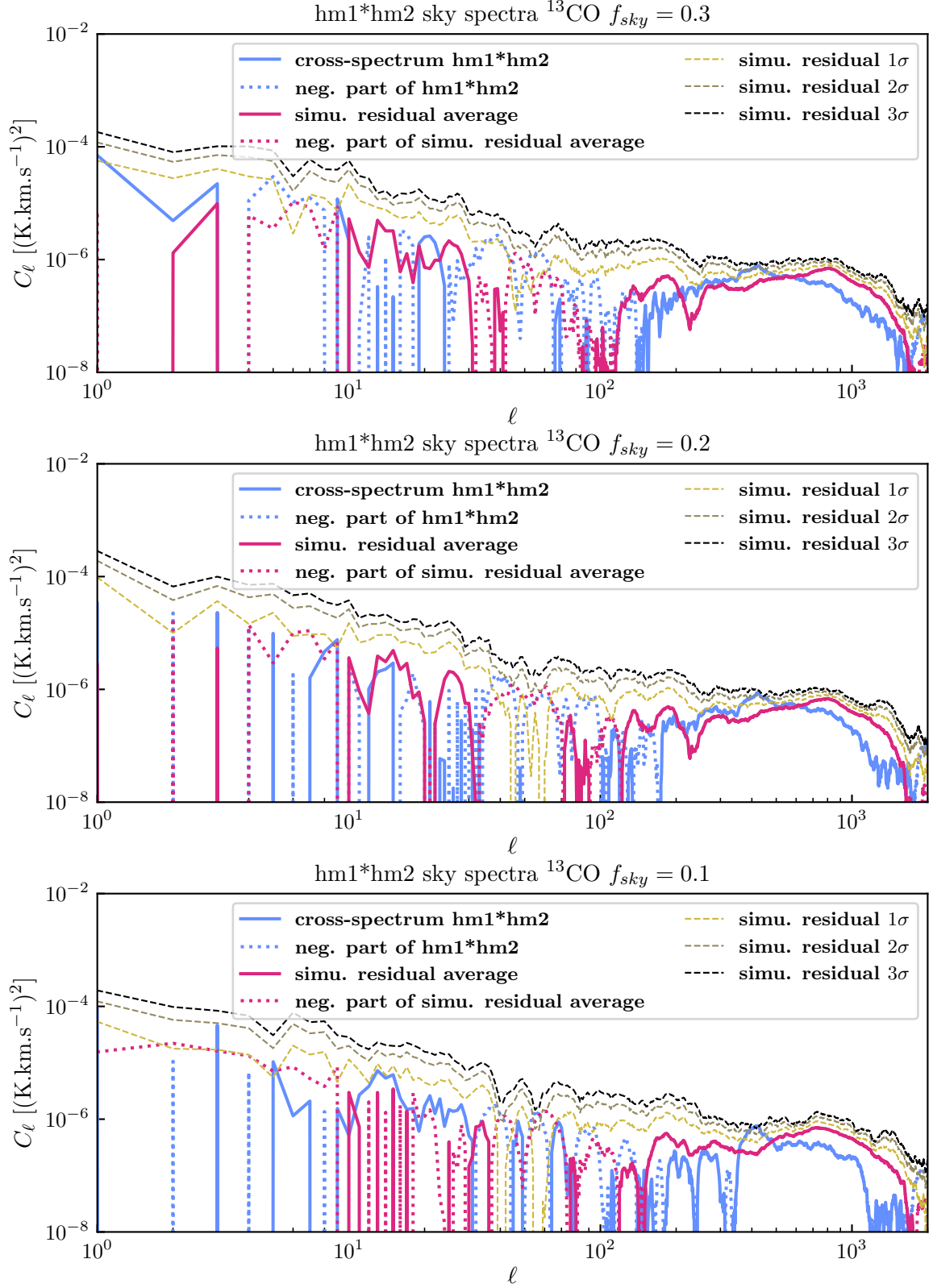


Fig. V.37 ^{13}CO HM1 * HM2 cross-spectra with various sky fraction masks, and log-smoothed with a multipole fraction of $\ell_{frac} = 10$ (cf. Appendix.C). **Top:** $f_{sky} = 30\%$ mask; **Middle:** $f_{sky} = 20\%$ mask; **Bottom:** $f_{sky} = 10\%$ mask. The plotted spectra are from the SRoll2.1 $^{13}\text{CO } J = 1 \rightarrow 0$ map (in blue), the average HM1 * HM2 cross-spectrum of ^{13}CO residual over the 10 SRoll2.1 simulations (in red), and the 1σ , 2σ , and 3σ deviations from said average cross-spectrum of ^{13}CO residual (respectively in light yellow, dark yellow, and black).

Conclusion

Summary Starting from the SRoll2 mapmaking algorithm, latest improvement of the *Planck* mapmaking method since the 2018 legacy release, I developed an altered version optimized for the determination of the bolometers' response to the ^{12}CO and ^{13}CO emissions: the SRoll2.1 version. This version has the peculiarity of operating not on *one* but **two** HFI frequency bands at the same time. This expansion of the implied sky model brings benefits to the determination of the response difference between bolometers to the ^{12}CO and ^{13}CO rotational lines, but also requires to adapt the fitted data model accordingly.

Applying this new version of the algorithm on the *Planck* HFI data on the 100GHz, 143GHz, and 217GHz bands, I qualified the quality of the resulting fitted data model parameters, especially the bolometers' foreground response coefficients associated to all astrophysical sources accounted for in the SRoll2.1 data model. Using simulations, I estimated the uncertainty level of this method, the consistency of its data model along the explored HFI frequency bands, as well as how it compares to the previous *Planck* results on the CO response coefficients. I showed a good consistency of the method with the previous results, with comparable levels of uncertainty on the sky fitted response coefficients *despite* my additional separation of the ^{12}CO and ^{13}CO isotopologues which was not attempted in the *Planck* collaboration.

Based on the outputs provided by the SRoll2.1 algorithm, i.e. cleaned sky emission maps for each bolometer and fitted foreground response coefficients, I devised a separation method to separate the ^{12}CO and ^{13}CO rotational lines' emission from the rest of the sky signal, leading to fullsky maps of the respective emissions for both the $J = 1 \rightarrow 0$ and $J = 2 \rightarrow 1$ lines. This computation is carried out using all bolometers from 2 HFI frequency bands: the 143GHz band, used as a reference *without any* CO emission, and another frequency containing the CO signal to be mapped. The 5 maps produced by this method are constructed at different resolutions to optimize the balance between resolution and the good invertibility of the defined system.

I proposed an iterative scheme on SRoll2.1 to improve the quality of the produced ^{12}CO and ^{13}CO maps, but the intermediate results did not display a stable convergence of the maps. I explored potential causes for this instability of the method, but did not have the time to develop and carry out the required correction to achieve its convergence.

The newly constructed ^{12}CO and ^{13}CO maps are then compared to state-of-the-art external CO surveys, as well as to the most recent CO maps provided by the *Planck* collaboration. The $J = 1 \rightarrow 0$ maps display a robust consistency with the external surveys, always either better or at least as good as the *Planck* release CO maps, with the added bonus of my additional separation of the 2 isotopologues. The quality of my new maps is only degraded in regions where the CO emission displays sharp spatial features and peaked signal. I proved that this is caused by some contamination stemming from the use of the 143GHz band in the construction procedure, and I suggested potential solutions to correct this effect. I presented the results from the most straightforward of these potential correc-

tions, yielding good results, but which can still be significantly improved. The $J = 2 \rightarrow 1$ maps yielded good agreement with the previous *Planck* CO maps, but were more impacted by the aforementioned effect, and display a consequent leakage of signal from ^{13}CO into the ^{12}CO map (as opposed to the $J = 1 \rightarrow 0$ maps displaying very low amount of such leakage), which I could not accurately quantify but is visible in the correlation with the *Planck* CO maps.

I also check for some potential sources of systematic errors and biases in my overall method, i.e. which could originate from either SRoll2.1 and/or my maps construction scheme. I showed that the most anticipated source of error, i.e. the signal leakage from dust into CO, is at most comparable to the aforementioned effect of the 143GHz contamination in peak signal regions. My understanding is that the dominant contributor of this contamination effect consists of the dust leakage in these regions. It was shown to be negligible in other regions of the sky. The zero-level of the bolometers was also proven to have negligible impact on the produced ^{12}CO and ^{13}CO maps, thanks to SRoll2.1 fitting *differential* quantities. I quantified the signal leakage between the ^{12}CO and ^{13}CO maps for the $J = 1 \rightarrow 0$ line, which is at most of the order of 1% of the ^{12}CO . I also explored the possibility that my method would detect the presence of molecular emission from the 143GHz frequency band, spatially correlated to the CO emission. My uncertainty level did not allow me to compute a map of such emission, neither could I conclusively attest of a detection of their emission from the average value of the CO response coefficients.

Finally, relying on the strength of these newly built maps, i.e. a complete coverage of the sky with acceptable noise level and low risk of contaminations, I used them to explore the CO emission at high Galactic latitudes and low column densities. Using results from simulations to estimate the level of *both* the random false alarm chances as well as the potential residues in the maps from systematic error or biases, I searched for a CO detection at increasingly high latitude and increasingly low column densities using cross-spectra computations. This search yielded a robust statistical detection of CO emission for multipoles $240 \lesssim \ell \lesssim 560$ ($44'.0 \gtrsim \text{ang. scale} \gtrsim 18'.8$) at high latitudes corresponding to gas column densities below $5.5 \times 10^{20} \text{cm}^{-2}$.

Future prospects The methods developed and presented in this work leave much room for improvements and extensions. One of the main improvements, with the widest range of expected effects, is the increase of the SRoll2.1 input template maps' resolution, ideally up to the full *Planck* resolution of $N_{\text{side}} = 2048$. I expect this resolution increase to improve:

- The efficiency of SRoll2.1 to separate the various foreground sources, leading to improved quality of the fitted RDCs, and a reduction of their determination uncertainty,
- The quality and accuracy of the *realignment* procedure, leading to better quality of the output maps,
- The quality of the constructed ^{12}CO and ^{13}CO maps, notably by a drastic reduction of the 143GHz band contamination in peaked signal region, if not a complete correction of the effect,
- Through the combination of the improved quality of the RDCs and the reduction of the 143GHz contamination, I expect the quality of the ^{12}CO and ^{13}CO separation to improve for the $J = 2 \rightarrow 1$ line,

- The chances to detect the molecular emission from the 143GHz band, thanks to the better determination of sharp emission features as well as the improved RDCs' quality,
- The convergence of the SRoll2.1 iterative scheme, by helping to mitigate many of the potential sources of the methods' non-convergence,
- The overall capacity of the method to detect the CO at high latitude and low column density, from the same type of computation, from the combination of all the aforementioned implied improvements.

However, increasing the input template maps' resolution requires the correction of the bolometers' beam differences effect. I believe that the first step to take to improve the overall quality of SRoll2.1 and the ^{12}CO and ^{13}CO maps should be the implementation of a correction of the bolometers beam differences and then an increase of the templates' resolution.

If this resolution increase was to be implemented, I believe that a new attempt at the iterative scheme on SRoll2.1 would be worth a shot. I expect that the convergence would be a lot easier to achieve, even though it might require a few additional investigative steps, such as the measure of the noise impact on the maps' drift, which could be estimated and then corrected within the iterative scheme at each step.

Another avenue worth exploring is the molecular emissions from the 143GHz. One potential approach I intended to investigate but did not have time to carry out is the exploration of the 3 maps of the remaining signal from my ^{12}CO and ^{13}CO maps computation, and more especially the I^{other} map. Indeed, I expect that if any molecular signal is present and detectable, given my computation method, it ought to be mixed in with the rest of the sky emission in the I^{other} map. It would then be interesting to explore this map for the presence of the molecular signal, either through the computation of some additional separation methods, or at least statistical exploration, using adapted tracers. One such tracer could be the very CO maps produced, and check their correlation with the I^{other} map to estimate the intensity level of the molecular emission from the 143GHz band. However, such exploration should be carefully thought out since an exploration based on correlation computation would inevitably be impacted by the presence of the dust and its high spatial correlation with the molecular lines' emission. This exploration would be interesting, *whether or not it yields actual detection*, since the molecular emission from the 143GHz band, but also at other frequencies, might be an additional source of concern for the next CMB surveys such as LiteBIRD.

The last expansion of my method I intended to attempt but did not have time to carry out during my PhD is the adaptation of my ^{12}CO and ^{13}CO maps computation algorithm to attempt the production of fullsky maps of the CO emission's **polarization**. Such fullsky mapping of the polarization of the CO emission has never been done before, and it would be of interest for many reasons. First, it would be a way to check the estimated polarization fraction of CO and provide a potential source of validation for the models attempting to account for this effect. Secondly, it would allow the future CMB experiments aiming to search for the B-modes of the CMB polarization, such as LiteBIRD, to correct their future data for this source of contamination, since it is expected to have non-negligible impact, as predicted by the model presented in the recent work of [Puglisi et al. \(2017\)](#).

Appendix A

HEALPix scheme: a pixelisation of the sphere

The Hierarchical Equal Area isoLatitude Pixelization (HEALPix), is a pixelisation scheme of the sphere surface, i.e. a discretization scheme of any arbitrary function or quantity defined on the sphere.

Origin This structure of sphere pixelisation, fully presented and demonstrated in the work of [Gorski et al. \(2005\)](#), was originally developed for the study, analysis and processing of the first CMB observation experiments (e.g. WMAP, BOOMERANG), and was later used for the following CMB and other large-scale sky surveys (e.g. *Planck*, Herschel), but is able to answer the many challenges of data representation and processing of any observational data on the sphere, and is, as such, of growing interest in the fields of oceanography and climatology where the observational data is taken at the level of the Earth’s surface, thereby facing similar challenges for the processing and analysis of their data.

Associated tools HEALPix was also developed in association with a full package of conjointly designed and optimized algorithms for data processing and analysis on said pixelated data, mostly falling into 3 categories:

- Data projection, representation, and plotting, following a wide range of projection of the sphere for an efficient display of the information embedded in the data, in a form most suitable to the nature of said data,
- Data handling and transformation on the sphere, with rotations, rescaling, kernel convolution, interpolation, etc,
- Statistical decomposition and analysis of the data, such as spherical harmonics decomposition, power spectrum and cross-spectrum computation (see [Appendix.C](#) for more details on such decomposition and analysis), for spatial and spectral auto-correlation and cross-correlation analysis.

Main characteristics To achieve the required quality and efficiency of the data representation and the aforementioned associated data processing tools, the scheme must meet certain mathematical conditions of the discretization. With the aim of achieving the goals set in the above description, and exhaustively listed and discussed in [Gorski et al. \(2005\)](#), the main characteristics of the pixelisation scheme are the following:

- ⇒ A **hierarchical structuring** of the database. This is essential to ensure the associated algorithms and data handling efficiency, especially necessary for large database processing. The main idea behind this hierarchical structuring is for neighbouring data on the sphere to conserve its neighbouring properties in the data tree structure,
- ⇒ **Equal surface area** covered by all the pixels on the sphere. This allows a constant definition of the noise per pixel on the whole sphere, and avoids the distortions of the data along its distribution in latitude, and especially at the poles, as can be seen in most other sky observation representations projected on the plane,
- ⇒ An **isolatitude distribution** of the pixels on the sphere, most critical for computational efficiency and speed in any algorithm involving spherical harmonics decomposition and handling.

Construction The pixelisation is finally constructed by starting from a subdivision of the sphere into **12 curvilinear quadrilaterals**, distributed with isolatitude centres and equal area. These 12 quadrilaterals are the base of the pixelisation and therefore correspond to the lowest resolution of the pixelisation, referred to as a HEALPix resolution of $N_{side} = 1$. Any further pixelisation, increasing the resolution of the discretisation, is performed by dividing each parent pixel into 4 subpixels of equal area positioned into the 4 corners of the parent pixel. Given the hereby defined pixelisation, a subdivision of the parent pixel into 4 subpixels doubles the number of sides along any given side of the parent pixel. For this reason, the incremental pixelisation is noted in resolution with a value of N_{side} being incremented by powers of 2, i.e. $N_{side} = 2^k$ with k the number of divisions into 4 subpixels performed on the 12 initial quadrilaterals. This leads to a total number of pixels in the map always equal to $N_{pix} = 12 \times N_{side}^2$.

Fig.A.1, taken from Fig.3 of [Gorski et al. \(2005\)](#) represents an orthographic view of the sphere pixelisation, starting from the first 12 curvilinear quadrilaterals, and incrementally subdividing its pixels to increase the resolution.

The angular size of a pixel from the HEALPix scheme, at resolution N_{side} , can be computed in arcmin unit from the following expression (cf. [Gorski et al. \(2005\)](#)):

$$\text{Pixel angular size}(N_{side}) = 60 \times \frac{180}{\pi} \sqrt{\frac{4\pi}{12N_{side}^2}} \text{ [arcmin]}. \quad (\text{A.1})$$

I listed in Table.A.1 the results from the above expression of Eq.A.1 for the first values of HEALPix resolutions, as well as the corresponding total number of pixels in the map, and the value of the spherical harmonics multipole of same angular size as the HEALPix pixel (see Appendix.C for the definition of the spherical harmonics and their multipoles) given by $\ell_{pixel \text{ size}} = 3N_{side} - 1$.

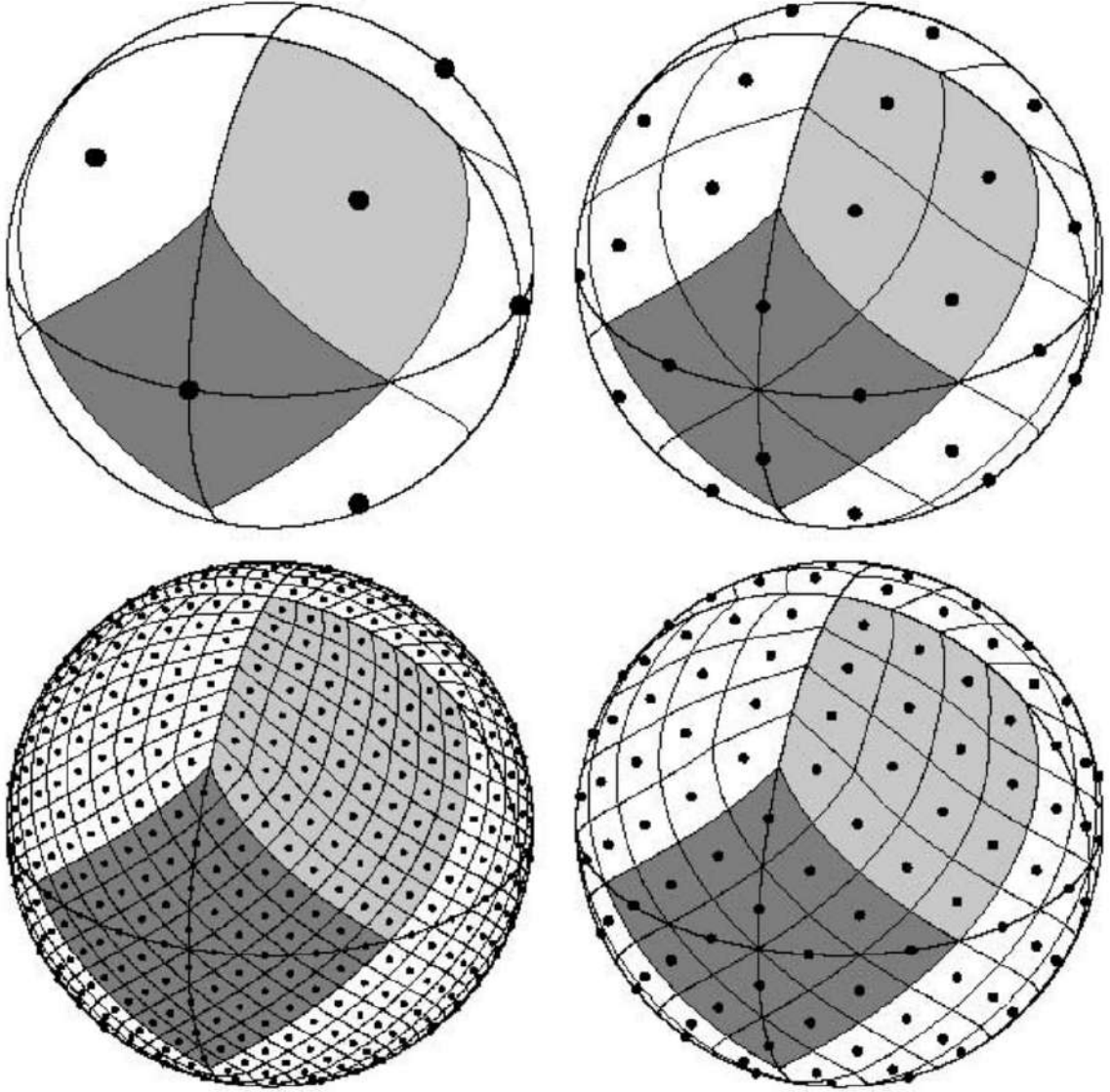


Fig. A.1 Orthographic view of the HEALPix partition of the sphere. The overplot of equator and meridians illustrates the octahedral symmetry of HEALPix. Light gray shading shows one of the 8 (4 north and 4 south) identical polar base-resolution pixels. Dark gray shading shows one of the 4 identical equatorial base-resolution pixels. Moving clockwise from the top left panel, the grid is hierarchically subdivided with the grid resolution parameter equal to $N_{side} = 1, 2, 4, 8$, and the corresponding total number of pixels equal to $N_{pix} = 12 \times N_{side}^2 = 12, 48, 192, 768$. All pixel centres are located on $N_{ring} = 4N_{side} - 1$ rings of constant latitude. Within each panel the areas of all pixels are identical. Taken from [Gorski et al. \(2005\)](#) Fig.3.

Table A.1: Summary of HEALPix total pixel number (N_{pix}), pixel angular size, and multipole of pixel size ($\ell_{pixel\ size}$) for the first 12 HEALPix resolution values (N_{side}).

N_{side}	N_{pix}	Pix. ang. size	$\ell_{pixel\ size}$
1	12	$58^\circ.6$	2
2	48	$29^\circ.3$	5
4	192	$14^\circ.7$	11
8	768	$7^\circ.33$	23
16	3072	$3^\circ.66$	47
32	12288	$1^\circ.83$	95
64	49152	$55'.0$	191
128	196608	$27'.5$	383
256	786432	$13'.7$	767
512	3145728	$6'.87$	1535
1024	12582912	$3'.44$	3071
2048	50331648	$1'.72$	6143

Appendix B

^{12}CO and ^{13}CO maps computation: complete matrix form

I detail here the full definition of the χ^2 minimization computed to build my ^{12}CO and ^{13}CO maps, as presented in Section IV.2. Especially, I detail how the computation is carried out at 3 different resolutions for the produced output maps.

I remind that the data model used to build the χ^2 minimization building the 5 output maps is defined as follows:

$$\begin{aligned}\tilde{S}_{b,p}^{out} = & I_p^{other} + \rho_b \cos(2\phi_{b,p}) Q_p^{other} + \rho_b \sin(2\phi_{b,p}) U_p^{other} \\ & + L_b^{12CO} M_p^{12CO} + L_b^{13CO} M_p^{13CO} + n_{b,p},\end{aligned}$$

see Eq. IV.8 for more details. I also remind that using this model I defined the following χ^2 to be minimised in each pixel:

$$\begin{aligned}\chi_p^2 = & \sum_{b \in \mathfrak{B}} w_{b,p} \left(I_p^{other} + \rho_b \cos(2\phi_{b,p}) Q_p^{other} + \rho_b \sin(2\phi_{b,p}) U_p^{other} \right. \\ & \left. + L_b^{12CO} M_p^{12CO} + L_b^{13CO} M_p^{13CO} - \tilde{S}_{b,p}^{out} \right)^2 \\ = & \sum_{b \in \mathcal{B}_{CO}} w_{b,p} \left(I_p^{other} + \rho_b \cos(2\phi_{b,p}) Q_p^{other} + \rho_b \sin(2\phi_{b,p}) U_p^{other} \right. \\ & \left. + L_b^{12CO} M_p^{12CO} + L_b^{13CO} M_p^{13CO} - \tilde{S}_{b,p}^{out} \right)^2 \\ & + \sum_{b \in \mathcal{B}_{143}} w_{b,p} \left(I_p^{other} + \rho_b \cos(2\phi_{b,p}) Q_p^{other} + \rho_b \sin(2\phi_{b,p}) U_p^{other} - \tilde{S}_{b,p}^{out} \right)^2.\end{aligned}$$

The computation of this χ^2 minimization leads to the following system of equations:

$$\begin{cases} \frac{\partial \chi^2}{\partial I_p^{other}} = 0 \\ \frac{\partial \chi^2}{\partial Q_p^{other}} = 0 \\ \frac{\partial \chi^2}{\partial U_p^{other}} = 0 \\ \frac{\partial \chi^2}{\partial M_p^{12CO}} = 0 \\ \frac{\partial \chi^2}{\partial M_p^{13CO}} = 0 \end{cases}.$$

By injecting the definition of the χ^2 given by Eq. IV.9, and reminded above, this system

becomes:

$$\left\{ \begin{array}{l} \sum_{b \in \mathfrak{B}} w_{b,p} \left(I_p^{other} + \rho_b \cos(2\phi_{b,p}) Q_p^{other} + \rho_b \sin(2\phi_{b,p}) U_p^{other} \right) \\ + \sum_{b \in \mathcal{B}_{CO}} w_{b,p} \left(L_b^{12CO} M_p^{12CO} + L_b^{13CO} M_p^{13CO} \right) \\ \sum_{b \in \mathfrak{B}} w_{b,p} \rho_b \cos(2\phi_{b,p}) \left(I_p^{other} + \rho_b \cos(2\phi_{b,p}) Q_p^{other} + \rho_b \sin(2\phi_{b,p}) U_p^{other} \right) \\ + \sum_{b \in \mathcal{B}_{CO}} w_{b,p} \rho_b \cos(2\phi_{b,p}) \left(L_b^{12CO} M_p^{12CO} + L_b^{13CO} M_p^{13CO} \right) \\ \sum_{b \in \mathfrak{B}} w_{b,p} \rho_b \sin(2\phi_{b,p}) \left(I_p^{other} + \rho_b \cos(2\phi_{b,p}) Q_p^{other} + \rho_b \sin(2\phi_{b,p}) U_p^{other} \right) \\ + \sum_{b \in \mathcal{B}_{CO}} w_{b,p} \rho_b \sin(2\phi_{b,p}) \left(L_b^{12CO} M_p^{12CO} + L_b^{13CO} M_p^{13CO} \right) \\ \sum_{b \in \mathcal{B}_{CO}} w_{b,p} L_b^{12CO} \left(I_p^{other} + \rho_b \cos(2\phi_{b,p}) Q_p^{other} + \rho_b \sin(2\phi_{b,p}) U_p^{other} \right. \\ \quad \left. + L_b^{12CO} M_p^{12CO} + L_b^{13CO} M_p^{13CO} \right) \\ \sum_{b \in \mathcal{B}_{CO}} w_{b,p} L_b^{13CO} \left(I_p^{other} + \rho_b \cos(2\phi_{b,p}) Q_p^{other} + \rho_b \sin(2\phi_{b,p}) U_p^{other} \right. \\ \quad \left. + L_b^{12CO} M_p^{12CO} + L_b^{13CO} M_p^{13CO} \right) \end{array} \right. = \begin{array}{l} \sum_{b \in \mathfrak{B}} w_{b,p} \tilde{S}_{b,p}^{out} \\ \sum_{b \in \mathfrak{B}} w_{b,p} \rho_b \cos(2\phi_{b,p}) \tilde{S}_{b,p}^{out} \\ \sum_{b \in \mathfrak{B}} w_{b,p} \rho_b \sin(2\phi_{b,p}) \tilde{S}_{b,p}^{out} \\ \sum_{b \in \mathcal{B}_{CO}} w_{b,p} L_b^{12CO} \tilde{S}_{b,p}^{out} \\ \sum_{b \in \mathcal{B}_{CO}} w_{b,p} L_b^{13CO} \tilde{S}_{b,p}^{out} \end{array} .$$

Using the notations introduced by Eq.III.30, which I remind were the following:

$$\begin{aligned} II_{b,p}^{out} &= w_{b,p}, \quad IQ_{b,p}^{out} = w_{b,p} \rho_b \cos(2\phi_{b,p}), \quad IU_{b,p}^{out} = w_{b,p} \rho_b \sin(2\phi_{b,p}), \\ QQ_{b,p}^{out} &= w_{b,p} \rho_b^2 \cos^2(2\phi_{b,p}), \quad QU_{b,p}^{out} = w_{b,p} \rho_b^2 \cos(2\phi_{b,p}) \sin(2\phi_{b,p}), \quad UU_{b,p}^{out} = w_{b,p} \rho_b^2 \sin^2(2\phi_{b,p}), \end{aligned}$$

we can then expressed the above system of equations in matrix form as $A \cdot X = V$ with:

$$A = \begin{pmatrix} \sum_{b \in \mathfrak{B}} II_{b,p}^{out} & \sum_{b \in \mathfrak{B}} IQ_{b,p}^{out} & \sum_{b \in \mathfrak{B}} IU_{b,p}^{out} & \sum_{b \in \mathcal{B}_{CO}} I^{12CO}_{b,p}^{out} & \sum_{b \in \mathcal{B}_{CO}} I^{13CO}_{b,p}^{out} \\ \sum_{b \in \mathfrak{B}} IQ_{b,p}^{out} & \sum_{b \in \mathfrak{B}} QQ_{b,p}^{out} & \sum_{b \in \mathfrak{B}} QU_{b,p}^{out} & \sum_{b \in \mathcal{B}_{CO}} Q^{12CO}_{b,p}^{out} & \sum_{b \in \mathcal{B}_{CO}} Q^{13CO}_{b,p}^{out} \\ \sum_{b \in \mathfrak{B}} IU_{b,p}^{out} & \sum_{b \in \mathfrak{B}} QU_{b,p}^{out} & \sum_{b \in \mathfrak{B}} UU_{b,p}^{out} & \sum_{b \in \mathcal{B}_{CO}} U^{12CO}_{b,p}^{out} & \sum_{b \in \mathcal{B}_{CO}} U^{13CO}_{b,p}^{out} \\ \sum_{b \in \mathcal{B}_{CO}} I^{12CO}_{b,p}^{out} & \sum_{b \in \mathcal{B}_{CO}} Q^{12CO}_{b,p}^{out} & \sum_{b \in \mathcal{B}_{CO}} U^{12CO}_{b,p}^{out} & \sum_{b \in \mathcal{B}_{CO}} {}^{12CO}{}^{12CO}_{b,p}^{out} & \sum_{b \in \mathcal{B}_{CO}} {}^{12CO}{}^{13CO}_{b,p}^{out} \\ \sum_{b \in \mathcal{B}_{CO}} I^{13CO}_{b,p}^{out} & \sum_{b \in \mathcal{B}_{CO}} Q^{13CO}_{b,p}^{out} & \sum_{b \in \mathcal{B}_{CO}} U^{13CO}_{b,p}^{out} & \sum_{b \in \mathcal{B}_{CO}} {}^{12CO}{}^{13CO}_{b,p}^{out} & \sum_{b \in \mathcal{B}_{CO}} {}^{13CO}{}^{13CO}_{b,p}^{out} \end{pmatrix},$$

$$X = \begin{pmatrix} I_p^{other} \\ Q_p^{other} \\ U_p^{other} \\ M_p^{12CO} \\ M_p^{13CO} \end{pmatrix}, \quad V = \begin{pmatrix} \sum_{b \in \mathfrak{B}} w_{b,p} \tilde{S}_{b,p}^{out} \\ \sum_{b \in \mathfrak{B}} w_{b,p} \rho_b \cos(2\phi_{b,p}) \tilde{S}_{b,p}^{out} \\ \sum_{b \in \mathfrak{B}} w_{b,p} \rho_b \sin(2\phi_{b,p}) \tilde{S}_{b,p}^{out} \\ \sum_{b \in \mathcal{B}_{CO}} w_{b,p} L_b^{12CO} \tilde{S}_{b,p}^{out} \\ \sum_{b \in \mathcal{B}_{CO}} w_{b,p} L_b^{13CO} \tilde{S}_{b,p}^{out} \end{pmatrix},$$

where all the data to compute the matrix A as well as the vector V are provided by SRoll2.1,

and where I additionally defined the following terms:

$$\begin{aligned}
I^{12}CO_{b,p}^{out} &= w_{b,p} L_b^{12CO}, \\
I^{13}CO_{b,p}^{out} &= w_{b,p} L_b^{13CO}, \\
Q^{12}CO_{b,p}^{out} &= w_{b,p} \rho_b \cos(2\phi_{b,p}) L_b^{12CO}, \\
Q^{13}CO_{b,p}^{out} &= w_{b,p} \rho_b \cos(2\phi_{b,p}) L_b^{13CO}, \\
U^{12}CO_{b,p}^{out} &= w_{b,p} \rho_b \sin(2\phi_{b,p}) L_b^{12CO}, \\
U^{13}CO_{b,p}^{out} &= w_{b,p} \rho_b \sin(2\phi_{b,p}) L_b^{13CO}, \\
^{12}CO^{12}CO_{b,p}^{out} &= w_{b,p} \left(L_b^{12CO} \right)^2, \\
^{12}CO^{13}CO_{b,p}^{out} &= w_{b,p} L_b^{12CO} L_b^{13CO}, \\
^{13}CO^{13}CO_{b,p}^{out} &= w_{b,p} \left(L_b^{13CO} \right)^2.
\end{aligned}$$

The above expression of the χ^2 minimization computed to build my maps only corresponds to the case where **all 5 maps** are computed *with the same* resolution. In the fully general case of my ^{12}CO and ^{13}CO maps computation, as described in subsection IV.2.1, this χ^2 minimization is computed at each pixel *of the maps with the lowest resolution*, i.e. the Q^{other} and U^{other} with resolution of $N_{side} = 256$ in my case. For each pixel of the aforementioned maps of lowest resolution, there are 16 pixels in the M^{12CO} and M^{13CO} maps, and 64 pixels in the I^{other} map, for the same corresponding region. Nevertheless, the χ^2 minimization computed at each pixel *of the maps with the lowest resolution* can still be written in matrix form as: $A \cdot X = V$, where A and V are respectively a matrix and a vector, both built from the outputs of a SRoll2.1 run, and where X is the vector of the maps we aim to determine.

In these conditions of multiple output maps resolution, the corresponding vectors X and V are then defined as follows:

$$X = \begin{pmatrix} I_{p_1}^{other} \\ \vdots \\ I_{p_{64}}^{other} \\ Q_p^{other} \\ U_p^{other} \\ M_{p_1}^{12CO} \\ \vdots \\ M_{p_{16}}^{12CO} \\ M_{p_1}^{13CO} \\ \vdots \\ M_{p_{16}}^{13CO} \end{pmatrix}, \quad V = \begin{pmatrix} \sum_{b \in \mathfrak{B}} w_{b,p} \tilde{S}_{b,p_1}^{out} \\ \vdots \\ \sum_{b \in \mathfrak{B}} w_{b,p} \tilde{S}_{b,p_{64}}^{out} \\ \sum_{b \in \mathfrak{B}} w_{b,p} \rho_b \cos(2\phi_{b,p}) \tilde{S}_{b,p}^{out} \\ \sum_{b \in \mathfrak{B}} w_{b,p} \rho_b \sin(2\phi_{b,p}) \tilde{S}_{b,p}^{out} \\ \sum_{b \in \mathcal{B}_{CO}} w_{b,p} L_b^{12CO} \tilde{S}_{b,p_1}^{out} \\ \vdots \\ \sum_{b \in \mathcal{B}_{CO}} w_{b,p} L_b^{12CO} \tilde{S}_{b,p_{16}}^{out} \\ \sum_{b \in \mathcal{B}_{CO}} w_{b,p} L_b^{13CO} \tilde{S}_{b,p_1}^{out} \\ \vdots \\ \sum_{b \in \mathcal{B}_{CO}} w_{b,p} L_b^{13CO} \tilde{S}_{b,p_{16}}^{out} \end{pmatrix},$$

where the index on the pixel p is the index of the subpixel in the output maps of higher resolution, since in our output maps, for each pixel in the Q^{other} and U^{other} , there are 16

pixels in the same region of the $M^{12\text{CO}}$ and $M^{13\text{CO}}$ maps, and 64 pixels in the same region of the I^{other} map.

As for the matrix A , it can be written as a block matrix:

$$A = \begin{pmatrix} \mathbf{D}_{II} & \mathbf{C}_{IQ} & \mathbf{C}_{IU} & \mathbf{B}_{I^{12}\text{CO}} & \mathbf{B}_{I^{13}\text{CO}} \\ \mathbf{C}_{IQ}^\top & \mathbf{V}_{QQ} & \mathbf{V}_{QU} & \mathbf{R}_{Q^{12}\text{CO}} & \mathbf{R}_{Q^{13}\text{CO}} \\ \mathbf{C}_{IU}^\top & \mathbf{V}_{QU} & \mathbf{V}_{UU} & \mathbf{R}_{U^{12}\text{CO}} & \mathbf{R}_{U^{13}\text{CO}} \\ \mathbf{B}_{I^{12}\text{CO}}^\top & \mathbf{R}_{Q^{12}\text{CO}}^\top & \mathbf{R}_{U^{12}\text{CO}}^\top & \mathbf{D}_{12\text{CO}^{12}\text{CO}} & \mathbf{D}_{12\text{CO}^{13}\text{CO}} \\ \mathbf{B}_{I^{13}\text{CO}}^\top & \mathbf{R}_{Q^{13}\text{CO}}^\top & \mathbf{R}_{U^{13}\text{CO}}^\top & \mathbf{D}_{12\text{CO}^{13}\text{CO}}^\top & \mathbf{D}_{13\text{CO}^{13}\text{CO}} \end{pmatrix},$$

where I used the following notations:

- $\mathbf{D}$ for a diagonal square block,
- $\mathbf{C}$ for a column block (therefore $\mathbf{C}^\top$ is a row block),
- $\mathbf{B}$ for a generic dimensions rectangle block,
- $\mathbf{V}$ for a single scalar value,
- $\mathbf{R}$ for a row block (therefore $\mathbf{R}^\top$ is a column block).

The matrix blocks introduced above are then defined as:

$$\mathbf{D}_{II} = \begin{pmatrix} \sum_{b \in \mathfrak{B}} II_{b,p_1}^{\text{out}} & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sum_{b \in \mathfrak{B}} II_{b,p_{64}}^{\text{out}} \end{pmatrix}, \quad \mathbf{C}_{IQ} = \begin{pmatrix} \sum_{b \in \mathfrak{B}} IQ_{b,p_1}^{\text{out}} \\ \vdots \\ \sum_{b \in \mathfrak{B}} IQ_{b,p_{64}}^{\text{out}} \end{pmatrix}, \quad \mathbf{C}_{IU} = \begin{pmatrix} \sum_{b \in \mathfrak{B}} IU_{b,p_1}^{\text{out}} \\ \vdots \\ \sum_{b \in \mathfrak{B}} IU_{b,p_{64}}^{\text{out}} \end{pmatrix},$$

$$\mathbf{R}_{Q^{12}\text{CO}} = \left(\sum_{b \in \mathcal{B}_{\text{CO}}} Q^{12}\text{CO}_{b,p_1}^{\text{out}} \quad \cdots \quad \sum_{b \in \mathcal{B}_{\text{CO}}} Q^{12}\text{CO}_{b,p_{16}}^{\text{out}} \right), \quad \mathbf{R}_{Q^{13}\text{CO}} = \left(\sum_{b \in \mathcal{B}_{\text{CO}}} Q^{13}\text{CO}_{b,p_1}^{\text{out}} \quad \cdots \quad \sum_{b \in \mathcal{B}_{\text{CO}}} Q^{13}\text{CO}_{b,p_{16}}^{\text{out}} \right),$$

$$\mathbf{R}_{U^{12}\text{CO}} = \left(\sum_{b \in \mathcal{B}_{\text{CO}}} U^{12}\text{CO}_{b,p_1}^{\text{out}} \quad \cdots \quad \sum_{b \in \mathcal{B}_{\text{CO}}} U^{12}\text{CO}_{b,p_{16}}^{\text{out}} \right), \quad \mathbf{R}_{U^{13}\text{CO}} = \left(\sum_{b \in \mathcal{B}_{\text{CO}}} U^{13}\text{CO}_{b,p_1}^{\text{out}} \quad \cdots \quad \sum_{b \in \mathcal{B}_{\text{CO}}} U^{13}\text{CO}_{b,p_{16}}^{\text{out}} \right),$$

$$\mathbf{B}_{I^{12}CO} = \left(\begin{array}{ccccccc} \sum_{b \in \mathcal{B}_{CO}} I^{12}CO_{b,p_1}^{out} & 0 & \cdots & & & & \\ \sum_{b \in \mathcal{B}_{CO}} I^{12}CO_{b,p_2}^{out} & 0 & \cdots & & & & \\ \sum_{b \in \mathcal{B}_{CO}} I^{12}CO_{b,p_3}^{out} & 0 & \cdots & & & & \\ \sum_{b \in \mathcal{B}_{CO}} I^{12}CO_{b,p_4}^{out} & 0 & \cdots & & & & \\ 0 & \sum_{b \in \mathcal{B}_{CO}} I^{12}CO_{b,p_5}^{out} & 0 & \cdots & & & \\ 0 & \sum_{b \in \mathcal{B}_{CO}} I^{12}CO_{b,p_6}^{out} & 0 & \cdots & & & \\ \vdots & \vdots & & \ddots & & & \\ & & & \vdots & & \vdots & \\ & \cdots & 0 & \sum_{b \in \mathcal{B}_{CO}} I^{12}CO_{b,p_{59}}^{out} & 0 & & \\ & \cdots & 0 & \sum_{b \in \mathcal{B}_{CO}} I^{12}CO_{b,p_{60}}^{out} & 0 & & \\ & & \cdots & 0 & \sum_{b \in \mathcal{B}_{CO}} I^{12}CO_{b,p_{61}}^{out} & & \\ & & \cdots & 0 & \sum_{b \in \mathcal{B}_{CO}} I^{12}CO_{b,p_{62}}^{out} & & \\ & & \cdots & 0 & \sum_{b \in \mathcal{B}_{CO}} I^{12}CO_{b,p_{63}}^{out} & & \\ & & \cdots & 0 & \sum_{b \in \mathcal{B}_{CO}} I^{12}CO_{b,p_{64}}^{out} & & \end{array} \right),$$

$$\mathbf{B}_{I^{13}CO} = \left(\begin{array}{ccccccc} \sum_{b \in \mathcal{B}_{CO}} I^{13}CO_{b,p_1}^{out} & 0 & \cdots & & & & \\ \sum_{b \in \mathcal{B}_{CO}} I^{13}CO_{b,p_2}^{out} & 0 & \cdots & & & & \\ \sum_{b \in \mathcal{B}_{CO}} I^{13}CO_{b,p_3}^{out} & 0 & \cdots & & & & \\ \sum_{b \in \mathcal{B}_{CO}} I^{13}CO_{b,p_4}^{out} & 0 & \cdots & & & & \\ 0 & \sum_{b \in \mathcal{B}_{CO}} I^{13}CO_{b,p_5}^{out} & 0 & \cdots & & & \\ 0 & \sum_{b \in \mathcal{B}_{CO}} I^{13}CO_{b,p_6}^{out} & 0 & \cdots & & & \\ \vdots & \vdots & & \ddots & & & \\ & & & \vdots & & \vdots & \\ & \cdots & 0 & \sum_{b \in \mathcal{B}_{CO}} I^{13}CO_{b,p_{59}}^{out} & 0 & & \\ & \cdots & 0 & \sum_{b \in \mathcal{B}_{CO}} I^{13}CO_{b,p_{60}}^{out} & 0 & & \\ & & \cdots & 0 & \sum_{b \in \mathcal{B}_{CO}} I^{13}CO_{b,p_{61}}^{out} & & \\ & & \cdots & 0 & \sum_{b \in \mathcal{B}_{CO}} I^{13}CO_{b,p_{62}}^{out} & & \\ & & \cdots & 0 & \sum_{b \in \mathcal{B}_{CO}} I^{13}CO_{b,p_{63}}^{out} & & \\ & & \cdots & 0 & \sum_{b \in \mathcal{B}_{CO}} I^{13}CO_{b,p_{64}}^{out} & & \end{array} \right),$$

$$\begin{aligned}
\mathbf{D}_{^{12}\text{CO}^{12}\text{CO}} &= \begin{pmatrix} \sum_{b \in \mathcal{B}_{\text{CO}}} ^{12}\text{CO}^{12}\text{CO}_{b,p_1}^{\text{out}} & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \sum_{b \in \mathcal{B}_{\text{CO}}} ^{12}\text{CO}^{12}\text{CO}_{b,p_{16}}^{\text{out}} \end{pmatrix}, \quad \mathbf{V}_{QQ} = \sum_{b \in \mathcal{B}} QQ_{b,p}^{\text{out}}, \\
\mathbf{D}_{^{12}\text{CO}^{13}\text{CO}} &= \begin{pmatrix} \sum_{b \in \mathcal{B}_{\text{CO}}} ^{12}\text{CO}^{13}\text{CO}_{b,p_1}^{\text{out}} & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \sum_{b \in \mathcal{B}_{\text{CO}}} ^{12}\text{CO}^{13}\text{CO}_{b,p_{16}}^{\text{out}} \end{pmatrix}, \quad \mathbf{V}_{QU} = \sum_{b \in \mathcal{B}} QU_{b,p}^{\text{out}}, \\
\mathbf{D}_{^{13}\text{CO}^{13}\text{CO}} &= \begin{pmatrix} \sum_{b \in \mathcal{B}_{\text{CO}}} ^{13}\text{CO}^{13}\text{CO}_{b,p_1}^{\text{out}} & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \sum_{b \in \mathcal{B}_{\text{CO}}} ^{13}\text{CO}^{13}\text{CO}_{b,p_{16}}^{\text{out}} \end{pmatrix}, \quad \mathbf{V}_{UU} = \sum_{b \in \mathcal{B}} UU_{b,p}^{\text{out}}.
\end{aligned}$$

Finally, the inverse matrix A^{-1} can be computed efficiently by first building the inverse matrix $\mathbf{D}_{II}^{-1}$ as the diagonal matrix produced from the inverse of the diagonal elements of $\mathbf{D}_{II}$, and by making use of a convenient property. Indeed, as demonstrated in [Lu & Shiou \(2002\)](#), for any matrix M built as a block matrix defined by:

$$M = \begin{pmatrix} \Delta & \Omega \\ \mathcal{U} & \Lambda \end{pmatrix},$$

where the block Δ is diagonal and Λ is a square block, its inverse can be computed from the following relation:

$$M^{-1} = \begin{pmatrix} \Delta^{-1} + \Delta^{-1}\Omega(\Lambda - \mathcal{U}\Delta^{-1}\Omega)^{-1}\mathcal{U}\Delta^{-1} & -\Delta^{-1}\Omega(\Lambda - \mathcal{U}\Delta^{-1}\Omega)^{-1} \\ -(\Lambda - \mathcal{U}\Delta^{-1}\Omega)^{-1}\mathcal{U}\Delta^{-1} & (\Lambda - \mathcal{U}\Delta^{-1}\Omega)^{-1} \end{pmatrix}.$$

From these notations, A^{-1} is computed by using the above property where $\Delta = \mathbf{D}_{II}$, as well as:

$$\begin{aligned}
\Lambda &= \begin{pmatrix} \mathbf{V}_{QQ} & \mathbf{V}_{QU} & \mathbf{R}_{Q^{12}\text{CO}} & \mathbf{R}_{Q^{13}\text{CO}} \\ \mathbf{V}_{QU} & \mathbf{V}_{UU} & \mathbf{R}_{U^{12}\text{CO}} & \mathbf{R}_{U^{13}\text{CO}} \\ \mathbf{R}_{Q^{12}\text{CO}}^\top & \mathbf{R}_{U^{12}\text{CO}}^\top & \mathbf{D}_{^{12}\text{CO}^{12}\text{CO}} & \mathbf{D}_{^{12}\text{CO}^{13}\text{CO}} \\ \mathbf{R}_{Q^{13}\text{CO}}^\top & \mathbf{R}_{U^{13}\text{CO}}^\top & \mathbf{D}_{^{12}\text{CO}^{13}\text{CO}}^\top & \mathbf{D}_{^{13}\text{CO}^{13}\text{CO}} \end{pmatrix}, \\
\mathcal{U} &= \begin{pmatrix} \mathbf{C}_{IQ}^\top \\ \mathbf{C}_{IU}^\top \\ \mathbf{B}_{I^{12}\text{CO}}^\top \\ \mathbf{B}_{I^{13}\text{CO}}^\top \end{pmatrix}, \quad \Omega = \begin{pmatrix} \mathbf{C}_{IQ} & \mathbf{C}_{IU} & \mathbf{B}_{I^{12}\text{CO}} & \mathbf{B}_{I^{13}\text{CO}} \end{pmatrix}.
\end{aligned}$$

Appendix C

Maps spectral analysis & sky spectra masking

A useful statistical tool to explore the data from the various maps used and constructed in this work are the various spectra, e.g. auto-spectrum (also called power spectrum) and cross spectrum, respectively describing the maps signal repartition and correlation in spatial scales space. It is equivalent to the power spectrum or cross-spectrum of 1-D time dependant signals, but applied to a 2-D signal in space.

Since all the maps considered in this work are 2-D signals on the sky, taken as spherical, the mathematical way of computing such spectra is from the maps decomposition, or expansion, into a linear combination of the **spherical harmonics functions**.

I will first define the spherical harmonics expansion, as well as the computation of the maps power spectrum and cross-spectrum from said decomposition. I will then present the masks I built for this work, and which I use in the computation of most spectra throughout this work.

C.1 Spherical harmonics decomposition

The spherical harmonics functions, also called the Laplace spherical harmonics, were defined as the angular part of the solution to Laplace's equation, i.e. $\Delta V = 0$, where Δ is the Laplacian operator and V a 3-D scalar field, i.e. $V : \mathbb{R}^3 \rightarrow \mathbb{R}$. The general solution is defined for complex valued scalar fields, i.e. $V : \mathbb{R}^3 \rightarrow \mathbb{C}$, and in this case, the spherical harmonics are defined as:

$$\forall \ell \in \mathbb{N}, \forall m \in \llbracket -\ell, \ell \rrbracket, Y_\ell^m : S^2 \rightarrow \mathbb{R}$$

$$(\theta, \phi) \rightarrow \sqrt{\frac{2\ell+1}{4\pi} \frac{(\ell-m)!}{(\ell+m)!}} P_\ell^m(\cos(\theta)) e^{im\phi},$$

where S^2 is the 2-D unit sphere, i.e. $S^2 = \{x \in \mathbb{R}^3 \mid \|x\| = 1\}$, using the spherical coordinates $\theta \in [0, \pi)$ and $\phi \in [0, 2\pi)$, l and m are respectively called the *degree* (or *multipole*) and *order* of the spherical harmonic Y_ℓ^m , and P_ℓ^m are the associated Legendre polynomials, defined as:

$$\forall \ell \in \mathbb{N}, \begin{cases} \forall m \in \mathbb{N} & P_\ell^m(x) = (-1)^m (1-x^2)^{\frac{m}{2}} \frac{d^m}{dx^m} (P_\ell(x)) \\ \forall m \in \mathbb{N}^* & P_\ell^{-m}(x) = (-1)^m \frac{(\ell-m)!}{(\ell+m)!} P_\ell^m(x) \end{cases},$$

where P_ℓ is the Legendre polynomial of degree l , which can be expressed using the Rodrigues' formula:

$$P_\ell(x) = \frac{1}{2^\ell \ell!} \frac{d^\ell}{dx^\ell} \left[(x^2 - 1)^\ell \right].$$

From this definition, these functions are orthonormal:

$$\forall(\ell, \ell') \in \mathbb{N}^2, \forall(m, m') \in \llbracket -\ell, \ell \rrbracket \times \llbracket -\ell', \ell' \rrbracket, \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} Y_{\ell}^m(\theta, \phi) Y_{\ell'}^{m'}{}^*(\theta, \phi) \sin(\theta) d\theta d\phi = \delta_{\ell\ell'} \delta_{mm'},$$

where $Y_{\ell}^m{}^*$ is the complex conjugate of Y_{ℓ}^m , and δ is the Kronecker delta.

The spherical harmonics hereby defined form a complete set of orthonormal functions, and therefore an orthonormal basis of the Hilbert space of square-integrable functions $L_{\mathbb{C}}^2(S^2)$, and as such any complex square-integrable function on the 2-D unit sphere S^2 can be expanded as a linear combination of the spherical harmonics.

For *real* functions of the 2-D unit sphere, a similar set of *real* spherical harmonics, noted $Y_{\ell m} : S^2 \rightarrow \mathbb{R}$, can be constructed to retain the main properties of the complex Y_{ℓ}^m . These *real* spherical harmonics are defined from the complex Y_{ℓ}^m by:

$$\forall \ell \in \mathbb{N}, \forall m \in \llbracket -\ell, \ell \rrbracket, Y_{\ell m} = \begin{cases} \sqrt{2}(-1)^m \text{Im}(Y_{\ell}^{|m|}), & \text{if } m < 0 \\ Y_{\ell}^0, & \text{if } m = 0 \\ \sqrt{2}(-1)^m \text{Re}(Y_{\ell}^m), & \text{if } m > 0 \end{cases}$$

where $\text{Re}(z)$ and $\text{Im}(z)$ respectively denote the real and imaginary parts of a complex number z .

Similarly to the complex spherical harmonics, the *real* spherical harmonics form a complete set of orthonormal functions, and therefore an orthonormal basis of the Hilbert space of square-integrable functions $L_{\mathbb{R}}^2(S^2)$, and as such any real square-integrable function on the 2-D unit sphere S^2 can be expanded as a linear combination of the spherical harmonics, which writes as:

$$\forall f \in L_{\mathbb{R}}^2(S^2), \forall(\theta, \phi) \in S^2, f(\theta, \phi) = \sum_{\ell \in \mathbb{N}} \sum_{m \in \llbracket -\ell, \ell \rrbracket} a_{\ell m} Y_{\ell m}(\theta, \phi), \quad (\text{C.1})$$

where the coefficients $a_{\ell m}$ can then be expressed as:

$$a_{\ell m} = \int_{\phi=0}^{2\pi} d\phi \int_{\theta=0}^{\pi} d\theta \sin(\theta) f(\theta, \phi) Y_{\ell m}(\theta, \phi)$$

Of course, since any signal in our maps is bounded and of real value, any of our maps represent a square-integrable signal of the sphere, and can as such also be expanded on the basis of the real spherical harmonics as defined by the equation above.

From the general definition of the total power of a function as the integral of said function squared divided by the area of said function's domain, we can write the total power in a map $M(\theta, \phi)$ as:

$$\frac{1}{4\pi} \int_{\Omega} |M(\theta, \phi)|^2 d\Omega,$$

where Ω denotes the unit sphere surface area. Introducing the maps expansion into spherical

harmonics, given by Eq.C.1, in the above expression yields:

$$\begin{aligned}
\frac{1}{4\pi} \int_{\Omega} |M(\theta, \phi)|^2 d\Omega &= \frac{1}{4\pi} \int_{\Omega} \left(\sum_{\ell=0}^{+\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \phi) \right)^2 d\Omega \\
&= \frac{1}{4\pi} \left(\sum_{\ell=0}^{+\infty} \sum_{m=-\ell}^{\ell} a_{\ell m}^2 \int_{\Omega} Y_{\ell m}(\theta, \phi)^2 d\Omega \right. \\
&\quad + \sum_{\substack{\ell \in \mathbb{N} \\ \ell' \in \mathbb{N}}} \sum_{\substack{m \in \llbracket -\ell, \ell \rrbracket \\ m' \in \llbracket -\ell', \ell' \rrbracket \\ m \neq m'}} a_{\ell m} a_{\ell' m'} \int_{\Omega} Y_{\ell m}(\theta, \phi) Y_{\ell' m'}(\theta, \phi) d\Omega \\
&\quad \left. + \sum_{\substack{\ell \in \mathbb{N} \\ \ell' \in \mathbb{N} \\ \ell \neq \ell'}} \sum_{\substack{m \in \llbracket -\ell, \ell \rrbracket \\ m' \in \llbracket -\ell', \ell' \rrbracket}} a_{\ell m} a_{\ell' m'} \int_{\Omega} Y_{\ell m}(\theta, \phi) Y_{\ell' m'}(\theta, \phi) d\Omega \right) \quad (C.2) \\
&= \frac{1}{4\pi} \sum_{\ell=0}^{+\infty} \sum_{m=-\ell}^{\ell} a_{\ell m}^2 + \frac{1}{4\pi} \sum_{\substack{\ell \in \mathbb{N} \\ \ell' \in \mathbb{N} \\ m \neq m'}} \sum_{\substack{m \in \llbracket -\ell, \ell \rrbracket \\ m' \in \llbracket -\ell', \ell' \rrbracket}} a_{\ell m} a_{\ell' m'} \delta_{\ell \ell'} \delta_{m m'} \\
&\quad + \frac{1}{4\pi} \sum_{\substack{\ell \in \mathbb{N} \\ \ell' \in \mathbb{N} \\ \ell \neq \ell'}} \sum_{\substack{m \in \llbracket -\ell, \ell \rrbracket \\ m' \in \llbracket -\ell', \ell' \rrbracket}} a_{\ell m} a_{\ell' m'} \delta_{\ell \ell'} \delta_{m m'} \\
&= \frac{1}{4\pi} \sum_{\ell=0}^{+\infty} \sum_{m=-\ell}^{\ell} a_{\ell m}^2.
\end{aligned}$$

Since the definition of the (angular) power spectrum is the distribution of the total power of a signal with respect to (angular) scales, we can start by writing said total power as the sum of the power spectrum, i.e. for a map M , as:

$$\frac{1}{4\pi} \int_{\Omega} |M(\theta, \phi)|^2 d\Omega = \sum_{\ell=0}^{+\infty} C_{\ell}(M),$$

where we can use the expression of the total power given by Eq.C.2 to identify the expression of the power spectrum of the map M as:

$$\forall \ell \in \mathbb{N}, C_{\ell}(M) = \frac{1}{4\pi} \sum_{m=-\ell}^{\ell} a_{\ell m}^2.$$

Similarly, the cross-spectrum between 2 maps M_1 and M_2 , of spherical harmonics decompositions $M_1(\theta, \phi) = \sum_{\ell \in \mathbb{N}} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \phi)$ and $M_2(\theta, \phi) = \sum_{\ell \in \mathbb{N}} \sum_{m=-\ell}^{\ell} b_{\ell m} Y_{\ell m}(\theta, \phi)$, is given by:

$$\forall \ell \in \mathbb{N}, C_{\ell}(M_1 * M_2) = \frac{1}{4\pi} \sum_{m=-\ell}^{\ell} a_{\ell m} b_{\ell m}.$$

Additionally, note that the power spectra and cross-spectra presented in this work may be represented not by their C_{ℓ} as a function of the multipole ℓ , but as the quantity noted D_{ℓ} , and defined as:

$$\forall \ell \in \mathbb{N}, D_{\ell} = \frac{\ell(\ell+1)}{2\pi} C_{\ell}.$$

This can prove easier to view power-spectra or cross-spectra of maps which evolve like $\propto \ell^{-2}$.

Finally, in most cases, the spectra computed in this work are presented after performing a log-smoothing: this is done by averaging, starting from a certain limit, all values of the C_ℓ or D_ℓ , on a fraction of the considered multipole. This log-smoothing procedure can be written:

$$\forall \ell > \max(\ell_{frac}, 10), \quad \mathcal{C}_\ell = \frac{1}{2\lfloor \frac{\ell}{\ell_{frac}} \rfloor + 1} \sum_{i=\ell-\lfloor \frac{\ell}{\ell_{frac}} \rfloor}^{\ell+\lfloor \frac{\ell}{\ell_{frac}} \rfloor} C_i,$$

where the notations $\mathcal{C}_\ell$ and $\mathcal{D}_\ell$ are used for the log-smoothed C_ℓ and D_ℓ , and where ℓ_{frac} is the smoothing fraction value which is given for each considered spectrum. The $\ell > \max(\ell_{frac}, 10)$ condition is used to ensure that the smoothing computation is defined, as well as to leave untouched the first multipoles corresponding to the largest scales.

C.2 The *sky fraction* masks

When computing power spectra and cross-spectra, it is often useful, if not necessary, to do it on a partially masked sky. This can happen because we need to characterize the signal scale distribution in specific parts of the sky, e.g. at high latitude, or because a part of the sky needs to be masked, such as the galactic disk in most cases as it is too bright and of lesser determination quality than the rest of the map.

I built a set of *sky fraction* masks, typically used to explore the high latitudes of the maps. However, more than high latitudes, I wanted it to be focused on the regions of *lower column density* in the map. To this end, I used the *Planck* legacy 2018 release HFI 857GHz frequency map, largely dominated by the thermal dust signal, which I smoothed with a Gaussian beam of FWHM = 5°. I then used this smoothed map to build masks of increasing intensity regions. More precisely, a mask characterized by a parameter $f_{sky} = X\%$, is a mask **keeping** the $X\%$ of the sky **with lowest intensity** in the smoothed 857GHz frequency map. The resulting masks, built for sky fractions increasing by intervals of 10%, are displayed in Fig.C.1.

In the work of [Planck Collaboration XI \(2014\)](#), their Eq.5 states that the dust emission of a given composition and structure, in thermal equilibrium, increases linearly with the gas column density N_H . Their Fig.5 also displays an affine fit of the *Planck* 857GHz frequency map intensity (dominated by dust emission) as a function of N_H . Using the fit from this plot, approximately giving the following law: $I_{857}(N_H) \simeq 0.093 + \frac{0.85}{2.1} \times N_H$ (where the 0.093 term is given in Table.1 from [Planck Collaboration XI \(2014\)](#), and the factor $\frac{0.85}{2.1}$ is estimated from their Fig.5), I can then estimate the gas column density limit for each of my sky fraction maps, i.e. the limit of the highest *unmasked* gas column density for each mask. I listed the corresponding results in Table.C.1.

Note that all power spectra and cross-spectra computed on such partially masked skies are always brought back to an *equivalent* full-sky spectrum, i.e. any spectrum of a sky masked by a mask of parameter $f_{sky} = X\%$ is then multiplied by a factor $\frac{100}{X}$. This is performed to allow later comparison of power spectra or cross-spectra of similar, if not identical maps, but with varying f_{sky} masking. To this end, any masked spectrum is brought to the *equivalent* total spectrum power of an *unmasked* map with identical spectrum distribution.

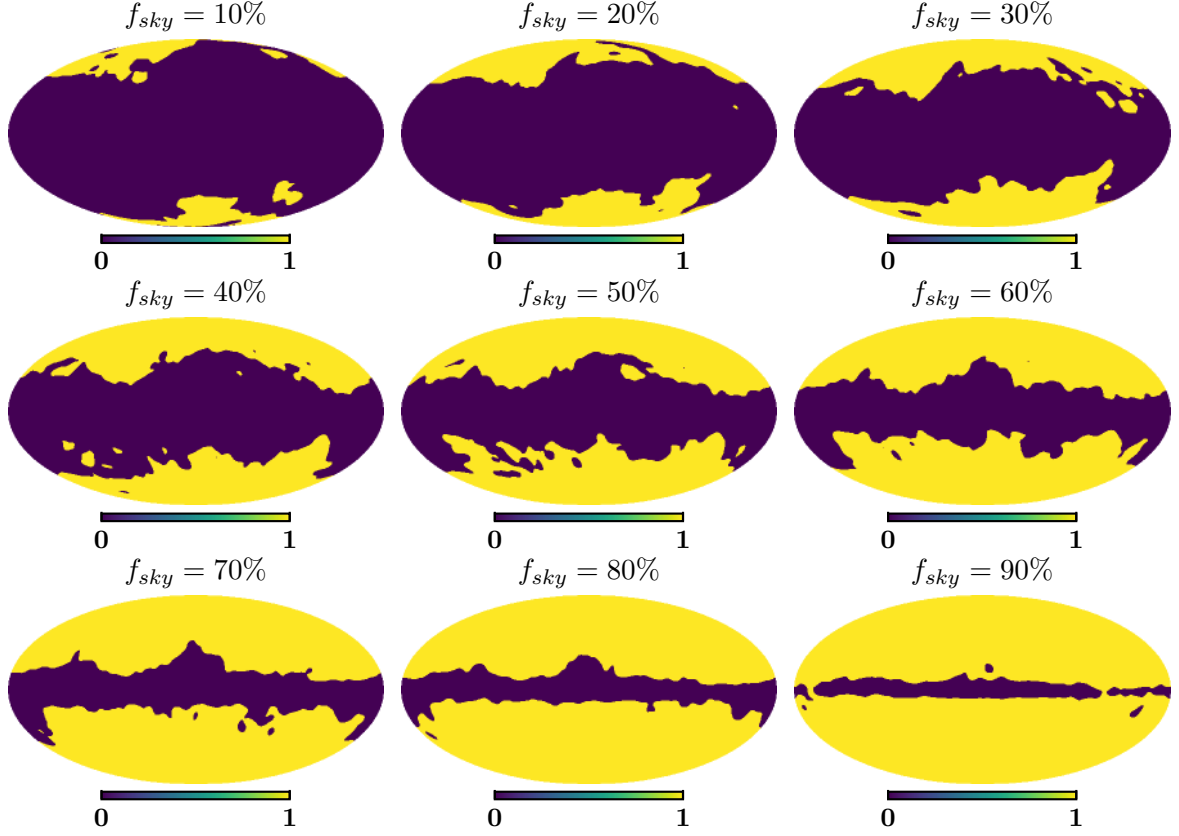


Fig. C.1 Sky fraction masks used in the various power spectra and cross-spectra computations throughout this work. The masks are built by keeping the fraction of the sky of lowest intensity in the *Planck* 2018 legacy release HFI 857GHz frequency map smoothed by a Gaussian beam of FWHM = 5° . The masks are built with kept sky fractions increasing by intervals of 10% (1 corresponds to **kept** pixels while 0 corresponds to **masked** pixels).

Table C.1: List of highest *unmasked* value of gas column density $N_{\text{H}}^{\text{max}}$ for each sky fraction mask.

f_{sky}	10%	20%	30%	40%	50%	60%	70%	80%	90%
$N_{\text{H}}^{\text{max}} [10^{20} \text{ cm}^{-2}]$	2.5	3.2	4.2	5.5	7.2	9.7	14.4	25.2	51.8

Appendix D

Foregrounds response *difference* coefficients from SRoll2.1

I list here all the additional results on the fitted RDCs from SRoll2.1, as described in subsection IV.1.4. First, I display the additional plots to check the method of evaluating the uncertainty on the RDCs determination from subset data runs. Then, I list all the RDCs fitted and output by SRoll2.1 in the case of both types of run: 100-143GHz and 217-143GHz. Finally, I show the plots of the RDCs for the free-free and synchrotron foregrounds.

Note that the order in which the bolometers are listed in Tables D.1, D.2, D.3, D.4, D.5, and D.6 below is the same as the one used in the plots of the corresponding RDCs, as seen in figures such as Figs IV.12, IV.13, IV.14, IV.15, D.3, and D.4.

D.1 Error from data: additional checks

I present here the validation checks of my method to evaluate the RDCs on the ^{13}CO and dust foregrounds. This is the same check as described in subsection IV.1.4.a) where it was discussed and plotted for the ^{12}CO in Fig. IV.10.

Figs. D.1 and D.2 therefore respectively display the RDCs distribution of the ^{13}CO and dust foregrounds for all the subset data runs with the RSTEP values of $\text{RSTEP} = \{10, 15, 20, 30\}$. We can see a stability of the subset results around the average values for the increasing values of RSTEP until $\text{RSTEP} = 20$ in the ^{13}CO . For the dust, the stability is seemingly kept for all RSTEP. They are also compatible with the results from the complete run with uncertainties determined on the simulations.

D.2 The 100-143GHz SRoll2.1 run

This section serves to list and catalogue all the foreground RDCs fitted by SRoll2.1 from the complete 100-143GHz data run. I also give, for each foreground component, 3 additional quantities characterizing the corresponding RDC distribution along the considered frequency band:

1. $\overline{\delta L^f}^{\mathcal{B}_{CO}}$ or $\overline{\delta L^f}^{\mathcal{B}_{143}}$ is the average value of the RDCs for the corresponding foreground f , along the corresponding frequency (note that, in any of the following tables, a given average is always computed on the frequency column located *above* it in said table),
2. $\sigma_{\delta L^f}$ is the standard deviation of the RDCs for the corresponding foreground f , along the corresponding frequency (note that, in any of the following tables, a given $\sigma_{\delta L^f}$ is

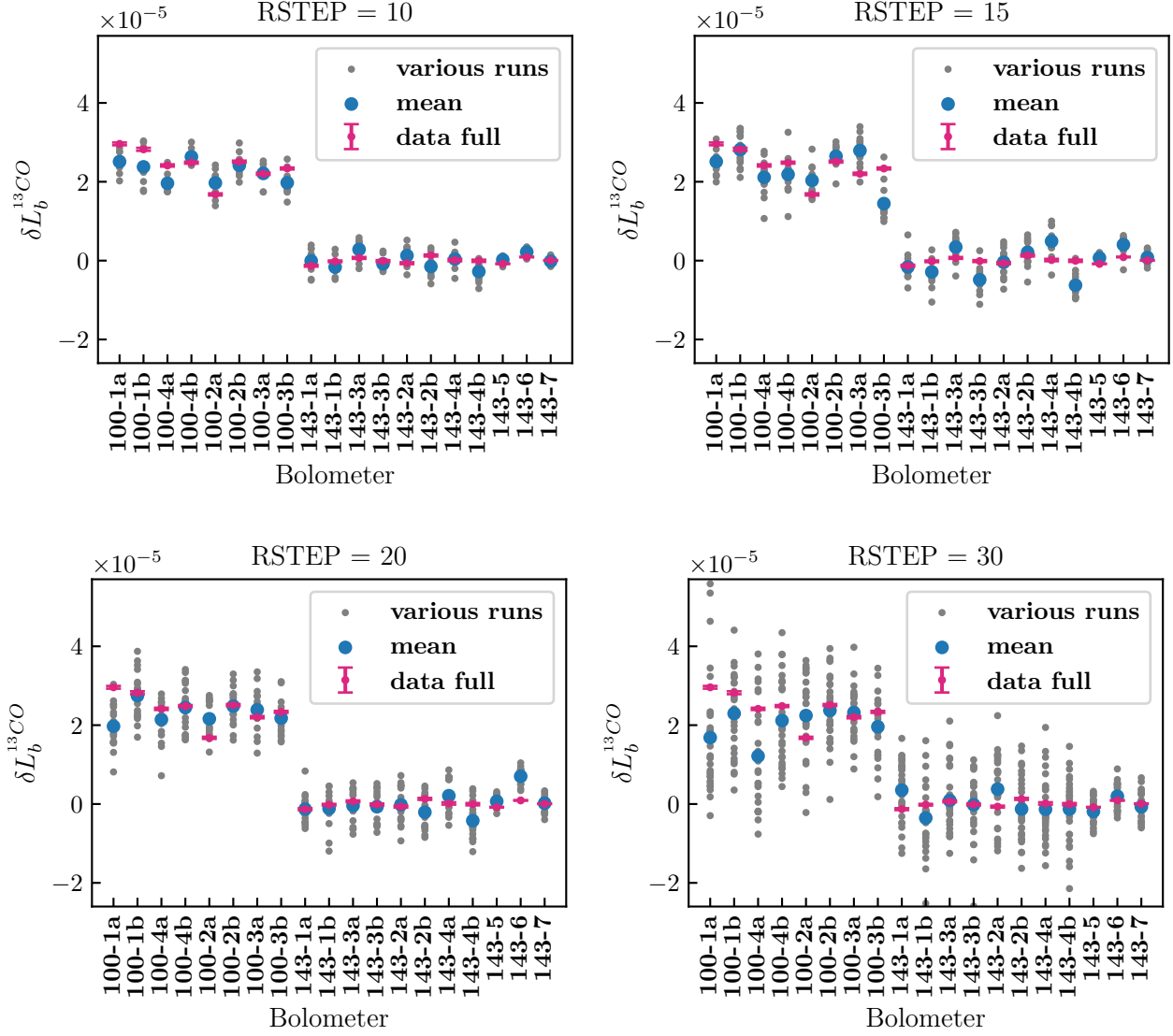


Fig. D.1 ^{13}CO RDCs ($\delta L_b^{13\text{CO}}$) distributions (in **grey**) for data subsets runs with: RSTEP = 10 (**upper left**); RSTEP = 15 (**upper right**); RSTEP = 20 (**lower left**); RSTEP = 30 (**lower right**). In each case, the average along the corresponding bolometer's RDC distribution is plotted in **blue**, and the corresponding RDC computed from the complete SROLL2.1 run is plotted in **red** (with uncertainty bars determined from the simulations).

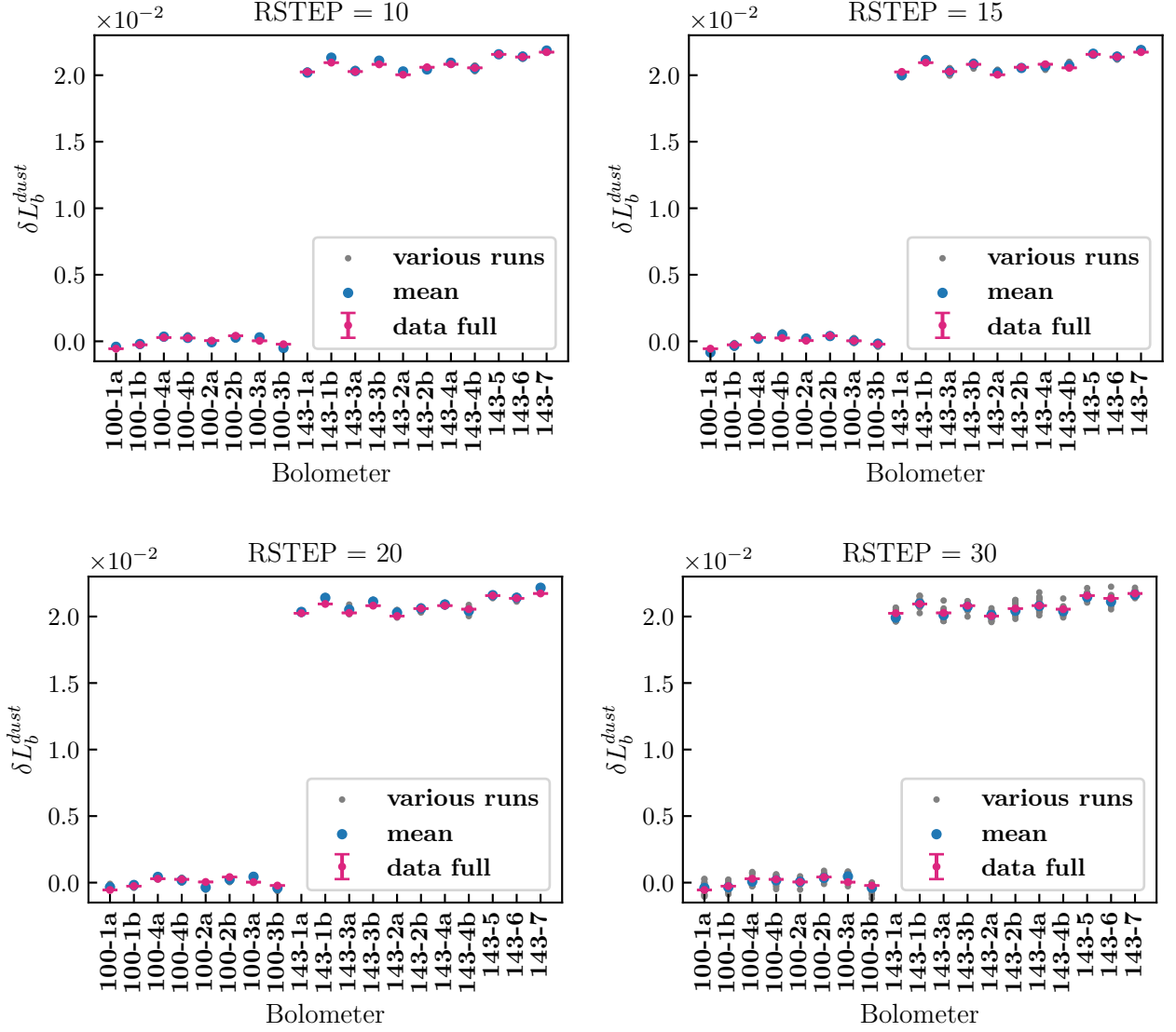


Fig. D.2 Dust RDCs (δL_b^{dust}) distributions (in **grey**) for data subsets runs with: RSTEP = 10 (**upper left**); RSTEP = 15 (**upper right**); RSTEP = 20 (**lower left**); RSTEP = 30 (**lower right**). In each case, the average along the corresponding bolometer's RDC distribution is plotted in **blue**, and the corresponding RDC computed from the complete SROLL2.1 run is plotted in **red** (with uncertainty bars determined from the simulations).

always computed on the frequency column located *above* it in said table),

3. $\sigma_{\delta Lf} / \overline{\Delta_{\delta Lf}}$ is the ratio between the standard deviation $\sigma_{\delta Lf}$ and the average uncertainty on the RDC determinations for the foreground f , along the corresponding frequency.

The value of the ratio $\sigma_{\delta Lf} / \overline{\Delta_{\delta Lf}}$ is used in discussions on the efficiency of the map building procedure in Section IV.2, as well as in discussing the potential presence of molecular emission signal in the 143GHz band, which can be found in subsection IV.3.5.

The results for the ^{12}CO and ^{13}CO foreground components are listed in Table.D.1. Those of the dust and DustVar foregrounds are given in Table.D.2. Finally, those corresponding to the free-free and synchrotron foregrounds are displayed in Table.D.3. In all 3 aforementioned tables, the uncertainty values given were computed from the set of 10 available SRoll2.1 simulation runs.

Note that all dust, DustVar, free-free, and synchrotron RDCs are dimensionless since the corresponding templates are in units of K_{CMB} , same as the signal HPRs.

D.3 The 217-143GHz SRoll2.1 run

This section serves to list and catalogue all the foreground RDCs fitted by SRoll2.1 from the complete 217-143GHz data run. The values presented in the 3 following tables are defined exactly as was presented in the case of the 100-143GHz run, as presented in Section.D.2 above.

The results for the ^{12}CO and ^{13}CO foreground components are listed in Table.D.4. Those of the dust and DustVar foregrounds are given in Table.D.5. Finally, those corresponding to the free-free and synchrotron foregrounds are displayed in Table.D.6. In all 3 aforementioned tables, the uncertainty values given were computed from the data subset runs, following the method described in subsection IV.1.4.a).

Note that all dust, DustVar, free-free, and synchrotron RDCs are dimensionless since the corresponding templates are in units of K_{CMB} , same as the signal HPRs.

D.4 Free-free and synchrotron SRoll2.1 RDCs

Here are presented the RDCs of the free-free and synchrotron foreground emissions resulting from the complete SRoll2.1 runs, complementary to what is presented in subsection IV.1.4.b).

Free-free I plotted in Fig.D.3 the results from both the 100-143GHz and 217-143GHz runs, for the free-free foreground component.

The visible uncertainty difference between the 100-143GHz run and the 217-143GHz one is most probably explained by the difference of uncertainty estimation methods (cf. subsection IV.1.4.a)).

Synchrotron I plotted in Fig.D.4 the results from both the 100-143GHz and 217-143GHz runs, for the synchrotron foreground component.

Table D.1: ^{12}CO and ^{13}CO foreground RDCs output by the SRoll2.1 100-143GHz run. $\overline{\delta L^f}^{\mathcal{B}_{CO}}$ and $\overline{\delta L^f}^{\mathcal{B}_{143}}$ are in each case the average value of the results displayed in the column above them, i.e. average of the corresponding δL_b^f along the corresponding frequency band. $\sigma_{\delta L^f}$ is the standard deviation value of the results displayed in the column above them, i.e. standard deviation of the corresponding δL_b^f along the corresponding frequency band. $\sigma_{\delta L^f}/\overline{\Delta_{\delta L^f}}$ is the ratio between the corresponding $\sigma_{\delta L^f}$ and the average uncertainty on the corresponding δL_b^f along the corresponding frequency band. All displayed uncertainty values are computed as the standard deviation along the 10 SRoll2.1 100-143GHz simulation runs. Values from this table are plotted in Figs. IV.10, IV.12, IV.16, IV.18, D.1.

Frequency	Bolometer	^{12}CO [$\mu\text{K}_{CMB}.\text{K}^{-1}.\text{km}^{-1}.\text{s}$]	^{13}CO [$\mu\text{K}_{CMB}.\text{K}^{-1}.\text{km}^{-1}.\text{s}$]
100GHz	100-1a	12.1 ± 0.35	29 ± 1.5
	100-1b	10.9 ± 0.37	28 ± 1.5
	100-4a	18.6 ± 0.29	24.1 ± 0.82
	100-4b	16.7 ± 0.25	24.8 ± 0.78
	100-2a	17.5 ± 0.27	16.8 ± 0.81
	100-2b	16.5 ± 0.25	25.1 ± 0.90
	100-3a	14.9 ± 0.28	22.0 ± 0.90
	100-3b	11.4 ± 0.25	23.4 ± 0.83
	$\overline{\delta L^f}^{\mathcal{B}_{CO}}$	14.8 ± 0.10	24.3 ± 0.37
	$\sigma_{\delta L^f}$	2.8 ± 0.11	3.7 ± 0.41
	$\sigma_{\delta L^f}/\overline{\Delta_{\delta L^f}}^{\mathcal{B}_{CO}}$	9.59	3.61
143GHz	143-1a	0.3 ± 0.24	-1.3 ± 0.67
	143-1b	0.04 ± 0.21	-0.2 ± 0.54
	143-3a	-0.08 ± 0.23	0.7 ± 0.91
	143-3b	0.04 ± 0.24	-0.1 ± 0.96
	143-2a	0.2 ± 0.24	-0.7 ± 0.67
	143-2b	-0.2 ± 0.25	1.3 ± 0.72
	143-4a	0.01 ± 0.3	0.2 ± 1
	143-4b	0.09 ± 0.29	-0.05 ± 1.2
	143-5	-0.04 ± 0.082	-0.8 ± 0.32
	143-6	-0.30 ± 0.083	0.9 ± 0.28
	143-7	-0.08 ± 0.071	0.06 ± 0.3
	$\overline{\delta L^f}^{\mathcal{B}_{143}}$	$6 \times 10^{-10} \pm 0.066$	$3 \times 10^{-9} \pm 0.23$
	$\sigma_{\delta L^f}$	0.15 ± 0.064	0.7 ± 0.19
	$\sigma_{\delta L^f}/\overline{\Delta_{\delta L^f}}^{\mathcal{B}_{143}}$	0.755	1.06

Table D.2: Dust and DustVar foreground RDCs (*dimensionless*) output by the SRoll2.1 100-143GHz run. $\overline{\delta L^f}^{\mathcal{B}_{CO}}$ and $\overline{\delta L^f}^{\mathcal{B}_{143}}$ are in each case the average value of the results displayed in the column above them, i.e. average of the corresponding δL_b^f along the corresponding frequency band. $\sigma_{\delta L^f}$ is the standard deviation value of the results displayed in the column above them, i.e. standard deviation of the corresponding δL_b^f along the corresponding frequency band. $\sigma_{\delta L^f}/\overline{\Delta_{\delta L^f}}$ is the ratio between the corresponding $\sigma_{\delta L^f}$ and the average uncertainty on the corresponding δL_b^f along the corresponding frequency band. All displayed uncertainty values are computed as the standard deviation along the 10 SRoll2.1 100-143GHz simulation runs. Values from this table are plotted in Figs. IV.14, IV.15, IV.18, D.2.

Frequency	Bolometer	Dust	DustVar
100GHz	100-1a	$-5 \times 10^{-4} \pm 1.1 \times 10^4$	-0.037 ± 0.0048
	100-1b	$-2.6 \times 10^{-4} \pm 9.1 \times 10^{-5}$	0.025 ± 0.0027
	100-4a	$2.9 \times 10^{-4} \pm 7.6 \times 10^{-5}$	-0.015 ± 0.0046
	100-4b	$2.5 \times 10^{-4} \pm 8.7 \times 10^{-5}$	0.057 ± 0.005
	100-2a	$5 \times 10^{-5} \pm 9.4 \times 10^{-5}$	0.007 ± 0.0039
	100-2b	$4.1 \times 10^{-4} \pm 9.8 \times 10^{-5}$	-0.009 ± 0.0036
	100-3a	$4 \times 10^{-5} \pm 5.4 \times 10^{-5}$	-0.005 ± 0.0031
	100-3b	$-2 \times 10^{-4} \pm 1.2 \times 10^{-4}$	-0.024 ± 0.0045
	$\overline{\delta L^f}^{\mathcal{B}_{CO}}$	$1 \times 10^{-13} \pm 3.3 \times 10^{-5}$	$-3 \times 10^{-11} \pm 0.0014$
	$\sigma_{\delta L^f}$	$3.1 \times 10^{-4} \pm 3.6 \times 10^{-5}$	0.028 ± 0.0016
	$\sigma_{\delta L^f}/\overline{\Delta_{\delta L^f}}^{\mathcal{B}_{CO}}$	3.32	6.89
143GHz	143-1a	$0.02025 \pm 4.1 \times 10^{-5}$	0.044 ± 0.0025
	143-1b	$0.02095 \pm 7.0 \times 10^{-5}$	0.052 ± 0.0029
	143-3a	$0.0203 \pm 1.1 \times 10^{-4}$	0.032 ± 0.0029
	143-3b	$0.02082 \pm 8.0 \times 10^{-5}$	0.046 ± 0.0036
	143-2a	$0.02004 \pm 7.4 \times 10^{-5}$	0.053 ± 0.0044
	143-2b	$0.0206 \pm 6.2 \times 10^{-5}$	0.032 ± 0.0019
	143-4a	$0.02082 \pm 9.3 \times 10^{-5}$	0.035 ± 0.0027
	143-4b	$0.02055 \pm 8.2 \times 10^{-5}$	0.045 ± 0.0044
	143-5	$0.02157 \pm 5.7 \times 10^{-5}$	0.056 ± 0.0023
	143-6	$0.02137 \pm 8.3 \times 10^{-5}$	0.040 ± 0.0038
	143-7	$0.02173 \pm 7.3 \times 10^{-5}$	0.045 ± 0.0032
	$\overline{\delta L^f}^{\mathcal{B}_{143}}$	$0.02081 \pm 2.3 \times 10^{-5}$	0.0440 ± 0.00098
	$\sigma_{\delta L^f}$	$5.3 \times 10^{-4} \pm 2.2 \times 10^{-5}$	0.0079 ± 0.00086
	$\sigma_{\delta L^f}/\overline{\Delta_{\delta L^f}}^{\mathcal{B}_{143}}$	7.06	2.5

Table D.3: Free-free and synchrotron foreground RDCs (*dimensionless*) output by the SRoll2.1 100-143GHz run. $\overline{\delta L^f}^{\mathcal{B}_{CO}}$ and $\overline{\delta L^f}^{\mathcal{B}_{143}}$ are in each case the average value of the results displayed in the column above them, i.e. average of the corresponding δL_b^f along the corresponding frequency band. $\sigma_{\delta L^f}$ is the standard deviation value of the results displayed in the column above them, i.e. standard deviation of the corresponding δL_b^f along the corresponding frequency band. $\sigma_{\delta L^f}/\overline{\Delta_{\delta L^f}}$ is the ratio between the corresponding $\sigma_{\delta L^f}$ and the average uncertainty on the corresponding δL_b^f along the corresponding frequency band. All displayed uncertainty values are computed as the standard deviation along the 10 SRoll2.1 100-143GHz simulation runs. Values from this table are plotted in Figs. IV.18, D.3, D.4.

Frequency	Bolometer	Free-free	Synchrotron
100GHz	100-1a	0.001 ± 0.012	-0.008 ± 0.087
	100-1b	0.0009 ± 0.0086	0.17 ± 0.089
	100-4a	-0.002 ± 0.0049	-0.03 ± 0.042
	100-4b	0.003 ± 0.0053	-0.07 ± 0.048
	100-2a	0.001 ± 0.0064	-0.06 ± 0.053
	100-2b	-0.006 ± 0.0057	0.08 ± 0.055
	100-3a	0.001 ± 0.0076	-0.04 ± 0.062
	100-3b	0.0003 ± 0.0051	-0.04 ± 0.038
	$\overline{\delta L^f}^{\mathcal{B}_{CO}}$	$8 \times 10^{-11} \pm 0.0026$	$4 \times 10^{-10} \pm 0.022$
	$\sigma_{\delta L^f}$	0.003 ± 0.0021	0.08 ± 0.027
	$\sigma_{\delta L^f}/\overline{\Delta_{\delta L^f}}^{\mathcal{B}_{CO}}$	0.383	1.3
143GHz	143-1a	-0.100 ± 0.0029	-0.46 ± 0.047
	143-1b	-0.103 ± 0.0071	-0.22 ± 0.067
	143-3a	-0.108 ± 0.0076	-0.44 ± 0.075
	143-3b	-0.107 ± 0.0054	-0.45 ± 0.032
	143-2a	-0.100 ± 0.0038	-0.49 ± 0.058
	143-2b	-0.105 ± 0.0062	-0.39 ± 0.043
	143-4a	-0.104 ± 0.0064	-0.43 ± 0.055
	143-4b	-0.104 ± 0.0061	-0.43 ± 0.052
	143-5	-0.105 ± 0.0024	-0.40 ± 0.021
	143-6	-0.107 ± 0.0035	-0.38 ± 0.053
	143-7	-0.105 ± 0.0042	-0.41 ± 0.033
	$\overline{\delta L^f}^{\mathcal{B}_{143}}$	-0.104 ± 0.0016	-0.41 ± 0.015
	$\sigma_{\delta L^f}$	0.003 ± 0.0015	0.07 ± 0.019
	$\sigma_{\delta L^f}/\overline{\Delta_{\delta L^f}}^{\mathcal{B}_{143}}$	0.493	1.38

Table D.4: ^{12}CO and ^{13}CO foreground RDCs output by the SRoll2.1 217-143GHz run. $\overline{\delta L^f}^{\mathcal{B}_{CO}}$ and $\overline{\delta L^f}^{\mathcal{B}_{143}}$ are in each case the average value of the results displayed in the column above them, i.e. average of the corresponding δL_b^f along the corresponding frequency band. $\sigma_{\delta L^f}$ is the standard deviation value of the results displayed in the column above them, i.e. standard deviation of the corresponding δL_b^f along the corresponding frequency band. $\sigma_{\delta L^f}/\overline{\Delta_{\delta L^f}}$ is the ratio between the corresponding $\sigma_{\delta L^f}$ and the average uncertainty on the corresponding δL_b^f along the corresponding frequency band. All displayed uncertainty values are computed from the data subset runs, following the method described in subsection IV.1.4.a). Values from this table are plotted in Figs. IV.13, IV.17, IV.18, IV.20.

Frequency	Bolometer	^{12}CO [$\mu\text{K}_{CMB} \cdot \text{K}^{-1} \cdot \text{km}^{-1} \cdot \text{s}$]	^{13}CO [$\mu\text{K}_{CMB} \cdot \text{K}^{-1} \cdot \text{km}^{-1} \cdot \text{s}$]
217GHz	217-5a	19.6 ± 0.29	13 ± 1.8
	217-5b	16.5 ± 0.32	18 ± 1.8
	217-7a	18.5 ± 0.32	11 ± 2.1
	217-7b	18.8 ± 0.34	18 ± 2.0
	217-6a	13.1 ± 0.32	12 ± 2.2
	217-6b	11.6 ± 0.43	15 ± 2.3
	217-8a	16.7 ± 0.37	17 ± 2.4
	217-8b	14.3 ± 0.37	17 ± 2.2
	217-1	18.5 ± 0.26	18 ± 1.5
	217-2	17.3 ± 0.35	12 ± 2.1
	217-3	22.0 ± 0.24	17 ± 1.5
	217-4	21.1 ± 0.36	18 ± 1.9
	$\overline{\delta L^f}^{\mathcal{B}_{CO}}$	17.35 ± 0.097	15.4 ± 0.57
	$\sigma_{\delta L^f}$	3.0 ± 0.10	2.7 ± 0.58
	$\sigma_{\delta L^f}/\overline{\Delta_{\delta L^f}}^{\mathcal{B}_{CO}}$	8.97	1.36
143GHz	143-1a	0.5 ± 0.20	-1.4 ± 0.9
	143-1b	-0.1 ± 0.17	-0.2 ± 0.79
	143-3a	0.007 ± 0.2	0.2 ± 1
	143-3b	-0.2 ± 0.14	1.4 ± 0.82
	143-2a	0.5 ± 0.19	-1.1 ± 0.81
	143-2b	-0.5 ± 0.17	1.7 ± 0.84
	143-4a	0.2 ± 0.2	-0.9 ± 0.97
	143-4b	-0.2 ± 0.21	1 ± 1.0
	143-5	0.07 ± 0.15	-1.2 ± 0.81
	143-6	-0.2 ± 0.23	0.7 ± 1.2
	143-7	0.05 ± 0.16	-0.5 ± 0.85
	$\overline{\delta L^f}^{\mathcal{B}_{143}}$	$-2 \times 10^{-8} \pm 0.056$	$7 \times 10^{-8} \pm 0.28$
	$\sigma_{\delta L^f}$	0.29 ± 0.056	1.1 ± 0.27
	$\sigma_{\delta L^f}/\overline{\Delta_{\delta L^f}}^{\mathcal{B}_{143}}$	1.58	1.18

Table D.5: Dust and DustVar foreground RDCs (*dimensionless*) output by the SRoll2.1 217-143GHz run. $\overline{\delta L^f}^{\mathcal{B}_{CO}}$ and $\overline{\delta L^f}^{\mathcal{B}_{143}}$ are in each case the average value of the results displayed in the column above them, i.e. average of the corresponding δL_b^f along the corresponding frequency band. $\sigma_{\delta L^f}$ is the standard deviation value of the results displayed in the column above them, i.e. standard deviation of the corresponding δL_b^f along the corresponding frequency band. $\sigma_{\delta L^f}/\overline{\Delta_{\delta L^f}}$ is the ratio between the corresponding $\sigma_{\delta L^f}$ and the average uncertainty on the corresponding δL_b^f along the corresponding frequency band. All displayed uncertainty values are computed from the data subset runs, following the method described in subsection IV.1.4.a). Values from this table are plotted in Figs. IV.14, IV.15, IV.18.

Frequency	Bolometer	Dust	DustVar
217GHz	217-5a	$-1.38 \times 10^{-3} \pm 2.7 \times 10^{-5}$	0.0094 ± 0.0048
	217-5b	$-9.2 \times 10^{-4} \pm 3.5 \times 10^{-5}$	0.014 ± 0.0075
	217-7a	$-1.76 \times 10^{-3} \pm 2.5 \times 10^{-5}$	$-7 \times 10^{-5} \pm 0.0056$
	217-7b	$-1.99 \times 10^{-3} \pm 3.7 \times 10^{-5}$	-0.020 ± 0.0068
	217-6a	$-1.23 \times 10^{-3} \pm 2.3 \times 10^{-5}$	$-8 \times 10^{-5} \pm 0.0062$
	217-6b	$-4.3 \times 10^{-4} \pm 2.2 \times 10^{-5}$	0.0006 ± 0.0071
	217-8a	$-2.37 \times 10^{-3} \pm 3.5 \times 10^{-5}$	-0.025 ± 0.0066
	217-8b	$-1.07 \times 10^{-3} \pm 2.9 \times 10^{-5}$	-0.010 ± 0.0068
	217-1	$2.08 \times 10^{-3} \pm 2.7 \times 10^{-5}$	0.007 ± 0.006
	217-2	$2.61 \times 10^{-3} \pm 2.3 \times 10^{-5}$	-0.011 ± 0.0075
	217-3	$3.85 \times 10^{-3} \pm 2.1 \times 10^{-5}$	0.028 ± 0.0062
	217-4	$2.62 \times 10^{-3} \pm 2.7 \times 10^{-5}$	0.006 ± 0.0083
	$\overline{\delta L^f}^{\mathcal{B}_{CO}}$	$6 \times 10^{-12} \pm 8.1 \times 10^{-6}$	$-2 \times 10^{-10} \pm 0.0019$
	$\sigma_{\delta L^f}$	$2.063 \times 10^{-3} \pm 7.7 \times 10^{-6}$	0.014 ± 0.0019
	$\sigma_{\delta L^f}/\overline{\Delta_{\delta L^f}}^{\mathcal{B}_{CO}}$	74.9	2.12
143GHz	143-1a	$-0.09875 \pm 2.2 \times 10^{-5}$	-0.391 ± 0.0073
	143-1b	$-0.09811 \pm 2.8 \times 10^{-5}$	-0.367 ± 0.0073
	143-3a	$-0.09885 \pm 3.5 \times 10^{-5}$	-0.39 ± 0.010
	143-3b	$-0.09818 \pm 2.6 \times 10^{-5}$	-0.384 ± 0.0076
	143-2a	$-0.09896 \pm 2.9 \times 10^{-5}$	-0.381 ± 0.0083
	143-2b	$-0.09851 \pm 2.1 \times 10^{-5}$	-0.385 ± 0.0069
	143-4a	$-0.09829 \pm 2.6 \times 10^{-5}$	-0.391 ± 0.0085
	143-4b	$-0.09840 \pm 3.7 \times 10^{-5}$	-0.384 ± 0.0086
	143-5	$-0.09750 \pm 1.6 \times 10^{-5}$	-0.376 ± 0.0074
	143-6	$-0.09769 \pm 2.1 \times 10^{-5}$	-0.391 ± 0.008
	143-7	$-0.09733 \pm 2.4 \times 10^{-5}$	-0.385 ± 0.0079
	$\overline{\delta L^f}^{\mathcal{B}_{143}}$	$-0.098233 \pm 8.0 \times 10^{-6}$	-0.384 ± 0.0024
	$\sigma_{\delta L^f}$	$5.18 \times 10^{-4} \pm 7.6 \times 10^{-6}$	0.007 ± 0.0024
	$\sigma_{\delta L^f}/\overline{\Delta_{\delta L^f}}^{\mathcal{B}_{143}}$	19.9	0.937

Table D.6: Free-free and synchrotron foreground RDCs (*dimensionless*) output by the SROLL2.1 217-143GHz run. $\overline{\delta L^f}^{\mathcal{B}_{CO}}$ and $\overline{\delta L^f}^{\mathcal{B}_{143}}$ are in each case the average value of the results displayed in the column above them, i.e. average of the corresponding δL_b^f along the corresponding frequency band. $\sigma_{\delta L^f}$ is the standard deviation value of the results displayed in the column above them, i.e. standard deviation of the corresponding δL_b^f along the corresponding frequency band. $\sigma_{\delta L^f}/\overline{\Delta_{\delta L^f}}$ is the ratio between the corresponding $\sigma_{\delta L^f}$ and the average uncertainty on the corresponding δL_b^f along the corresponding frequency band. All displayed uncertainty values are computed from the data subset runs, following the method described in subsection IV.1.4.a). Values from this table are plotted in Figs. IV.18, D.3, D.4.

Frequency	Bolometer	Free-free	Synchrotron
217GHz	217-5a	0.004 ± 0.0012	0.14 ± 0.042
	217-5b	-0.002 ± 0.0011	-0.11 ± 0.052
	217-7a	0.0006 ± 0.00089	0.06 ± 0.031
	217-7b	-0.002 ± 0.0013	-0.17 ± 0.040
	217-6a	0.0022 ± 0.00095	0.16 ± 0.029
	217-6b	0.0007 ± 0.0009	-0.06 ± 0.039
	217-8a	0.002 ± 0.0011	0.20 ± 0.034
	217-8b	-0.001 ± 0.0012	-0.08 ± 0.038
	217-1	0.0045 ± 0.0009	-0.02 ± 0.040
	217-2	0.0020 ± 0.00095	0.03 ± 0.029
	217-3	-0.0018 ± 0.00067	-0.08 ± 0.022
	217-4	-0.009 ± 0.0011	-0.07 ± 0.03
	$\overline{\delta L^f}^{\mathcal{B}_{CO}}$	$1 \times 10^{-11} \pm 0.00030$	$-2 \times 10^{-10} \pm 0.010$
	$\sigma_{\delta L^f}$	0.0034 ± 0.0003	0.11 ± 0.011
	$\sigma_{\delta L^f}/\overline{\Delta_{\delta L^f}}^{\mathcal{B}_{CO}}$	3.29	3.13
143GHz	143-1a	0.0453 ± 0.00096	-0.03 ± 0.034
	143-1b	0.042 ± 0.0011	0.26 ± 0.029
	143-3a	0.037 ± 0.0011	-0.06 ± 0.032
	143-3b	0.039 ± 0.0013	0.06 ± 0.033
	143-2a	0.0433 ± 0.00098	-0.05 ± 0.046
	143-2b	0.040 ± 0.001	0.06 ± 0.022
	143-4a	0.040 ± 0.0011	-0.07 ± 0.034
	143-4b	0.040 ± 0.0011	0.09 ± 0.036
	143-5	0.0406 ± 0.00063	0.06 ± 0.025
	143-6	0.0375 ± 0.00074	0.06 ± 0.030
	143-7	0.0394 ± 0.00075	0.04 ± 0.026
	$\overline{\delta L^f}^{\mathcal{B}_{143}}$	0.0403 ± 0.00030	0.037 ± 0.0097
	$\sigma_{\delta L^f}$	0.0024 ± 0.0003	0.088 ± 0.0097
	$\sigma_{\delta L^f}/\overline{\Delta_{\delta L^f}}^{\mathcal{B}_{143}}$	2.43	2.8

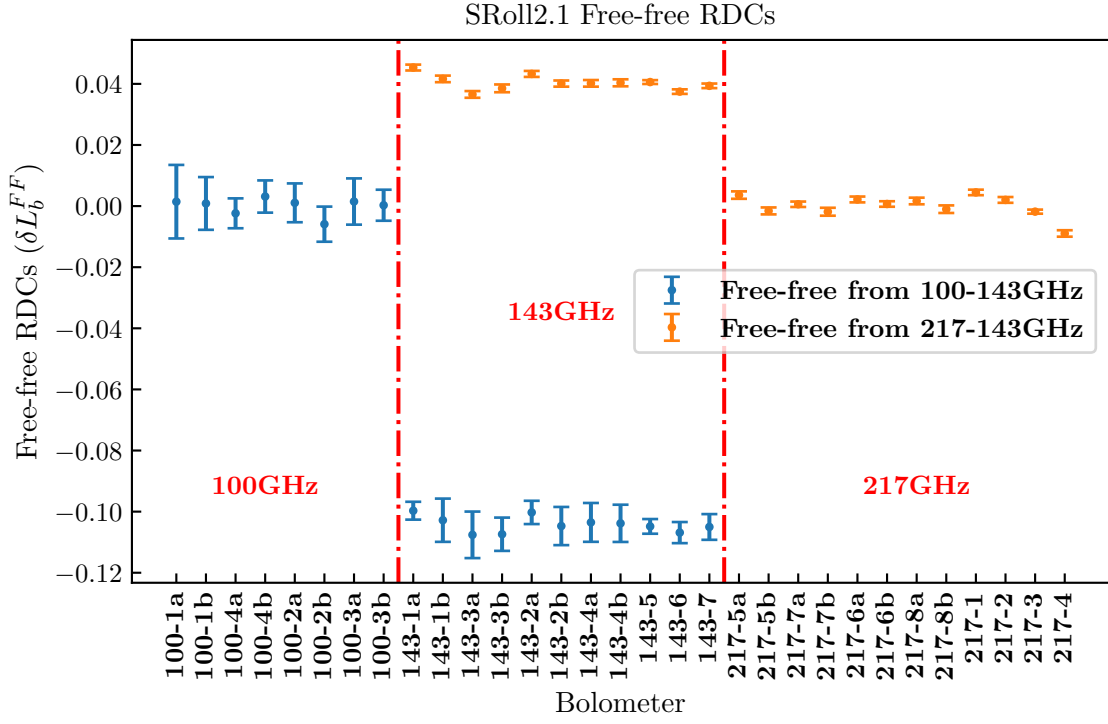


Fig. D.3 Free-free bolometer RDCs determined by SRoll2.1 from both 100-143GHz and 143-217GHz runs. The first 8 bolometers are from the 100GHz band, the middle 11 bolometers are from the 143GHz band, and the 12 last are from the 217GHz band.

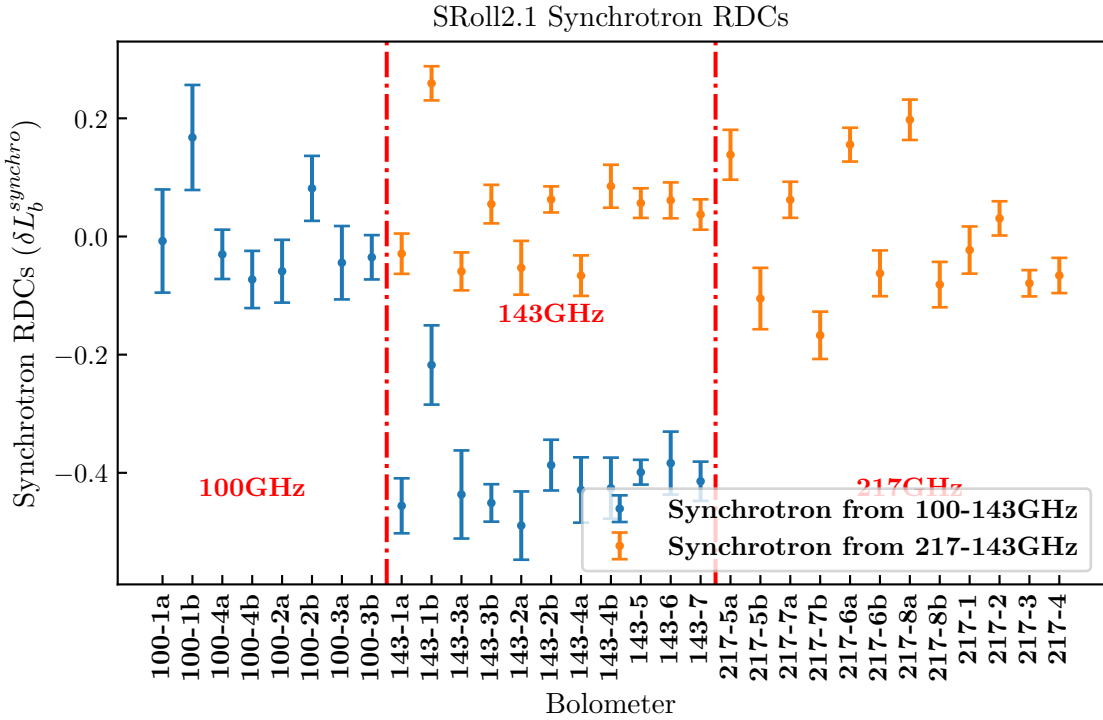


Fig. D.4 Synchrotron bolometer RDCs determined by SRoll2.1 from both 100-143GHz and 143-217GHz runs. The first 8 bolometers are from the 100GHz band, the middle 11 bolometers are from the 143GHz band, and the 12 last are from the 217GHz band.

D.5 Monopole effect: limits of the uncertainty estimation

I reported here the figure from the check of the bolometers' monopole effect on the RDCs' determination (see subsection IV.3.3). The estimated uncertainty is far too high for 2 reasons:

- The uncertainty on the RDCs estimated from the simulations is probably overestimated (see subsection IV.1.4.a),
- The uncertainty is multiplied by a factor $\sqrt{10}$ because the test is computed from runs with $RSTEP = 10$ (necessary to limit the computational load of the test).

This leads to the estimated levels of uncertainty displayed in Fig.D.5.

This uncertainty seems unreasonably high, especially since it is much higher than the variations of the results distribution between bolometers.

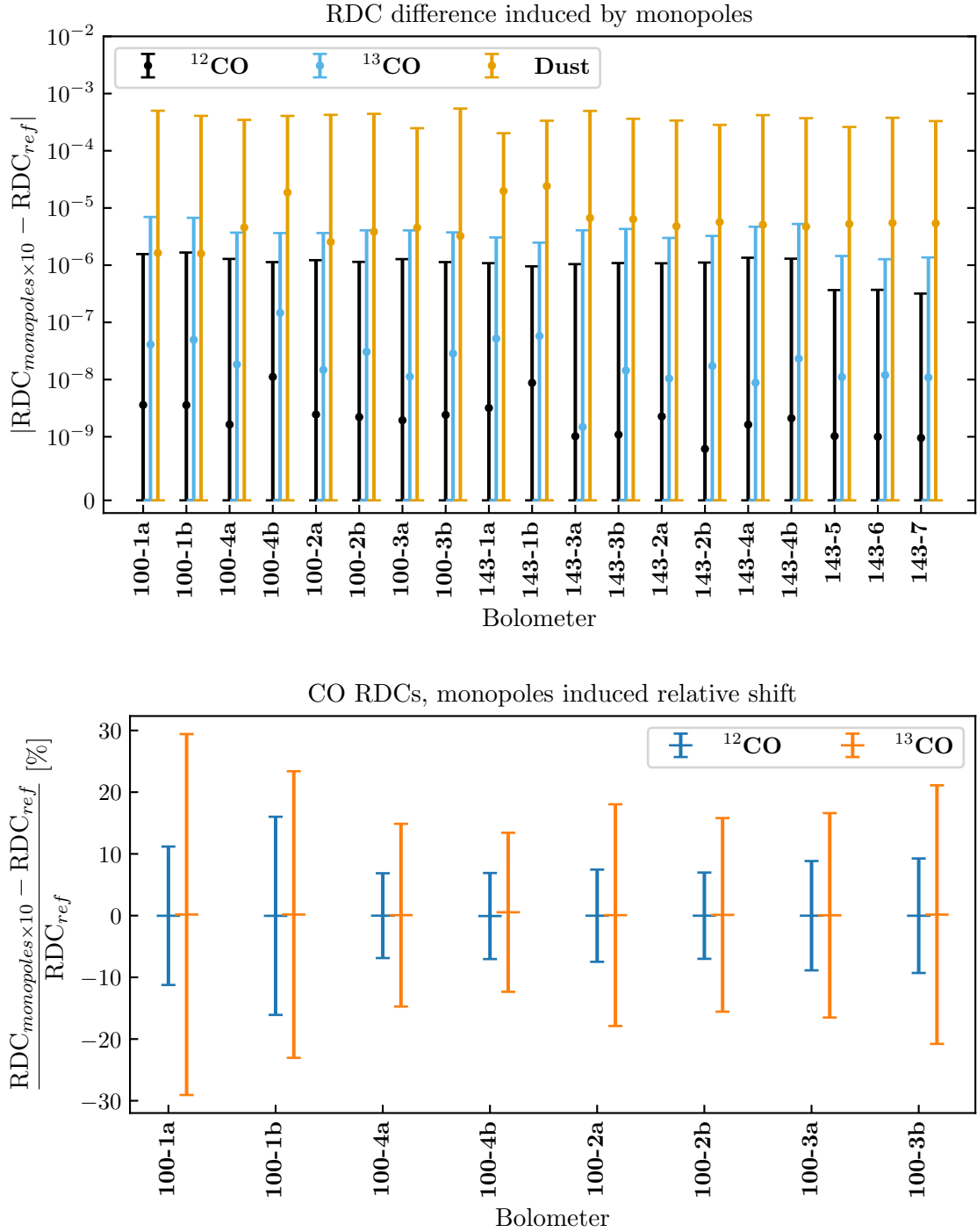


Fig. D.5 Variation of ^{12}CO , ^{13}CO , and dust RDCs when the bolometers' monopoles used as SRoll2.1 input are multiplied by 10 in a 100-143GHz run. **Top:** Difference between RDCs from modified monopoles and the reference RDCs determined from the default SRoll2.1 run; **Bottom:** 100GHz band relative difference between RDCs from modified monopoles and the reference RDCs (i.e. divided by the value of the respective reference RDC).

Appendix E

The Stokes parameters: the 3 map components

The Stokes parameters are a mathematical parametrization of the polarization state of any electromagnetic radiation. It was first defined by George Gabriel Stokes (cf. [Stokes \(1851\)](#)), and is composed of a set of 4 parameters called I , Q , U , and V , and constructed from the values of the total radiation intensity (I_{tot}), its polarization fraction - or degree of polarization - ($0 < p < 1$), and the 2 angle parameters defining the polarization ellipse of the radiation (noted ψ and χ in this Appendix).

The polarization state of a radiation is entirely defined by its corresponding polarization ellipse defined from the 2 angles ψ and χ as represented in the plot of a generic polarization ellipse in Fig.E.1. ψ is the angle between the x-axis and the major axis of the ellipse, and χ is defined by $\chi = \arctan\left(\frac{b}{a}\right)$ with a and b the lengths of respectively the semi-major and semi-minor axes of the ellipse. The Stokes parameters of such a radiation are then defined by:

$$\begin{aligned} I &= I_{tot}, \\ Q &= I_{tot}p \cos(2\psi) \cos(2\chi), \\ U &= I_{tot}p \sin(2\psi) \cos(2\chi), \\ V &= I_{tot}p \sin(2\chi). \end{aligned}$$

In the above equation, the polarization state is then fully determined when the angles ψ and χ span the ranges $\psi \in [0, \pi]$ and $\chi \in [-\frac{\pi}{4}, \frac{\pi}{4}]$, since a polarization ellipse cannot be differentiated from the same ellipse rotated by a 180° angle, and neither can it be distinguished from an ellipse rotated by a 90° angle with the a and b lengths swapped. Also, no time information is included in the Stokes parameters, so the information of the phase is lost, but every *geometrical* properties of the radiation's polarization are contained.

One immediate property stemming directly from the above definition is the following:

$$I^2 \geq Q^2 + U^2 + V^2,$$

and this inequality becomes an *equality* if and only if the radiation is purely polarized; i.e. if and only if $p = 1$.

In the *Planck* mission, as described in Section II.1, the bolometers from HFI are either SWBs almost indiscriminate of all incoming linearly polarized radiation, or PSBs which can determine the direction of incoming linearly polarized radiation. In any case, HFI is only sensitive to linearly polarized radiations, which means that in the *Planck* data we always have $V = 0$. In the general case, maps computed from the projection of the *Planck* TOIs

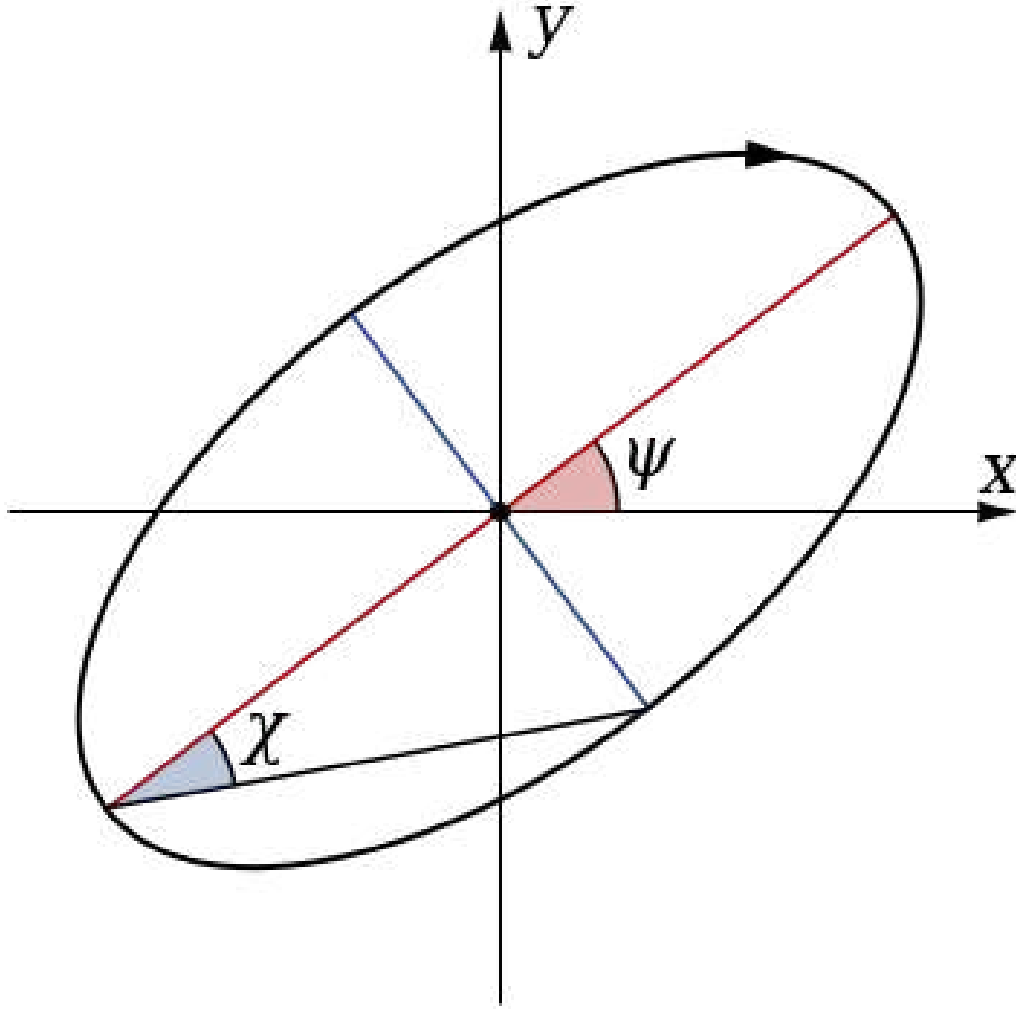


Fig. E.1 Representation of a generic polarization ellipse. The angles ψ and χ are used in the definition of the Stokes parameters of the associated radiation. The major axis length is determined by the total radiation intensity. Taken from [here](#)*.

onto the sky result in 3 component maps which correspond the maps of the I , Q , and U Stokes parameters of the sky emission.

Also, in the *Planck* maps computed following the HEALPix pixelisation scheme (see Appendix.A) the polarization angle is defined with respect to the North-South direction in the Galactic coordinate system. This implies an *ill-definition* of the polarization exactly at the poles. Thankfully, this issue is mitigated by the pixelisation scheme leading to the poles of the sphere **never** falling *inside* any pixels, but only at the vertex of 4 neighbour pixels.

Finally, note that following the notation from the work of [Delouis et al. \(2019\)](#), the polarization angle of the linearly polarized radiation is usually noted ϕ throughout this work.

*Image source link: https://commons.wikimedia.org/wiki/File:Polarisation_ellipse2.svg

Bibliography

- Ade, P. A. R., Savini, G., Sudiwala, R., et al. 2010, *A&A*, 520, A11 21, 24
- Alpher, R. A. & Herman, R. 1948, *Nature*, 162, 774 1
- Bennett, C., Boggess, N., Cheng, E., et al. 1993, *Advances in Space Research*, 13, 409 1
- Bennett, C. L., Larson, D., Weiland, J. L., et al. 2013, *Astrophysical Journal, Supplement Series*, 208 [arXiv:1212.5225] 1, 3
- Bersanelli, M., Mandolesi, N., Butler, R. C., et al. 2010, *A&A*, 520, A4 19
- Blitz, L., Magnani, L., & Mundy, L. 1984, *The Astrophysical Journal*, 282, L9 13, 15
- Canny, J. 1986, *IEEE Transactions on Pattern Analysis and Machine Intelligence*, PAMI-8, 679 220
- COMPLETE team. 2011a, FCRAO Ophiuchus 12CO cubes and map 202, 206, 207, 208, 209, 226, 227
- COMPLETE team. 2011b, FCRAO Ophiuchus 13CO cubes and map 202, 206, 207, 208, 209, 226, 227
- COMPLETE team. 2011c, FCRAO Perseus 12CO cubes and map 202, 203, 204, 205, 224, 225
- COMPLETE team. 2011d, FCRAO Perseus 13CO cubes and map 202, 203, 204, 205, 224, 225
- Cord, M. S., Lojko, M. S., & Petersen, J. D. 1968, Microwave spectral tables: Spectral line Listing, Tech. rep., National Bureau of Standards, Gaithersburg, MD 170
- Crill, B. P., Ade, P. A. R., Artusa, D. R., et al. 2003, *The Astrophysical Journal Supplement Series*, 148, 527 1
- Dame, T. M., Hartmann, D., & Thaddeus, P. 2001, *ApJ*, 547, 792 47, 100, 135
- Dame, T. M. & Thaddeus, P. 2022, *The Astrophysical Journal Supplement Series*, 262, 5 vii, xi, 13, 111, 115, 124, 191, 202, 210, 211, 212, 228
- De Bernardis, P., Ade, P. A., Bock, J. J., et al. 2000, *Nature*, 404, 955 1
- Delabrouille, J., Betoule, M., Melin, J.-B., et al. 2013, *A&A*, 553, A96 97, 100, 124
- Dell’Ova, P. 2021, *Exploring the interstellar contents of a TeVatron : molecules, dust and star formation in the evolved supernova remnant IC443*, Theses, Université Paris sciences et lettres 13, 14, 15

- Delouis, J.-M., Pagano, L., Mottet, S., Puget, J.-L., & Vibert, L. 2019, *Astronomy and Astrophysics*, 629, A38 5, 53, 57, 67, 68, 69, 80, 81, 82, 83, 89, 95, 96, 97, 103, 104, 108, 114, 176, 278
- Delouis, J.-M., Puget, J.-L., & Vibert, L. 2021, *A&A*, 650, A82 43, 44, 53, 67, 68, 103, 108, 110, 112, 113, 167, 219
- Désert, F.-X. 2022, *Astronomy & Astrophysics*, 659, A70 10
- Dicke, R. H., Peebles, P. J. E., Roll, P. G., & Wilkinson, D. T. 1965, *The Astrophysical Journal*, 142, 414 1
- Dickinson, C., Davies, R. D., & Davis, R. J. 2003, *Monthly Notices of the Royal Astronomical Society*, 341, 369 100
- Draine, B. T. 2011, *Physics of the Interstellar and Intergalactic Medium* (Princeton University Press), 567 10, 14
- Draine, B. T. & Lazarian, A. 1998, *The Astrophysical Journal*, 494, L19 10
- Falgarone, E. & Puget, J.-L. 1995, *Astronomy and Astrophysics*, 293, 840 16, 234
- Fixsen, D. J. 2009, *The Astrophysical Journal*, 707, 916 2, 42, 45
- Fixsen, D. J., Bennett, C. L., & Mather, J. C. 1999, *The Astrophysical Journal*, 526, 207 111
- Gir, B.-Y., Blitz, L., & Magnani, L. 1994, *The Astrophysical Journal*, 434, 162 13, 15
- Godard, B., Falgarone, E., & Pineau Des Forêts, G. 2014, *Astronomy and Astrophysics*, 570 [arXiv:1408.3716] 16, 234
- Goldreich, P. & Kylafis, N. D. 1982, *The Astrophysical Journal*, 253, 606 15
- Goldsmith, P. F., Heyer, M., Narayanan, G., et al. 2008, *The Astrophysical Journal*, 680, 428 vii, xi, 13, 71, 73, 74, 75, 76, 107, 111, 114, 115, 117, 135, 136, 138, 140, 141, 142, 149, 165, 166, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 220, 222, 223, 229
- Gorski, K. M., Hivon, E., Banday, A. J., et al. 2005, *The Astrophysical Journal*, 622, 759 56, 72, 147, 247, 248, 249
- Greaves, J. S., Holland, W. S., Friberg, P., & Dent, W. R. F. 1999, *The Astrophysical Journal*, 512, L139 16, 116
- Hartmann, D., Magnani, L., & Thaddeus, P. 1998, *The Astrophysical Journal*, 492, 205 13
- Hazumi, M., Ade, P. A., Akiba, Y., et al. 2019, *Journal of Low Temperature Physics*, 194, 443 7
- Hennebelle, P. & Falgarone, E. 2012, *Astronomy and Astrophysics Review*, 20 [arXiv:1211.0637] 13, 14, 16
- Heyer, M. & Dame, T. M. 2015, *Annual Review of Astronomy and Astrophysics*, 53, 583 13
- Holmes, W. A., Bock, J. J., Crill, B. P., et al. 2008, *Appl. Opt.*, 47, 5996 21

- Hurier, G. 2019, Extraction of ^{12}CO and ^{13}CO maps from *Planck* data [145](#), [192](#)
- Hurier, G., Macías-Pérez, J. F., & Hildebrandt, S. 2013, *Astronomy and Astrophysics*, 558, 1 [47](#)
- Johnson, D. 2018, in *Comprehensive Remote Sensing*, ed. S. Liang (Oxford: Elsevier), 376–411 [20](#)
- Jones, W. C., Bhatia, R., Bock, J. J., & Lange, A. E. 2003, in *Millimeter and Submillimeter Detectors for Astronomy*, ed. T. G. Phillips & J. Zmuidzinas, Vol. 4855, International Society for Optics and Photonics (SPIE), [227 – 238](#) [21](#), [22](#)
- Kamionkowski, M. & Kovetz, E. D. 2016, *Annual Review of Astronomy and Astrophysics*, 54, 227 [7](#)
- Klypin, A. A., Strukov, I. A., & Skulachev, D. P. 1992, *Monthly Notices of the Royal Astronomical Society*, 258, 71 [1](#)
- Lamarre, J., Puget, J., Ade, P. A. R., et al. 2010, *A&A*, 520, A9 [21](#), [22](#), [23](#), [24](#), [27](#), [32](#), [35](#)
- Lang, K. R. 1980, *Astrophysical Formulae* (Berlin, Heidelberg: Springer Berlin Heidelberg) [170](#), [171](#)
- Lequeux, J., Falgarone, E., & Ryter, C. 2002, *Le milieu interstellaire, Savoirs actuels* (EDP Sciences) [13](#), [14](#), [15](#), [72](#), [138](#)
- Lesaffre, P., Todorov, P., Levrier, F., et al. 2020, *Monthly Notices of the Royal Astronomical Society*, 495, 816 [16](#), [234](#)
- Lu, T. T. & Shiou, S. H. 2002, *Computers and Mathematics with Applications*, 43, 119 [256](#)
- Macellari, N., Pierpaoli, E., Dickinson, C., & Vaillancourt, J. E. 2011, *Monthly Notices of the Royal Astronomical Society*, 418, 888 [11](#)
- Maffei, B., Noviello, F., Murphy, J. A., et al. 2010, *A&A*, 520, A12 [21](#), [23](#)
- Magnani, L., Blitz, L., & Mundy, L. 1985, *The Astrophysical Journal*, 295, 402 [13](#)
- Magnani, L., Hartmann, D., Holcomb, S. L., Smith, L. E., & Thaddeus, P. 2000, *The Astrophysical Journal*, 535, 167 [13](#)
- Mennella, A., Butler, R. C., Curto, A., et al. 2011, *A&A*, 536, A3 [19](#)
- Moreno, R. 2010, Neptune and Uranus planetary brightness temperature tabulation, available from <ftp://ftp.sciops.esa.int/pub/hsc-calibration/PlanetaryModels/ESA244>
- Narayanan, G., Heyer, M. H., Brunt, C., et al. 2008, *The Astrophysical Journal Supplement Series*, 177, 341 [194](#)
- Notari, A. & Quartin, M. 2015, *Journal of Cosmology and Astroparticle Physics*, 2015, 047 [101](#)
- Pagano, L., Delouis, J.-M., Mottet, S., Puget, J.-L., & Vibert, L. 2020, *Astronomy & Astrophysics*, 635, A99 [5](#), [53](#)

- Pajot, F., Ade, P. A. R., Beney, J., et al. 2010, *A&A*, 520, A10 21, 23, 24
- Peebles, P. J. E. & Wilkinson, D. T. 1968, *Physical Review*, 174, 2168 42
- Peebles, P. J. E. & Yu, J. T. 1970, *The Astrophysical Journal*, 162, 815 5
- Penzias, A. A. & Wilson, R. W. 1965, *The Astrophysical Journal*, 142, 419 1
- Planck HFI Core Team. 2011a, *A&A*, 536, A4 23, 63
- Planck HFI Core Team. 2011b, *A&A*, 536, A6 23, 27, 36, 70
- Planck Collaboration II. 2011, *A&A*, 536, A2 24
- Planck Collaboration XX. 2011, *A&A*, 536, A20 100
- Planck Collaboration I. 2014, *A&A*, 571, A1 30, 38, 116
- Planck Collaboration VI. 2014, *A&A*, 571, A6 21, 23, 32, 35, 36, 37, 38, 70
- Planck Collaboration VII. 2014, *A&A*, 571, A7 27, 28, 29, 32, 37, 39, 41, 42, 78
- Planck Collaboration VIII. 2014, *A&A*, 571, A8 30, 33, 44, 58, 59
- Planck Collaboration IX. 2014, *A&A*, 571, A9 24, 26, 32, 43, 44, 61, 143
- Planck Collaboration X. 2014, *A&A*, 571, A10 35, 63
- Planck Collaboration XI. 2014, *A&A*, 571, A11 167, 260
- Planck Collaboration XII. 2014, *A&A*, 571, A12 49, 50
- Planck Collaboration XIII. 2014, *A&A*, 571, A13 24, 45, 46, 47, 48, 49, 50, 51, 61, 101, 111, 116, 133, 136, 143, 144, 145, 149, 156, 159, 173, 196
- Planck Collaboration XIV. 2014, *A&A*, 571, A14 38
- Planck Collaboration XVII. 2014, *A&A*, 571, A17 5
- Planck Collaboration XIX. 2014, *A&A*, 571, A19 5
- Planck Collaboration XX. 2014, *A&A*, 571, A20 5
- Planck Collaboration XXX. 2014, *A&A*, 571, A30 12
- Planck Collaboration VII. 2016, *A&A*, 594, A7 23, 31, 33, 35, 37, 58, 71, 78, 79, 117
- Planck Collaboration VIII. 2016, *A&A*, 594, A8 42, 43, 44, 58, 70, 167
- Planck Collaboration X. 2016, *A&A*, 594, A10 9, 10, 11, 46, 50, 73, 76, 77, 100, 108, 216, 218
- Planck Collaboration XII. 2016, *A&A*, 594, A12 98, 100
- Planck Collaboration XXI. 2016, *A&A*, 594, A21 7
- Planck Collaboration XXII. 2016, *A&A*, 594, A22 6
- Planck Collaboration I. 2020, *A&A*, 641, A1 4, 5, 6, 8, 9, 10

- Planck Collaboration III. 2020, *A&A*, 641, A3 23, 27, 31, 34, 43, 53, 58, 63, 65, 66, 67, 71, 72, 78, 89, 95, 98, 99, 100, 105, 108, 109, 114, 117, 123
- Planck Collaboration IV. 2020, *A&A*, 641, A4 8, 44, 45, 162, 183
- Planck Collaboration XII. 2020, *A&A*, 641, A12 10
- Planck Collaboration Int. XIV. 2014, *A&A*, 564, A45 44
- Planck Collaboration Int. XV. 2014, *A&A*, 565, A103 10
- Planck Collaboration Int. XLVI. 2016, *A&A*, 596, A107 53, 63, 65, 66, 69, 72, 78, 89
- Planck Collaboration Int. XLVIII. 2016, *A&A*, 596, A109 12
- Puglisi, G., Fabbian, G., & Baccigalupi, C. 2017, *Monthly Notices of the Royal Astronomical Society*, 469, 2982 16, 245
- Remazeilles, M., Delabrouille, J., & Cardoso, J.-F. 2011, *Monthly Notices of the Royal Astronomical Society*, 418, 467 100
- Richard, T., Lesaffre, P., Falgarone, E., & Lehmann, A. 2022, *Astronomy and Astrophysics*, 664 [arXiv:2206.03045] 16
- Richards, P. L. 1994, *Journal of Applied Physics*, 76, 1 20, 21
- Ridge, N. A., Di Francesco, J., Kirk, H., et al. 2006, *The Astronomical Journal*, 131, 2921 191, 202, 206, 224, 226
- Rosset, C., Tristram, M., Ponthieu, N., et al. 2010, *A&A*, 520, A13 21, 22, 23, 57, 72
- Rybicki, G. B. & Lightman, A. P. 1979, in *Radiative processes in astrophysics* 11, 73
- Sachs, R. K. & Wolfe, A. M. 1967, *The Astrophysical Journal*, 147, 73 5
- Stephán, G. 2016, *Modélisation de la chimie dans les régions de formation d'étoiles massives avec des PDRs internes*, PhD thesis, thèse de doctorat dirigée par Le Boulot, Jacques et Schilke, Peter Astronomie et Astrophysique Paris Sciences et Lettres (ComUE) 2016 14
- Stokes, G. G. 1851, *Transactions of the Cambridge Philosophical Society*, 9, 399 277
- Sugai, H., Ade, P. A., Akiba, Y., et al. 2020, *Journal of Low Temperature Physics*, 199, 1107 7
- Sunyaev, R. A. & Zeldovich, Y. B. 1970, *Astrophysics and Space Science*, 7, 3 5
- Sunyaev, R. A. & Zeldovich, Y. B. 1972, *Comments on Astrophysics and Space Physics*, 4, 173 5
- T., S. P. 1881, *Nature*, 25, 14 19
- Tauber, J. A., Mandolesi, N., Puget, J., et al. 2010, *A&A*, 520, A1 30, 31, 38
- van der Tak, F. F. S., Black, J. H., Schöier, F. L., Jansen, D. J., & van Dishoeck, E. F. 2007, *Astronomy and Astrophysics*, 468, 627 152
- Vansyngel, F., Boulanger, F., Ghosh, T., et al. 2017, *Astronomy and Astrophysics*, 603 100

- Wilson, R. W., Jefferts, K. B., & Penzias, A. A. 1970, [The Astrophysical Journal](#), 161, L43
[12](#)
- Wolfire, M. G., Hollenbach, D., & McKee, C. F. 2010, [Astrophysical Journal](#), 716, 1191 [13](#)

RÉSUMÉ

Le satellite *Planck* a cartographié l'ensemble du ciel sur neuf bandes de fréquences dans les domaines millimétriques et submillimétriques. Son objectif principal était de caractériser les très petites anisotropies de température du fond diffus cosmologique (CMB), une émission de corps noir presque parfaite produite à un redshift $z \sim 1100$. Ses anisotropies sont considérées comme les graines à partir desquelles les structures à grande échelle de l'Univers ont évolué.

Cependant, ce signal primordial est masqué sur de grandes parties du ciel par les émissions d'avant-plan provenant de notre propre galaxie (émissions du continuum tel que l'émission thermique des poussières, le rayonnement synchrotron, et le rayonnement free-free, plus les raies rotationnelles du CO). Pour caractériser correctement le signal cosmologique, il faut donc séparer avec précision les composantes de l'émission.

Dans le pipeline d'analyse des données de *Planck*, cette séparation des composantes a été calculée à partir de cartes du ciel, elles-mêmes produites en projetant les données ordonnées dans le temps collectées par les 11 récepteurs radiométriques de l'instrument basse fréquence (LFI) et les 52 bolomètres de l'instrument haute fréquence (HFI) à bord du satellite. L'algorithme utilisé pour produire ces cartes ("mapmaking") à partir des données HFI, après nettoyage des systématiques instrumentales restantes, est appelé SRoll. Cet algorithme exploite la redondance des observations pour ajuster les systématiques afin de supprimer leur effet sur les données et de calibrer le signal. Une nouvelle version améliorée de cet algorithme, appelée SRoll2, a été développée et publiée par la suite.

Le travail de cette thèse vise à construire de nouvelles cartes plein-ciel des raies d'émission rotationnelles du CO ($J = 1 \rightarrow 0$ and $J = 2 \rightarrow 1$) à partir des données *Planck*, en exploitant les différences de bande passante ("mismatch") entre les différents détecteurs fonctionnant à la même fréquence. Ceci est réalisé en développant une version alternative de l'algorithme SRoll2, appelée SRoll2.1, optimisée pour la détermination des différences de réponse des bolomètres à l'émission de CO. En utilisant les produits de SRoll2.1 (cartes du ciel nettoyées et divers paramètres ajustés à partir de son modèle de données), je présente une méthode pour extraire l'émission de CO du reste du signal, tout en séparant les isotopologues ^{12}CO et ^{13}CO .

Après avoir caractérisé la méthode à l'aide de simulations, et vérifié les différentes sources d'erreurs et de biais potentiels, je qualifie les cartes en les comparant à l'état de l'art en termes de relevé radioastronomique de CO, ainsi qu'aux précédentes cartes de CO provenant des derniers résultats de *Planck*. Après avoir confirmé la qualité de ces nouvelles cartes, je les utilise pour explorer l'émission de CO aux hautes latitudes galactiques et aux faibles densités de colonne. En effet, la force de ces cartes réside dans leur couverture complète du ciel, permettant l'utilisation d'outils statistiques pour rassembler toutes les informations disponibles sur le ciel. Ceci conduit à une détection statistique robuste de l'émission de CO pour les multipoles $240 \lesssim \ell \lesssim 560$ ($44'.0 \gtrsim$ échelle ang. $\gtrsim 18'.8$) aux hautes latitudes correspondant à des densités de colonne de gaz inférieures à $5.5 \times 10^{20} \text{ cm}^{-2}$.

MOTS CLÉS

CO, Avant-plans, Données *Planck*.

ABSTRACT

The *Planck* satellite has mapped the entire sky over nine frequency bands in the millimetre and submillimetre domains. Its primary objective was to characterize the very small temperature anisotropies of the Cosmic Microwave Background (CMB), a nearly-perfect blackbody emission produced at a redshift $z \sim 1100$. Its anisotropies are thought to be the seeds from which the large-scale structures of the Universe have evolved.

However, this primordial signal is masked over large parts of the sky by the emission from foregrounds originating in our own Galaxy (thermal dust, synchrotron, and free-free continua, plus CO rotational lines). To properly characterize the cosmological signal thus requires an accurate separation of the components of the emission.

In the *Planck* data analysis pipeline, this component separation was computed from sky maps, themselves produced by projecting the time-ordered data collected by the 11 radiometric receivers of the Low Frequency Instrument (LFI) and the 52 bolometers from the High Frequency Instrument (HFI) onboard the satellite. The algorithm made to produce these maps ("mapmaking") from the HFI data, after cleaning the remaining instrumental systematics affecting it, is called SRoll. This algorithm exploits the redundancy of the observations to fit the systematics in order to remove their effect from the data and to calibrate the signal. A new and improved version of this algorithm, called SRoll2, was later developed and published.

The work of this PhD aims to build new fullsky maps of the CO rotational line emission ($J = 1 \rightarrow 0$ and $J = 2 \rightarrow 1$) from the *Planck* data, by exploiting the bandpass differences ("mismatch") between different detectors operating at the same frequency. This is achieved by developing an alternate version of the SRoll2 algorithm, called SRoll2.1, optimized for the determination of the bolometers response differences to the CO emission. Using the products of SRoll2.1 (clean sky maps and various parameters fitted from its data model) I present a method to extract the CO emission from the rest of the signal, while also separating the ^{12}CO and ^{13}CO isotopologues.

After characterizing the method using simulations, and checking for various sources of potential errors and biases, I qualify the maps by comparing them to the state of the art in terms of CO radio-surveys, as well as to the previous CO maps from the *Planck* legacy release. After confirming the quality of these newly constructed maps, I use them to explore the CO emission at high Galactic latitudes and low column densities. Indeed, the strength of these maps is their complete sky coverage, allowing the use of statistical tools to gather all the available information over the sky. This leads to a robust statistical detection of CO emission for multipoles $240 \lesssim \ell \lesssim 560$ ($44'.0 \gtrsim$ ang. scale $\gtrsim 18'.8$) at high latitudes corresponding to gas column densities below $5.5 \times 10^{20} \text{ cm}^{-2}$.

KEYWORDS

CO, Foregrounds, *Planck* data.